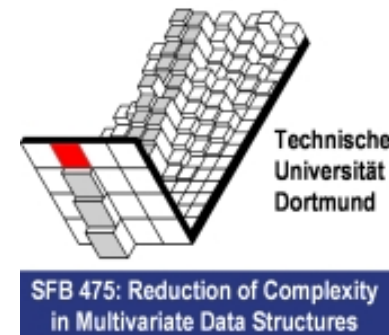


# robfilter: An R-Package for Robust Time Series Filters

Karen Schettlinger, Roland Fried, Ursula Gather

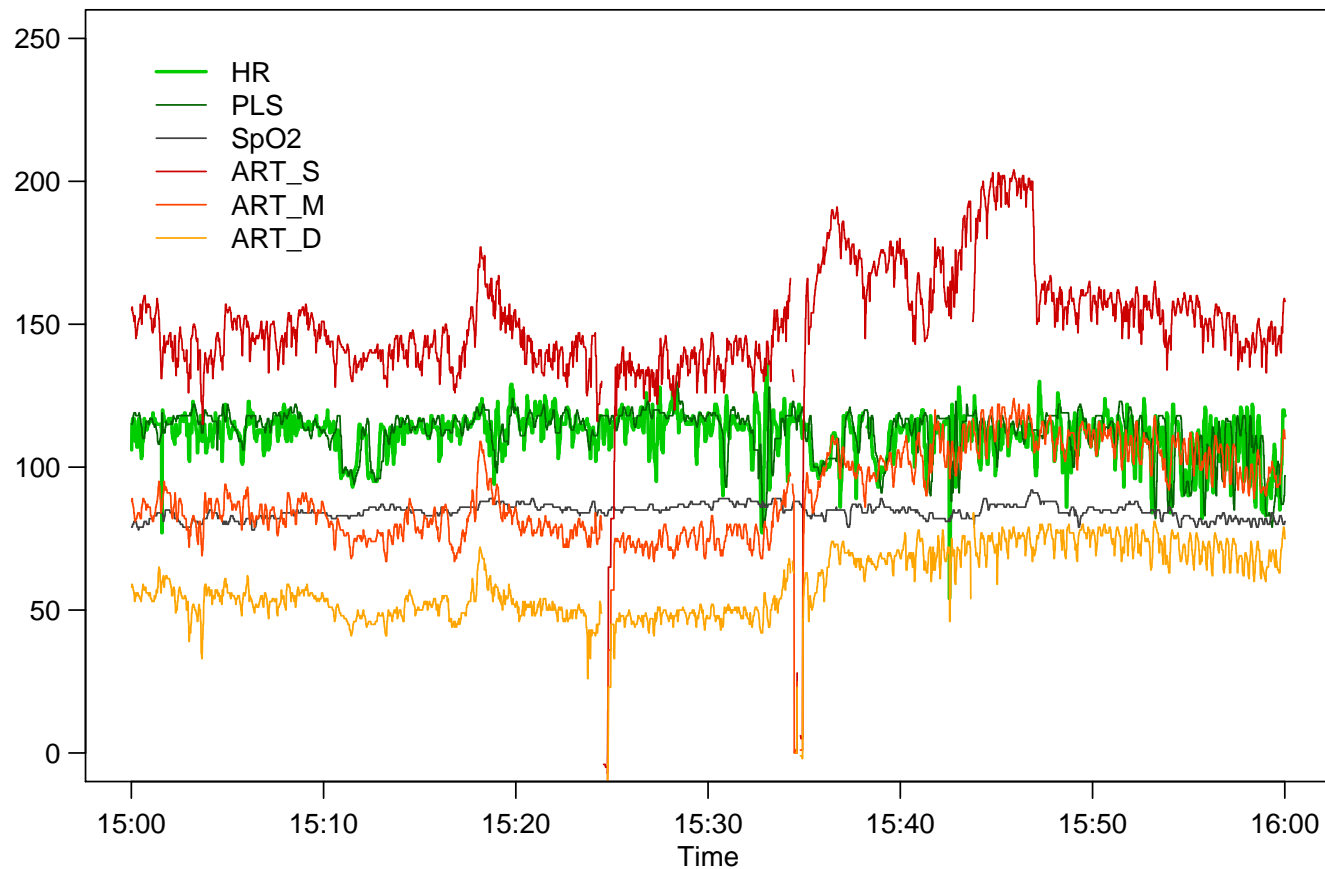
**tu** technische universität  
dortmund

Fakultät Statistik



# Motivation

## Multivariate physiological time series



➔ Filter signals to improve ICU monitoring systems

# Contents of `robfilter` Version 2.0

## 7 time series filters

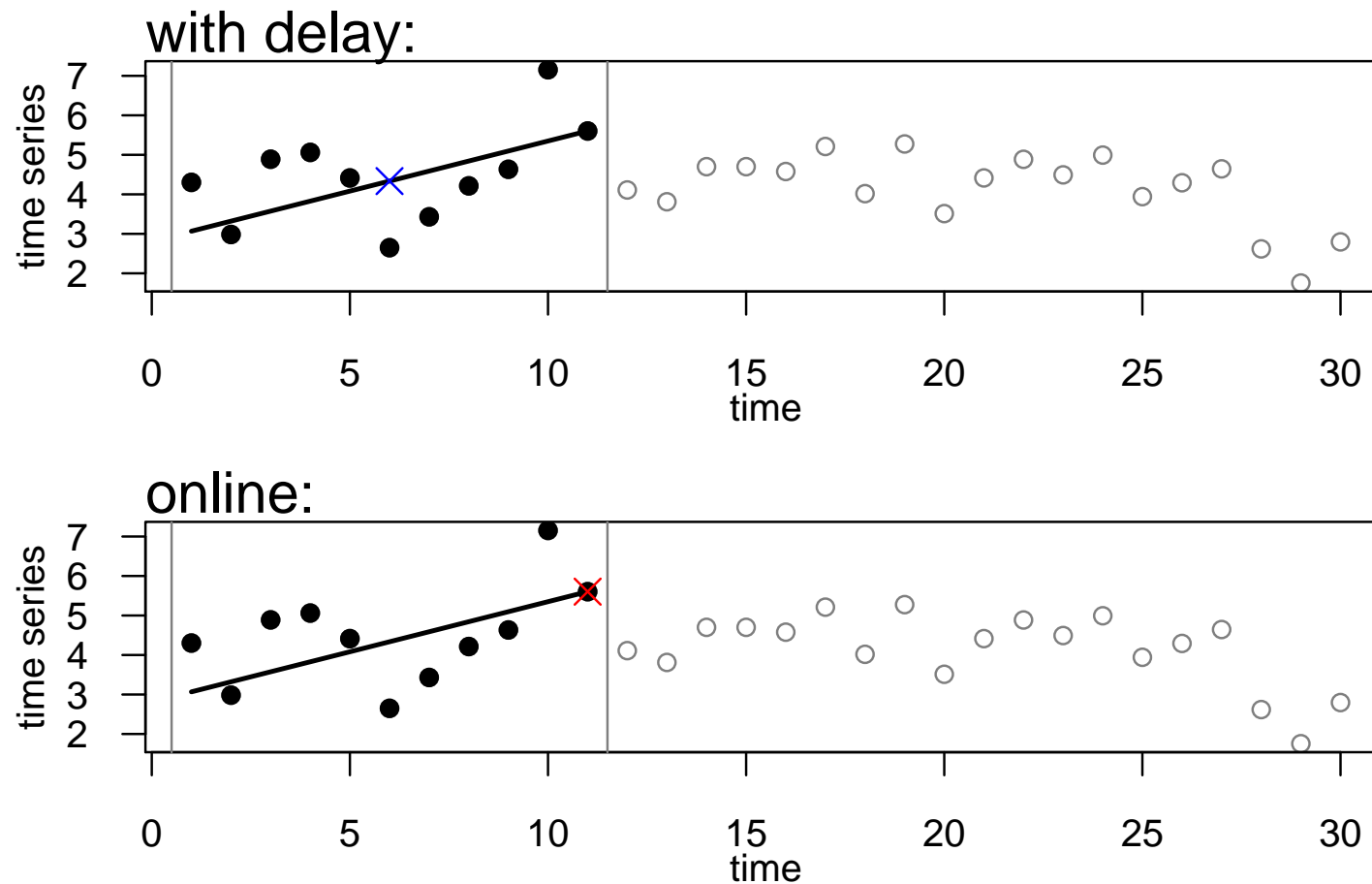
<code>robreg.filter</code>	Simple regression filters
<code>hybrid.filter</code>	Median and repeated median hybrid filters
<code>dw.filter</code>	Two-step location-/regression-based filters
<code>wrm.filter</code>	Weighted repeated median filters
<code>robust.filter</code>	Regression filters with additional rules (outlier & level shift detection)
<code>adore.filter</code>	Adaptive repeated median filters
<code>madore.filter</code>	Multivariate adaptive repeated median filters

## 1 univariate smoothing method

<code>wrm.smooth</code>	Weighted repeated median smoothing
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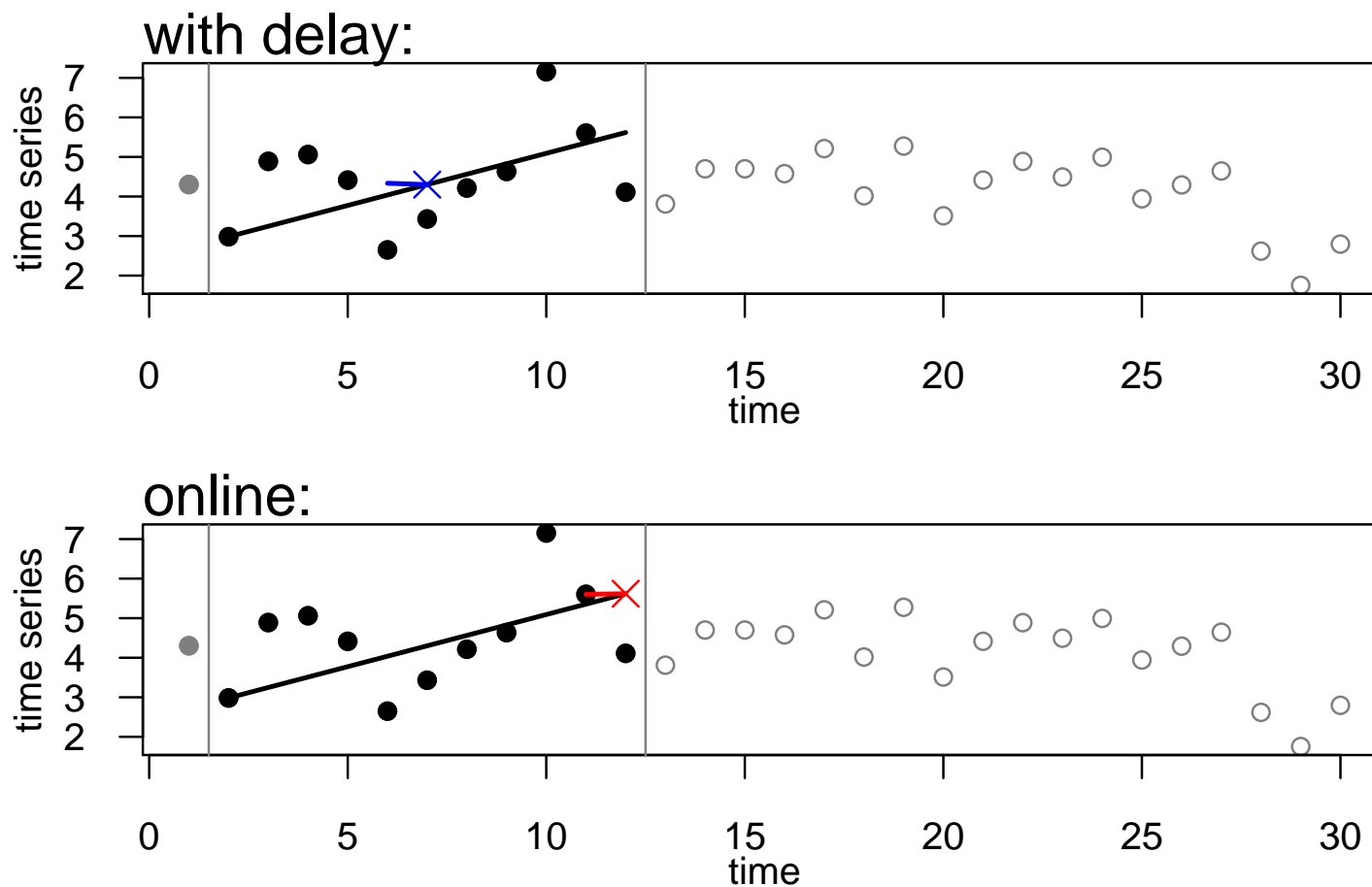
# robreg.filter – Illustration

**Robust regression** in a moving time window:



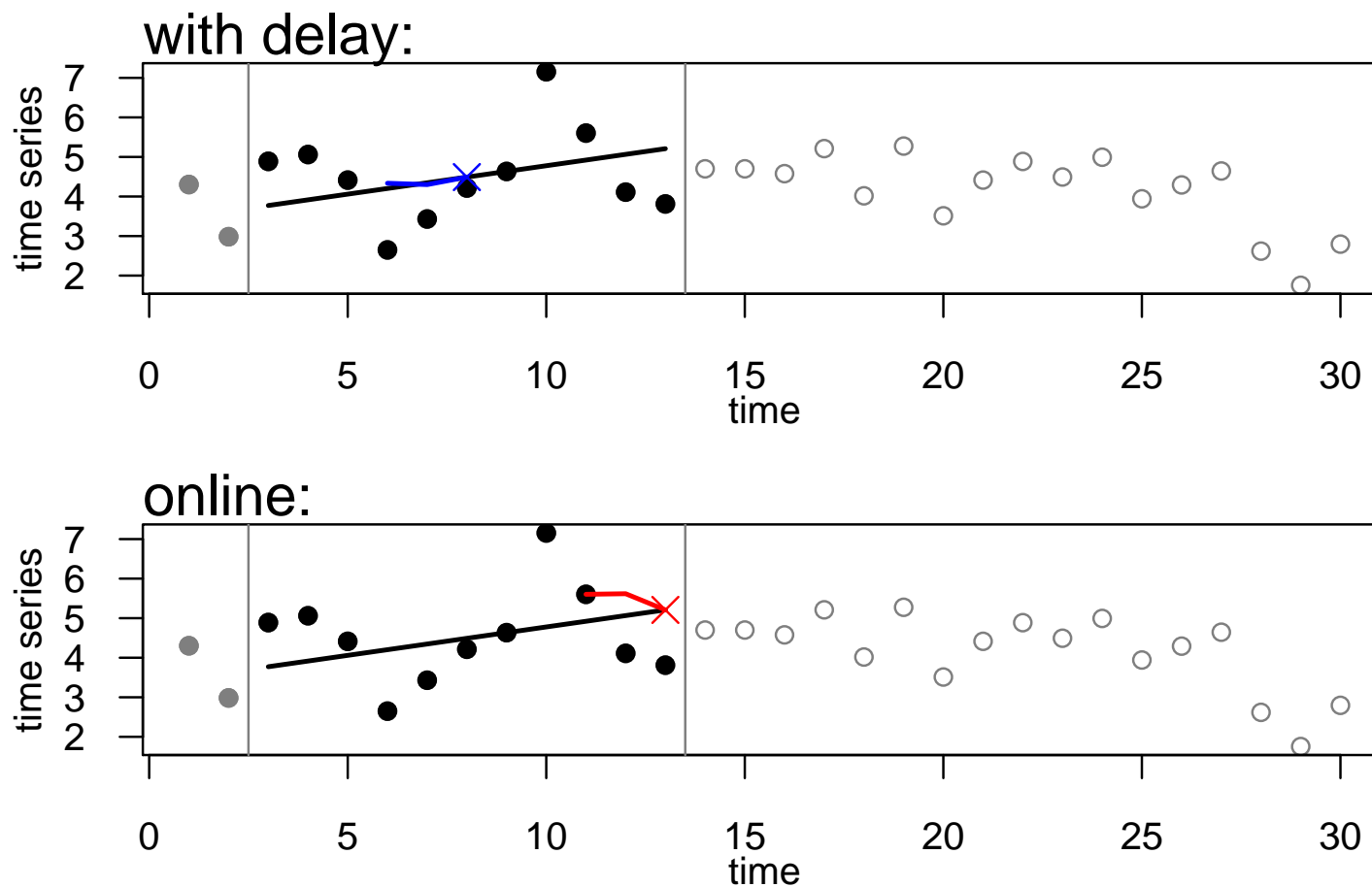
# robreg.filter – Illustration

**Robust regression** in a moving time window:



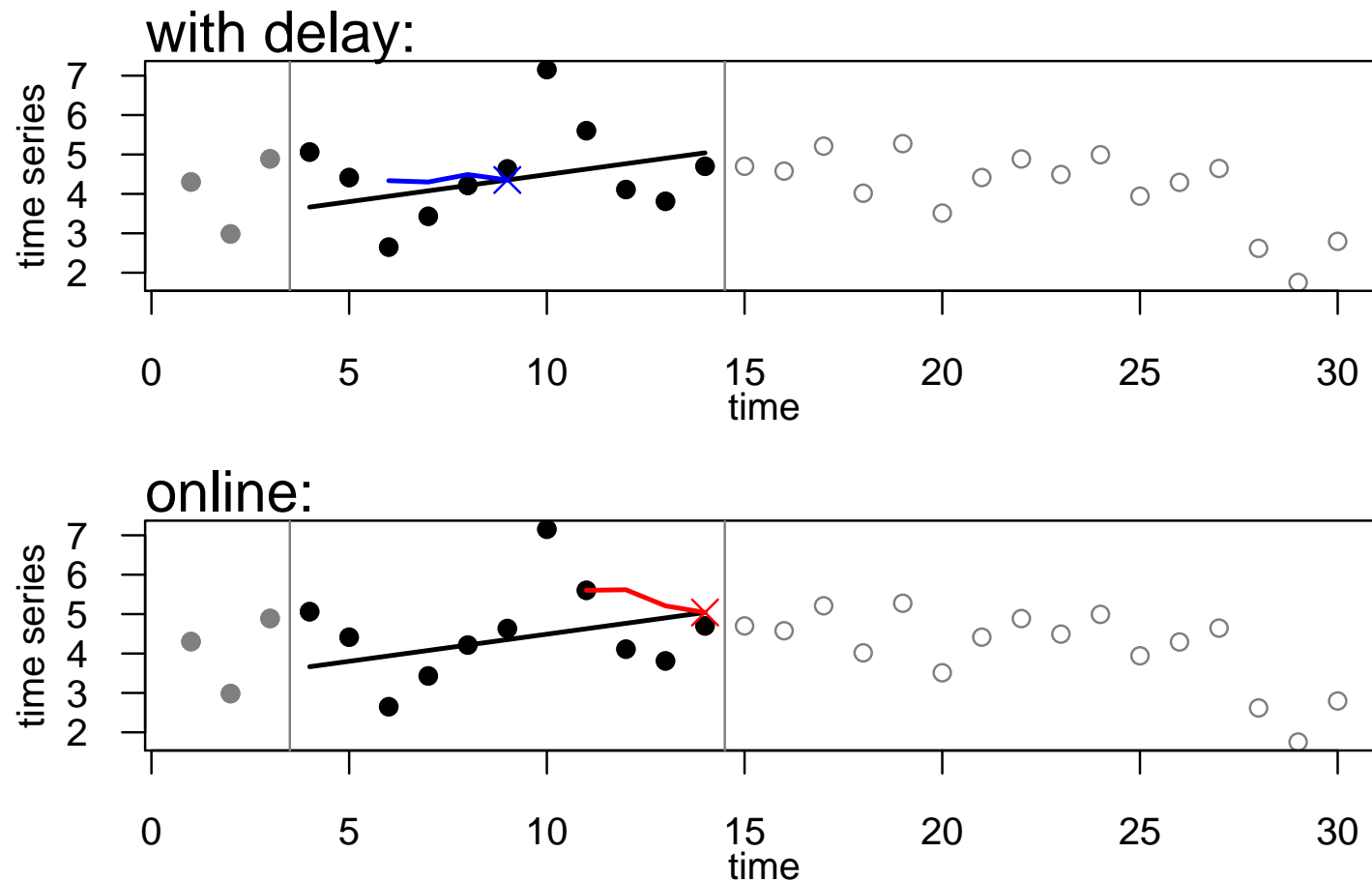
# robreg.filter – Illustration

**Robust regression** in a moving time window:



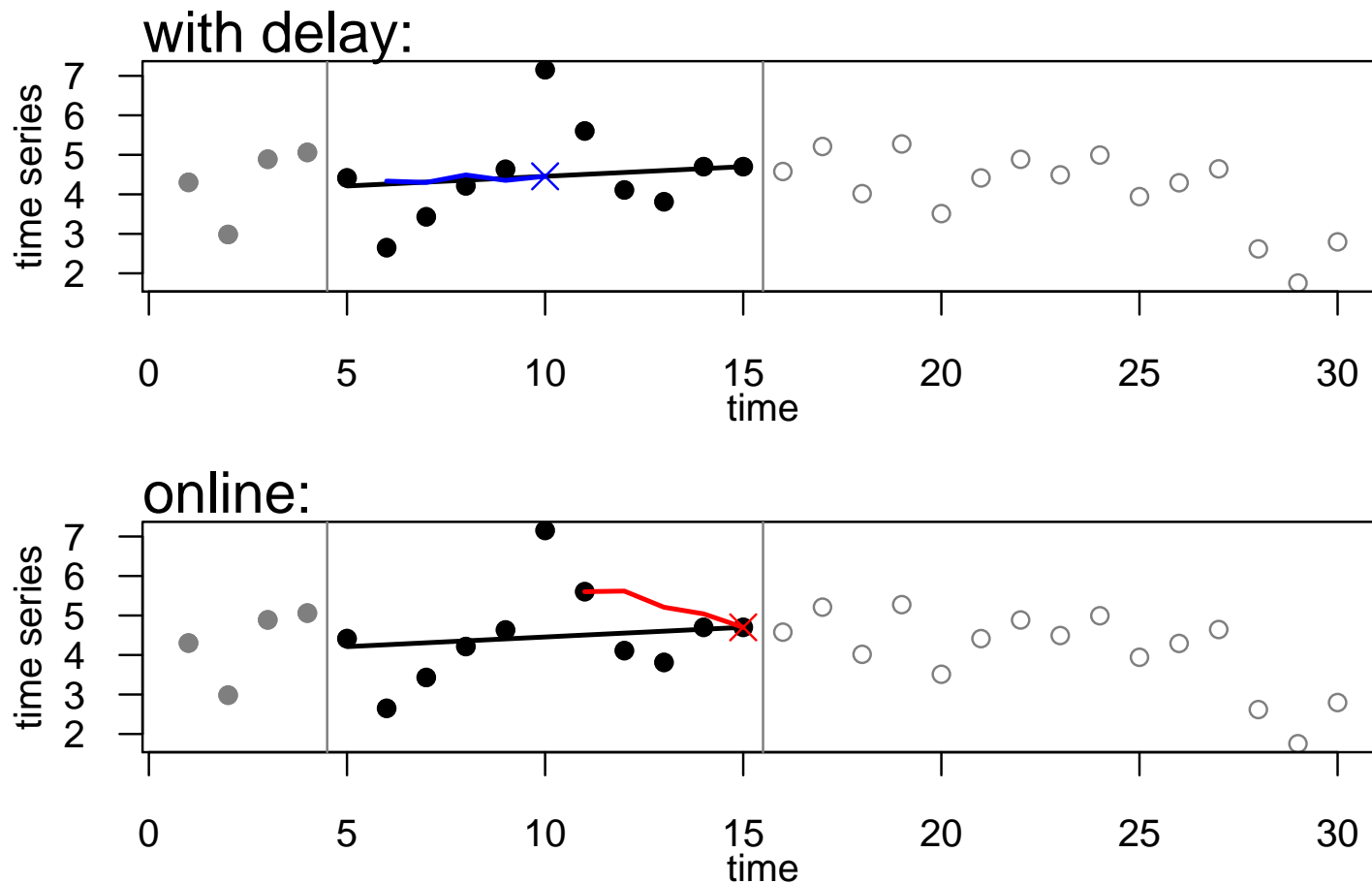
# robreg.filter – Illustration

**Robust regression** in a moving time window:



# robreg.filter – Illustration

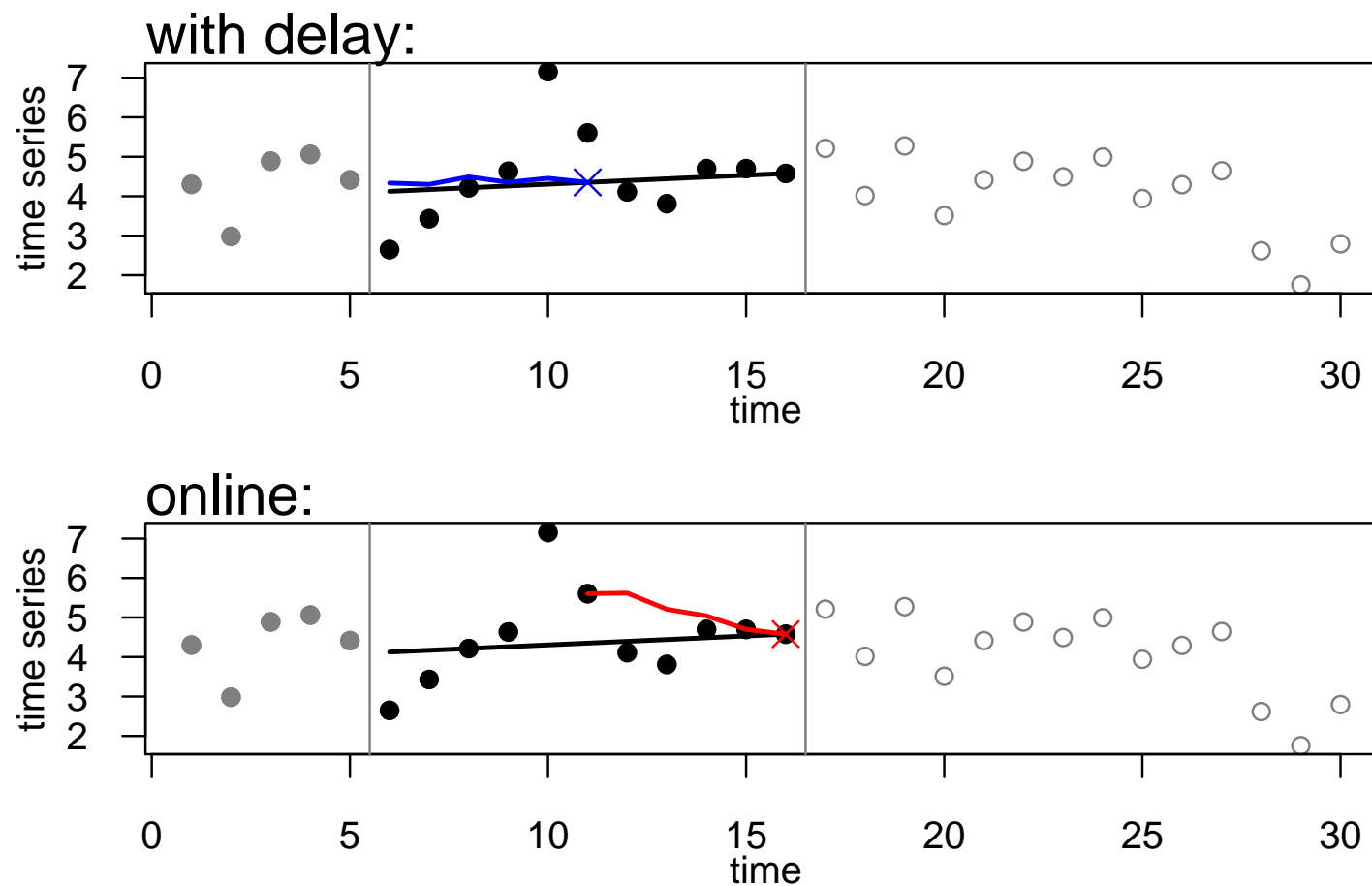
**Robust regression** in a moving time window:





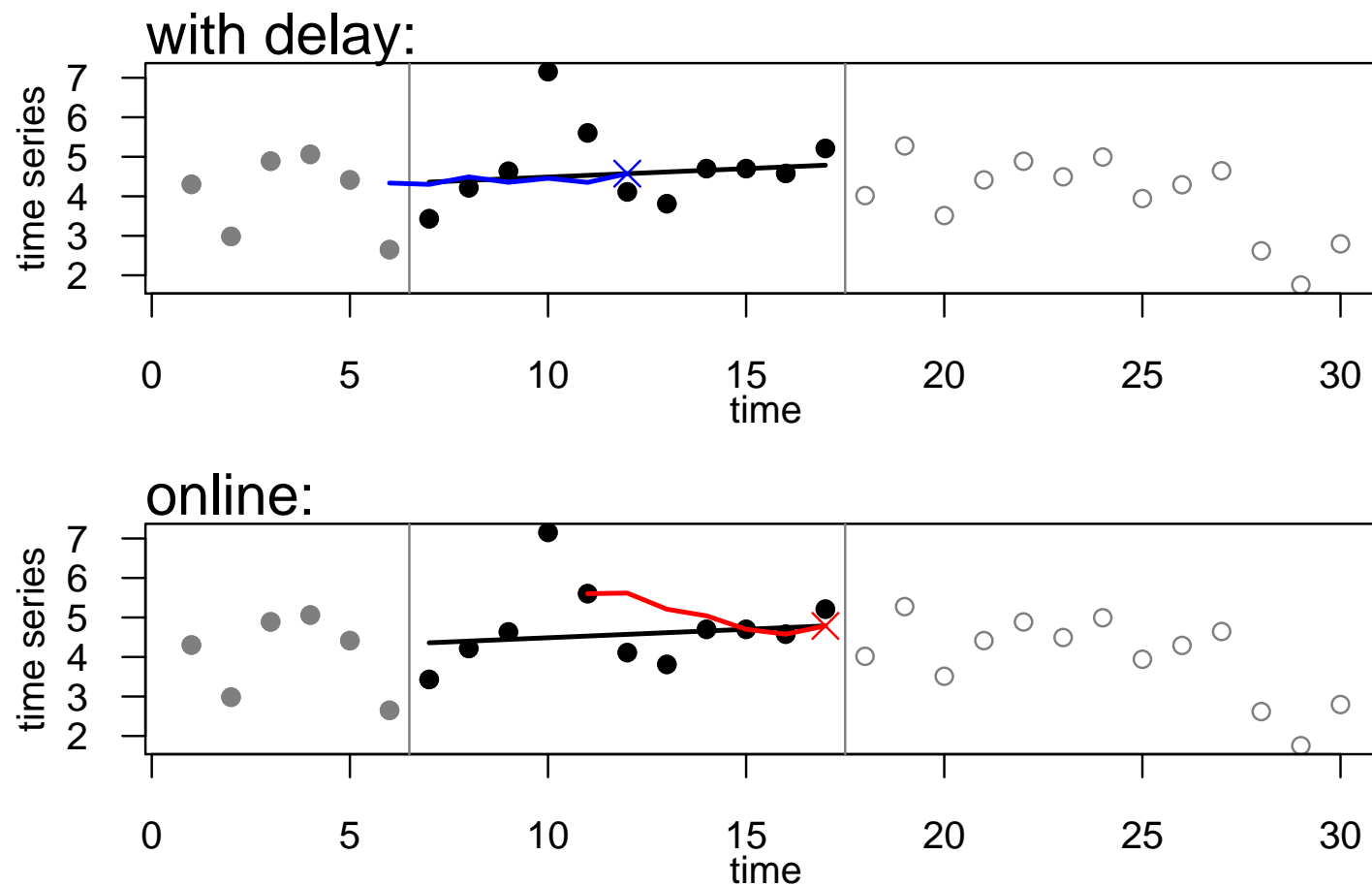
# robreg.filter – Illustration

**Robust regression** in a moving time window:



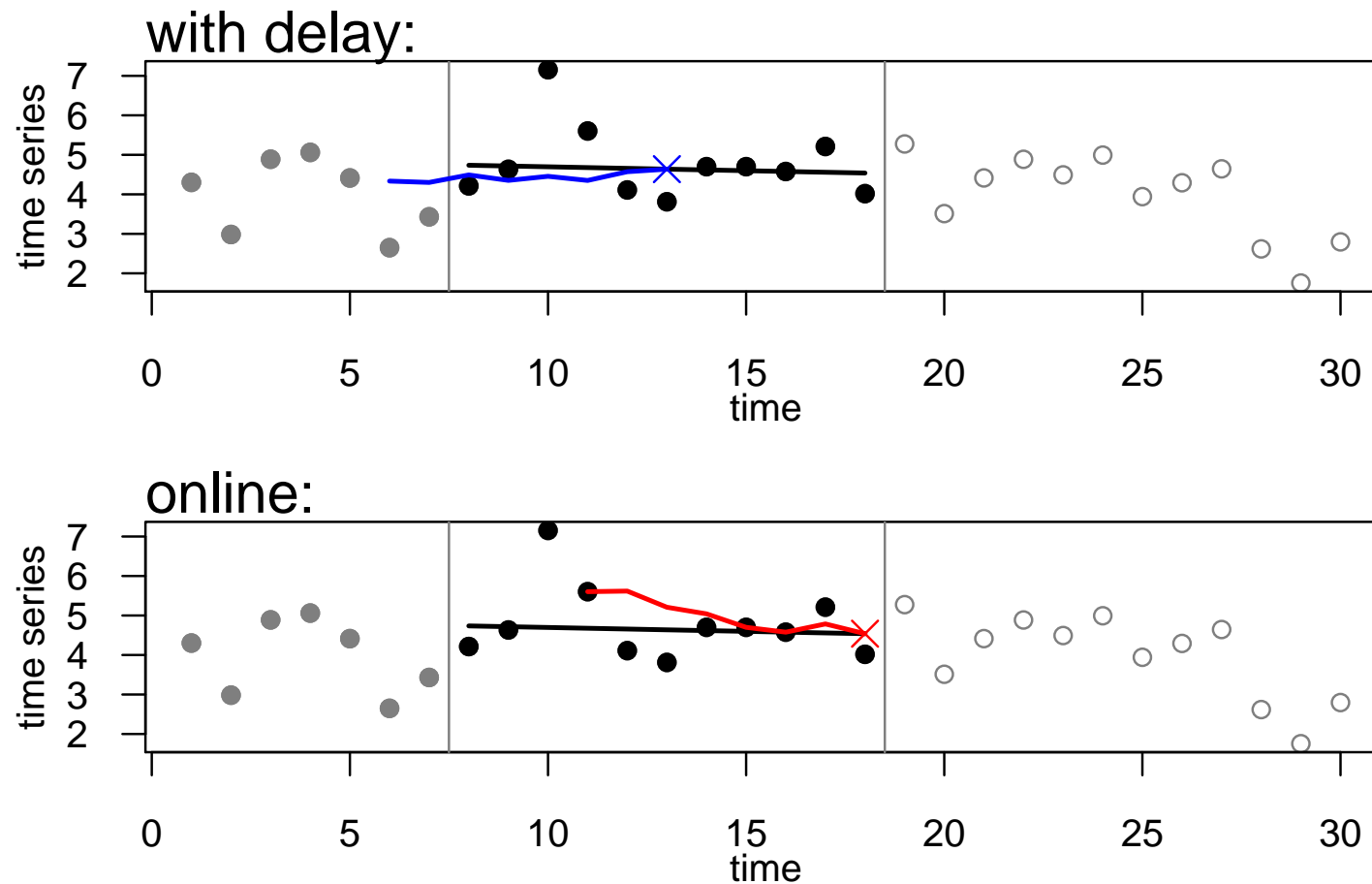
# robreg.filter – Illustration

**Robust regression** in a moving time window:



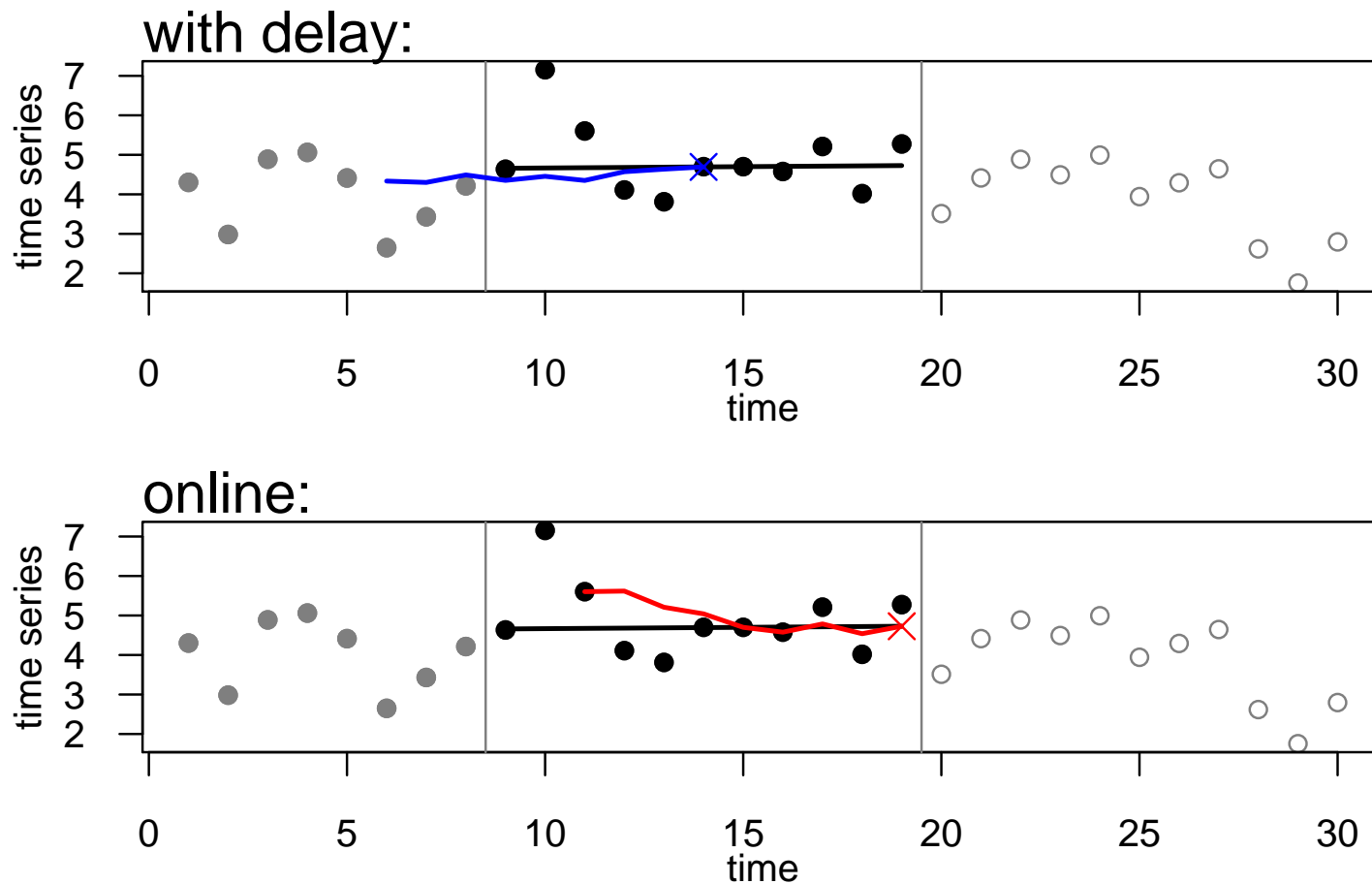
# robreg.filter – Illustration

**Robust regression** in a moving time window:



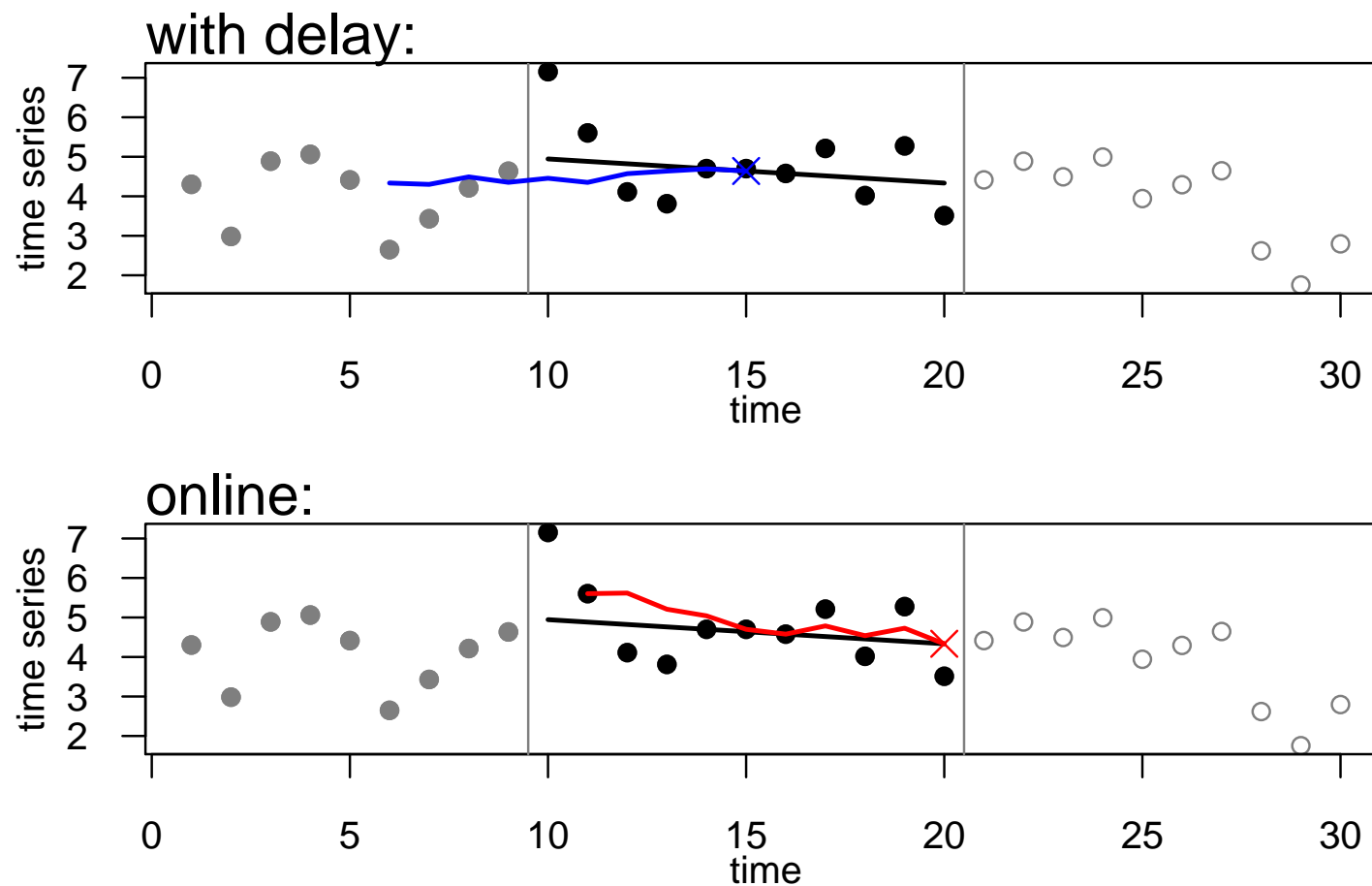
# robreg.filter – Illustration

**Robust regression** in a moving time window:



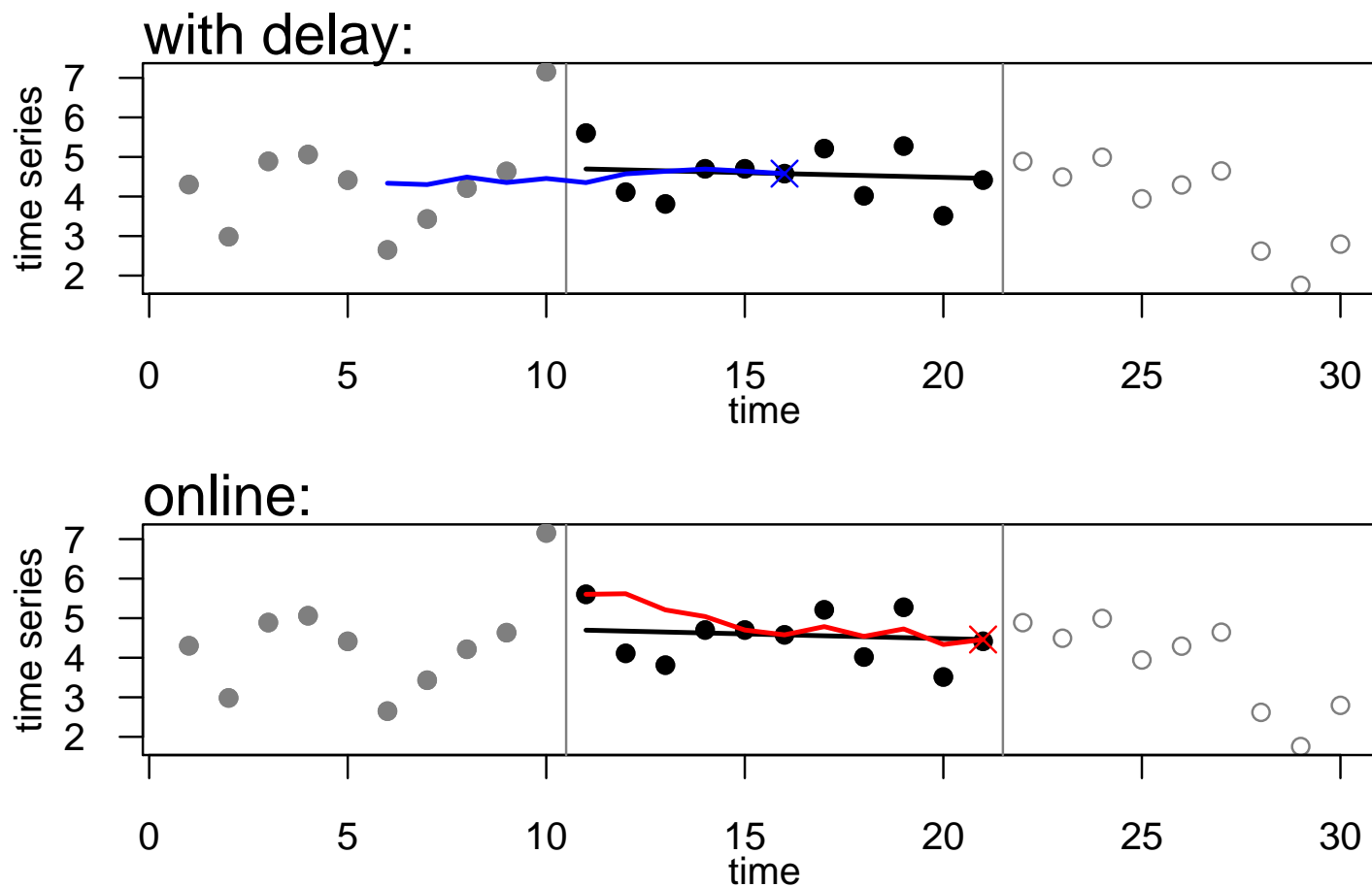
# robreg.filter – Illustration

**Robust regression** in a moving time window:



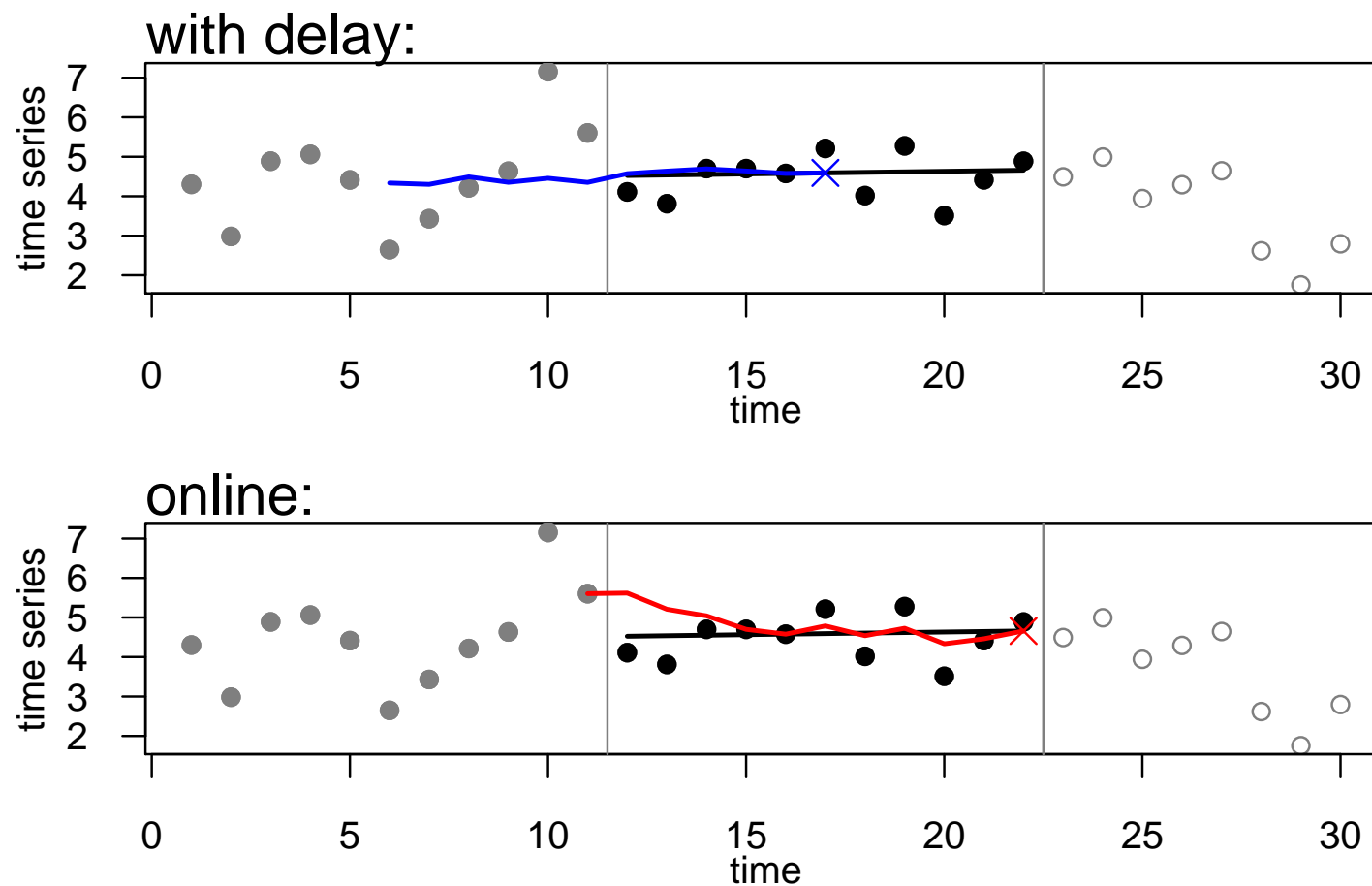
# robreg.filter – Illustration

**Robust regression** in a moving time window:



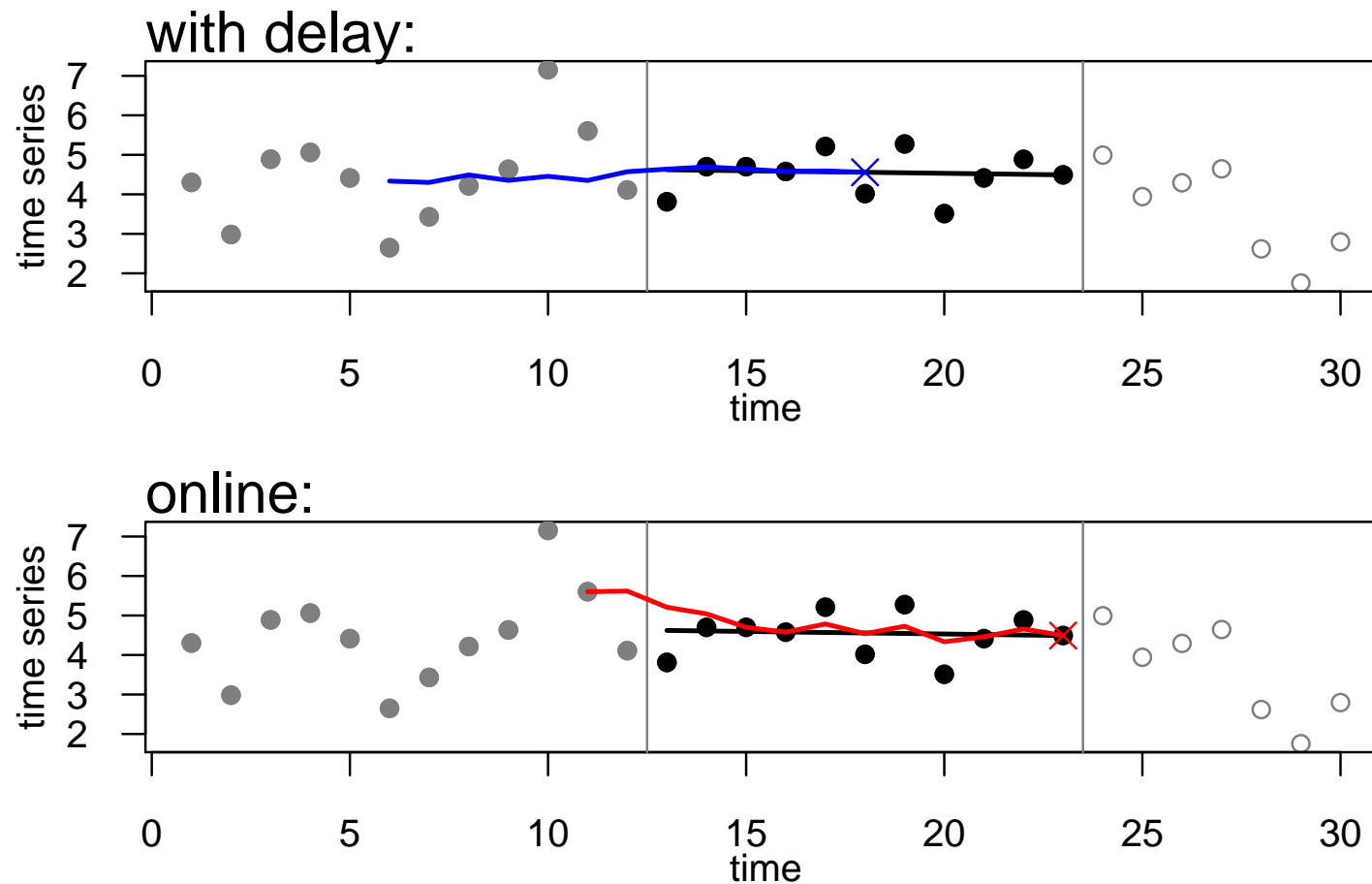
# robreg.filter – Illustration

**Robust regression** in a moving time window:



# robreg.filter – Illustration

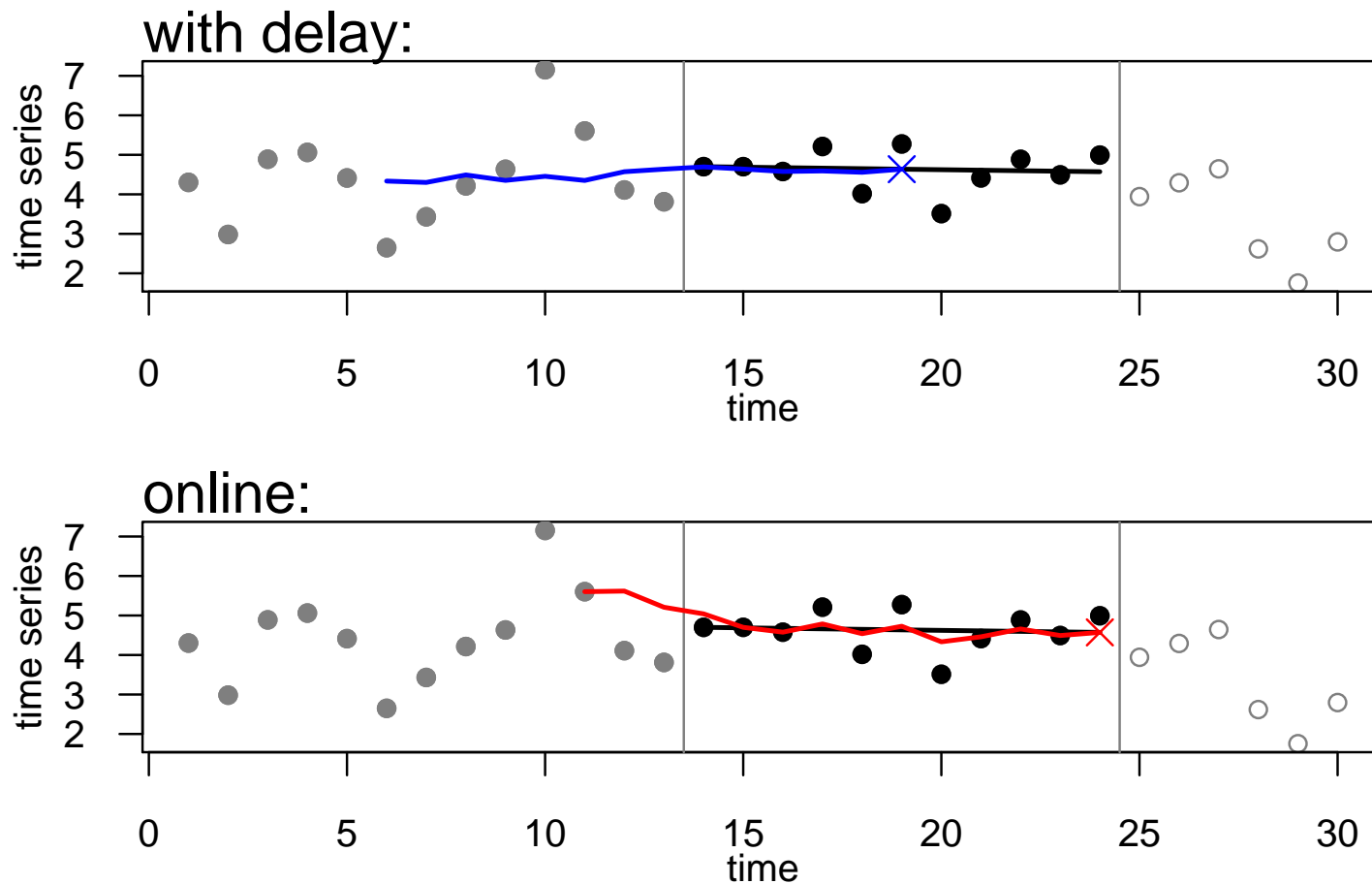
**Robust regression** in a moving time window:





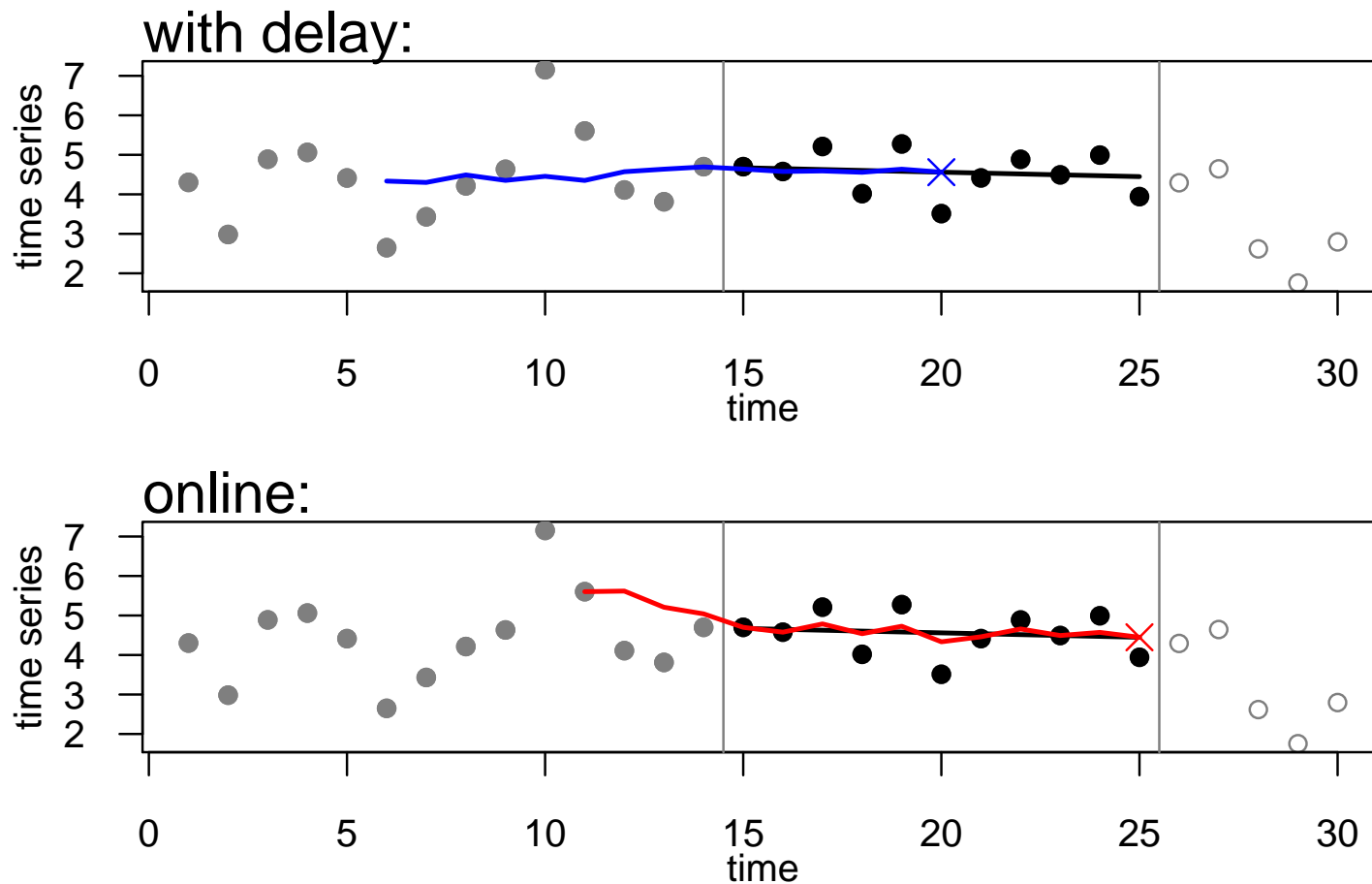
# robreg.filter – Illustration

Robust regression in a moving time window:



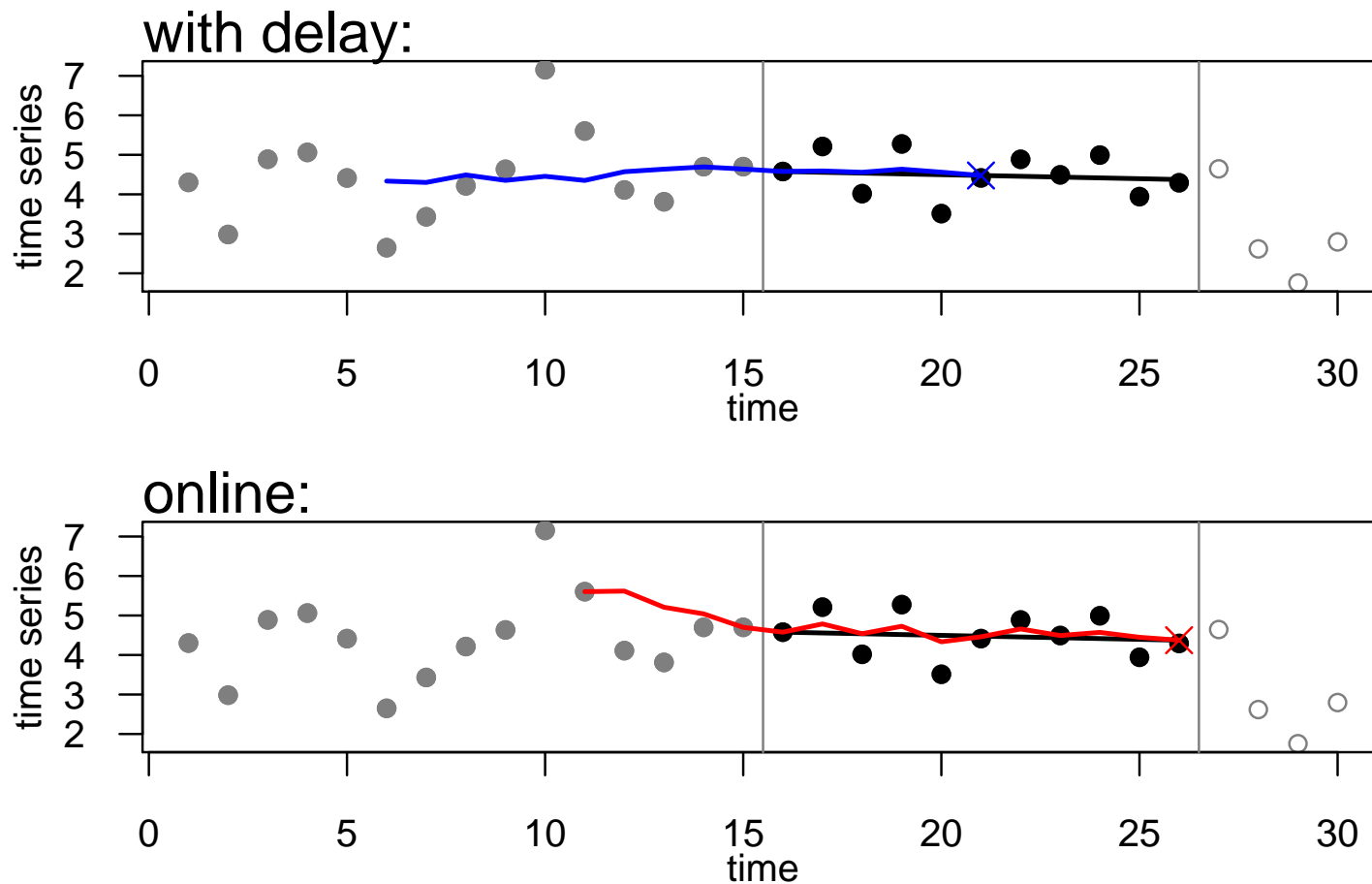
# robreg.filter – Illustration

**Robust regression** in a moving time window:



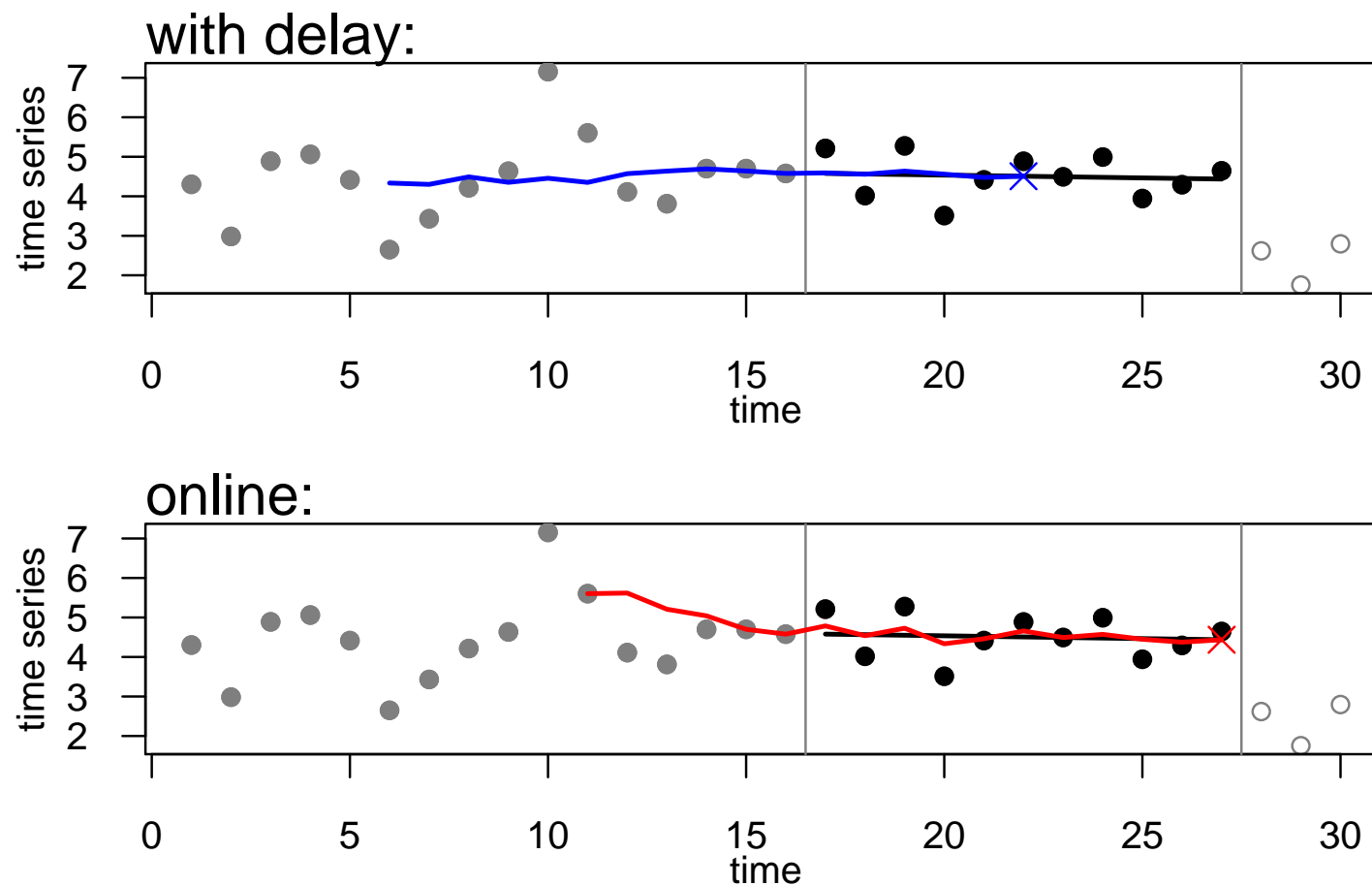
# robreg.filter – Illustration

Robust regression in a moving time window:



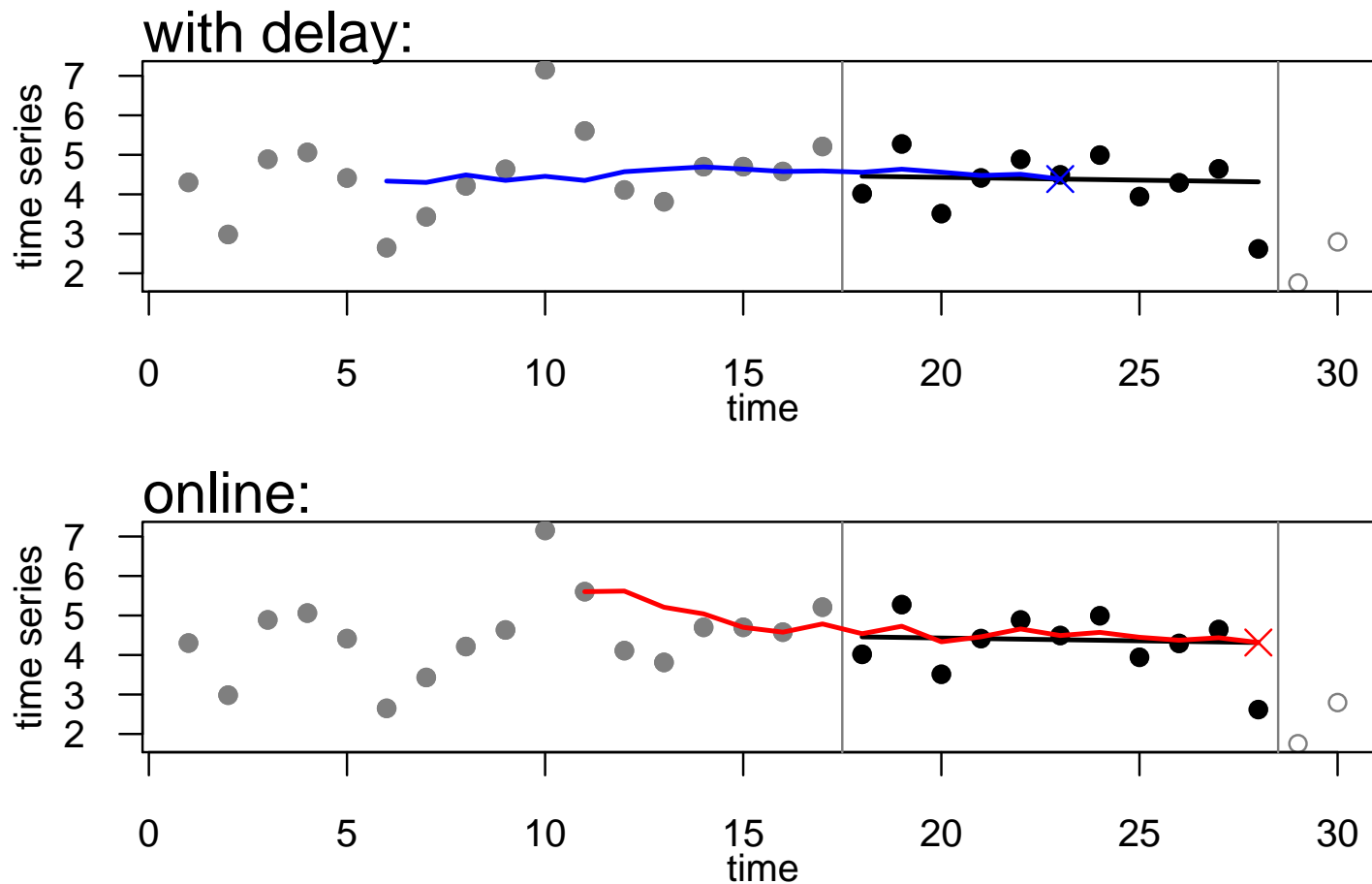
# robreg.filter – Illustration

Robust regression in a moving time window:



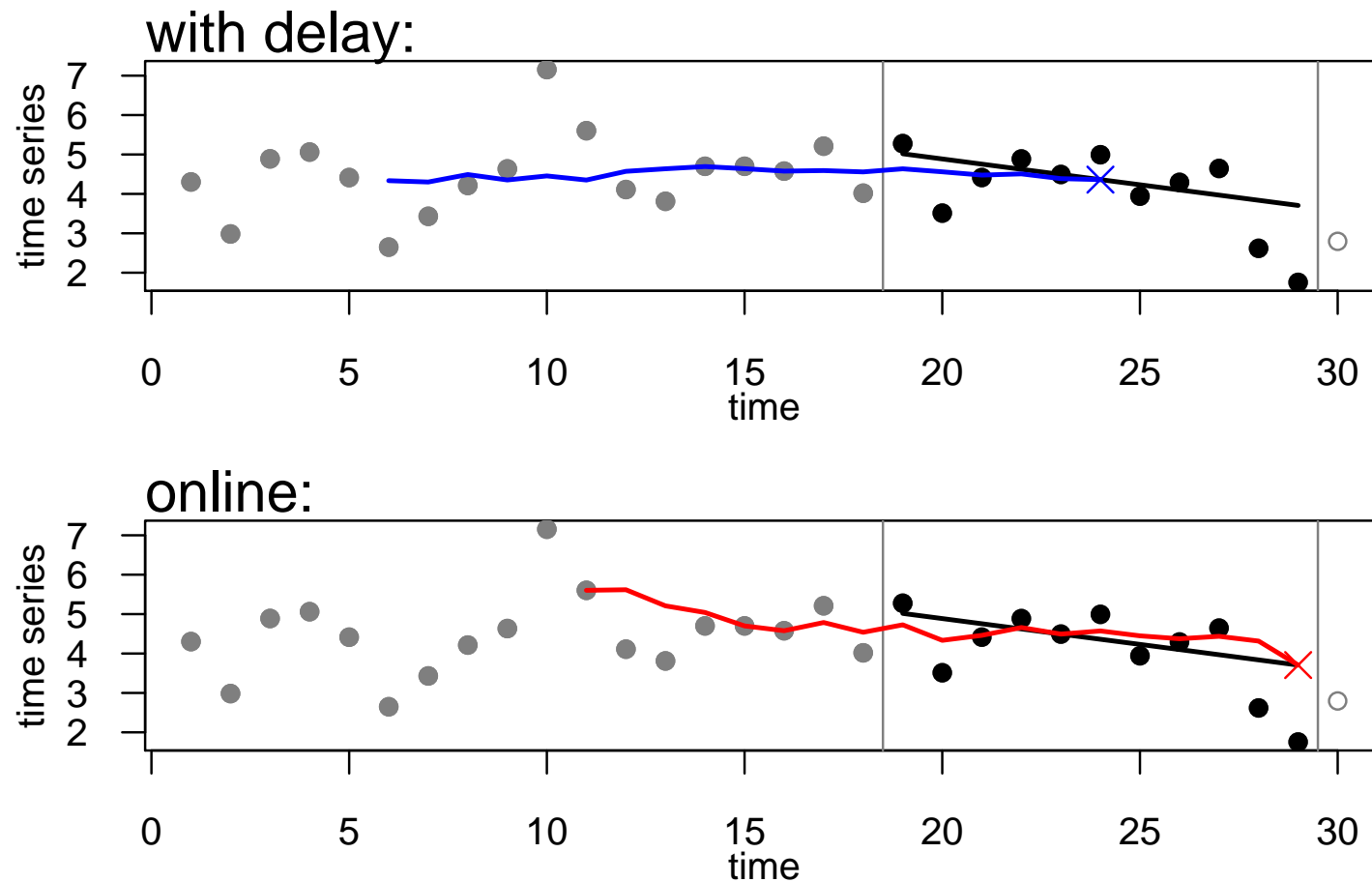
# robreg.filter – Illustration

Robust regression in a moving time window:



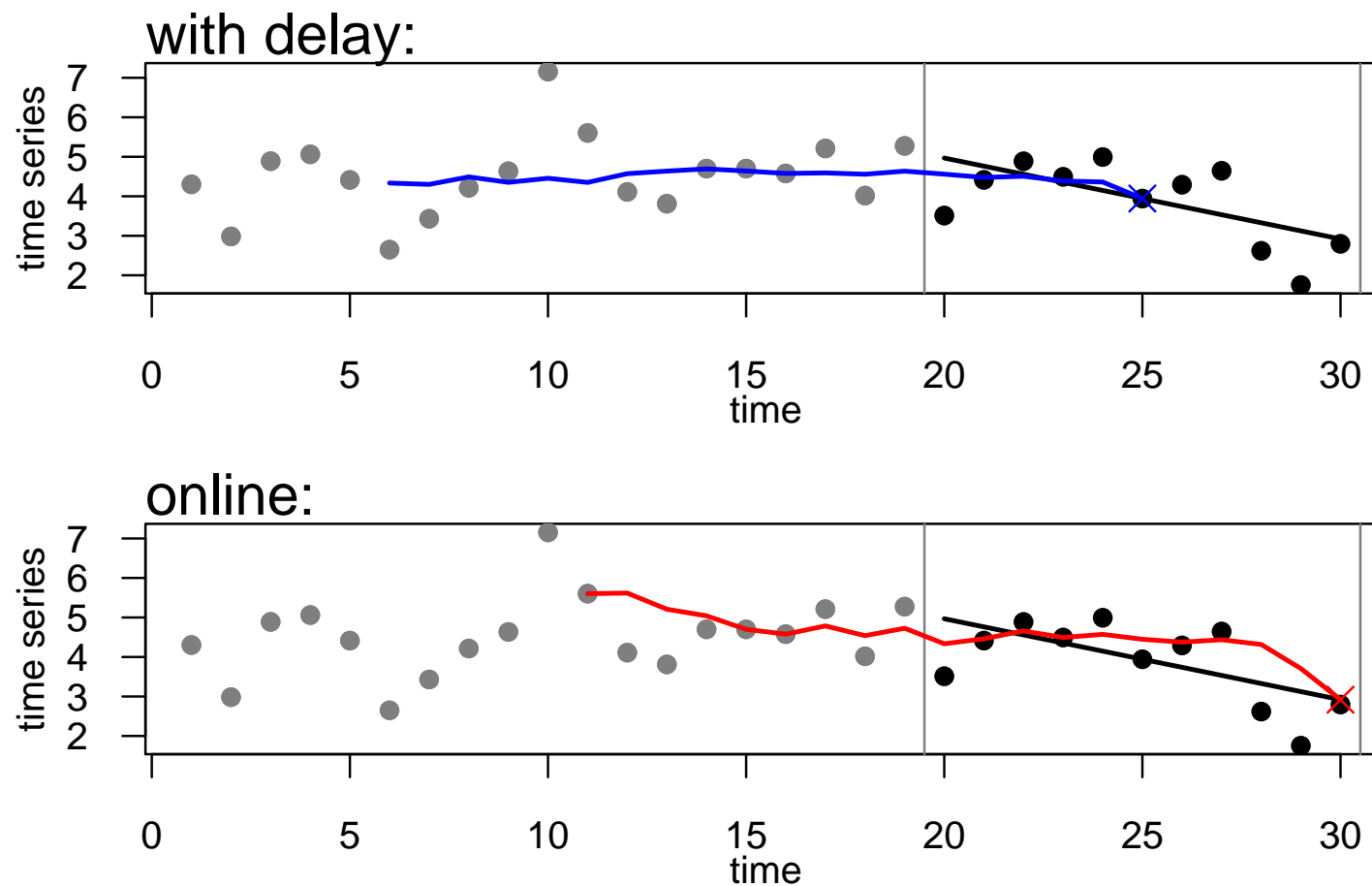
# robreg.filter – Illustration

Robust regression in a moving time window:



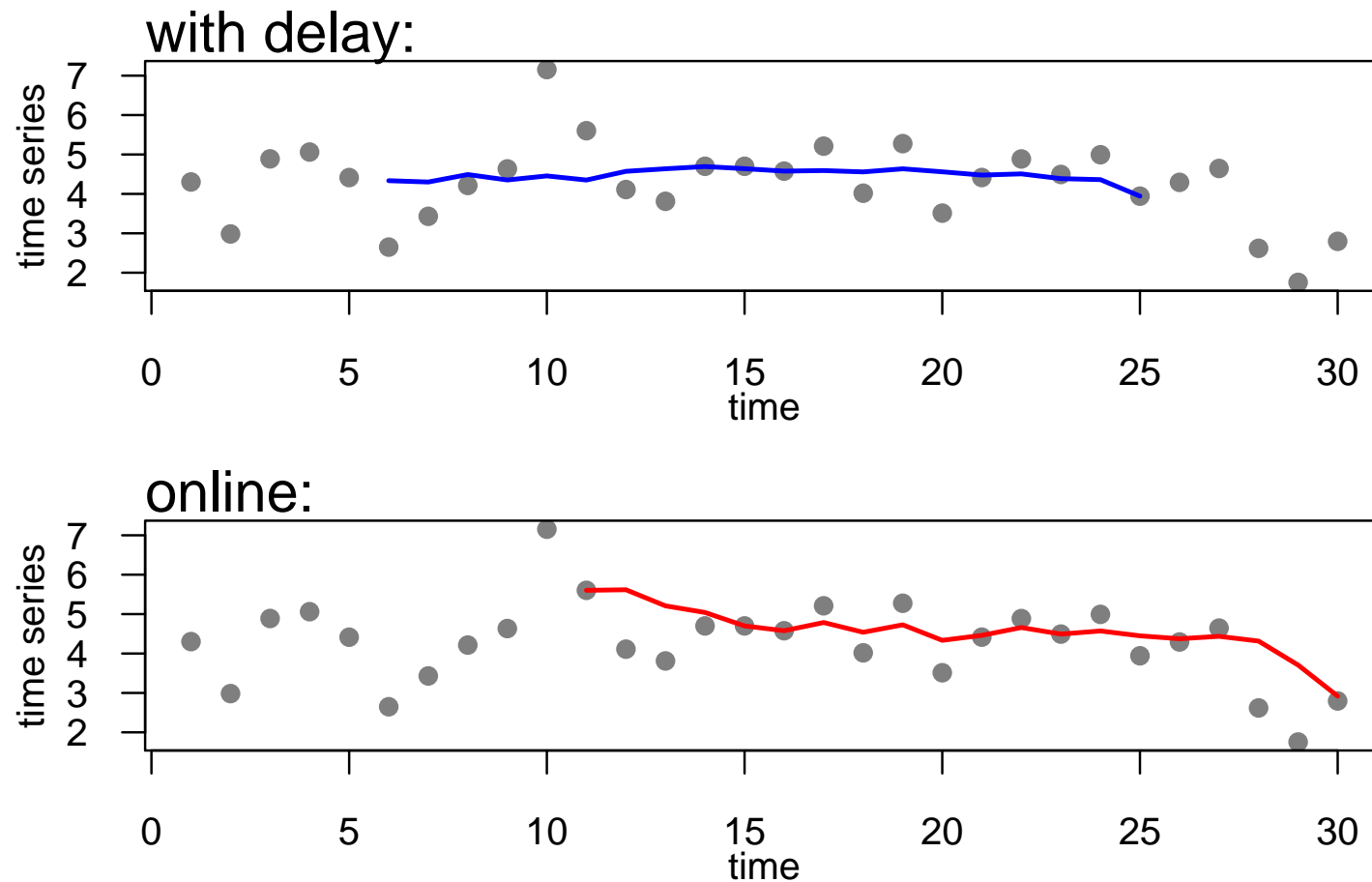
# robreg.filter – Illustration

Robust regression in a moving time window:



## robreg.filter – Illustration

**Robust regression** in a moving time window:





## robreg.filter – Parameter Options

- Robust regression techniques

```
robreg.filter(...,method=" ")
```

<b>RM</b>	Repeated Median	(Siegel, 1982)
<b>LMS</b>	Least Median of Squares	(Hampel, 1975; Rousseeuw, 1984)
<b>LTS</b>	Least Trimmed Squares	(Rousseeuw, 1983)
<b>LQD</b>	Least Quartile Difference	(Croux, Rousseeuw, Hössjer, 1994)
<b>DR</b>	Deepest Regression	(Rousseeuw and Hubert, 1999)

## robreg.filter – Parameter Options

- Robust regression techniques

```
robreg.filter(...,method=" ")
```

**MED** Median

**RM** Repeated Median

(Siegel, 1982)

**LMS** Least Median of Squares

(Hampel, 1975; Rousseeuw, 1984)

**LTS** Least Trimmed Squares

(Rousseeuw, 1983)

**LQD** Least Quartile Difference

(Croux, Rousseeuw, Hössjer, 1994)

**DR** Deepest Regression

(Rousseeuw and Hubert, 1999)

## robreg.filter – Parameter Options

- Robust regression techniques

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robreg.filter(...,method=" ")
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(Siegel, 1982)

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**DR** Deepest Regression

(Rousseeuw and Hubert, 1999)

- Window width  $\in \mathbb{N}$  (`width`)

## robreg.filter – Parameter Options

- Robust regression techniques

```
robreg.filter(...,method=" ")
```

**MED** Median

**RM** Repeated Median

(Siegel, 1982)

**LMS** Least Median of Squares

(Hampel, 1975; Rousseeuw, 1984)

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**LQD** Least Quartile Difference

(Croux, Rousseeuw, Hössjer, 1994)

**DR** Deepest Regression

(Rousseeuw and Hubert, 1999)

- Window width  $\in \mathbb{N}$  (`width`)

- Minimum number of non-missing observations within one window (`minNonNAs`)

## robreg.filter – Parameter Options

- Robust regression techniques

```
robreg.filter(...,method=" ")
```

**MED** Median

**RM** Repeated Median (Siegel, 1982)

**LMS** Least Median of Squares (Hampel, 1975; Rousseeuw, 1984)

**LTS** Least Trimmed Squares (Rousseeuw, 1983)

**LQD** Least Quartile Difference (Croux, Rousseeuw, Hössjer, 1994)

**DR** Deepest Regression (Rousseeuw and Hubert, 1999)

- Window width  $\in \mathbb{N}$  (`width`)
- Minimum number of non-missing observations within one window (`minNonNAs`)
- Online estimation (`online = TRUE / FALSE`)

## robreg.filter – Parameter Options

- Robust regression techniques

```
robreg.filter(...,method=" ")
```

### Aliases

**MED** Median

`med.filter`

**RM** Repeated Median

`rm.filter`

**LMS** Least Median of Squares

`lms.filter`

**LTS** Least Trimmed Squares

`lts.filter`

**LQD** Least Quartile Difference

`lqd.filter`

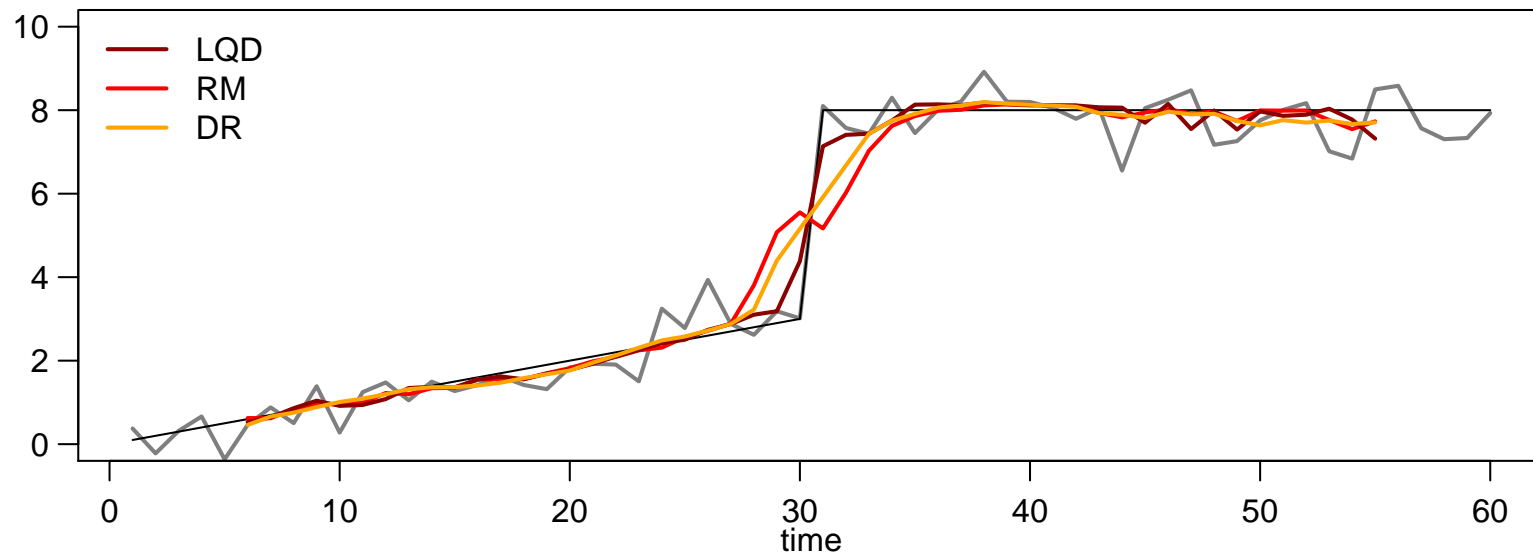
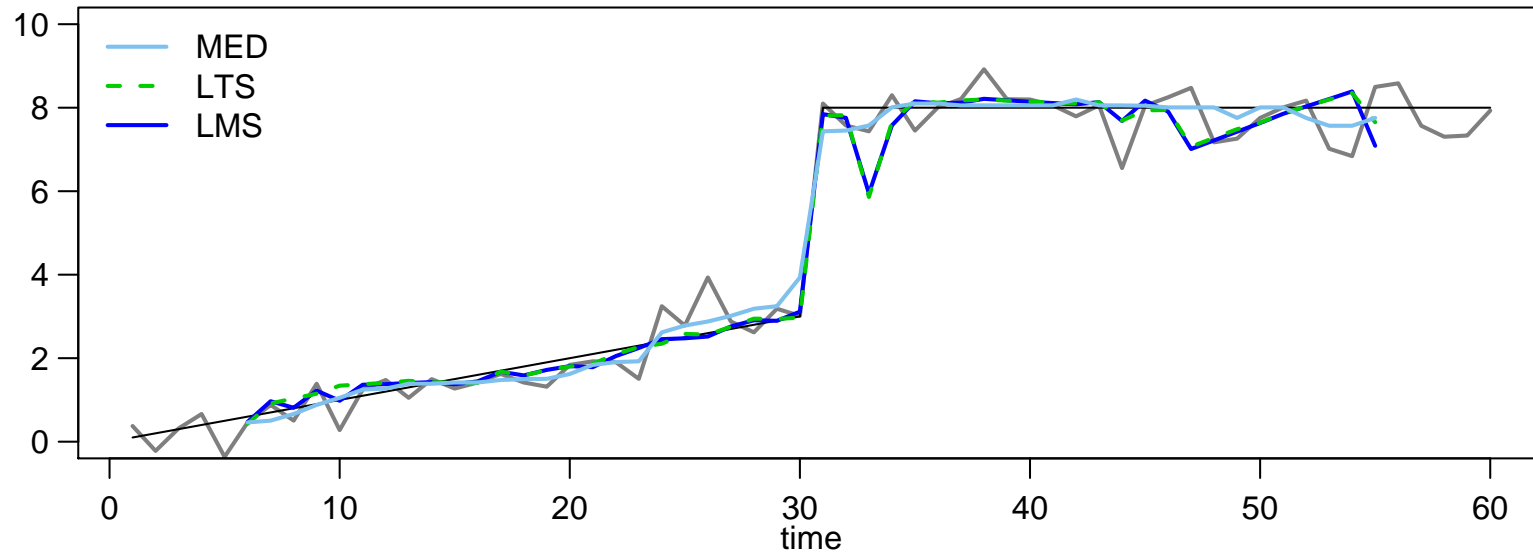
**DR** Deepest Regression

`dr.filter`

- Window width  $\in \mathbb{N}$  (`width`)
- Minimum number of non-missing observations within one window (`minNonNAs`)
- Online estimation (`online = TRUE / FALSE`)

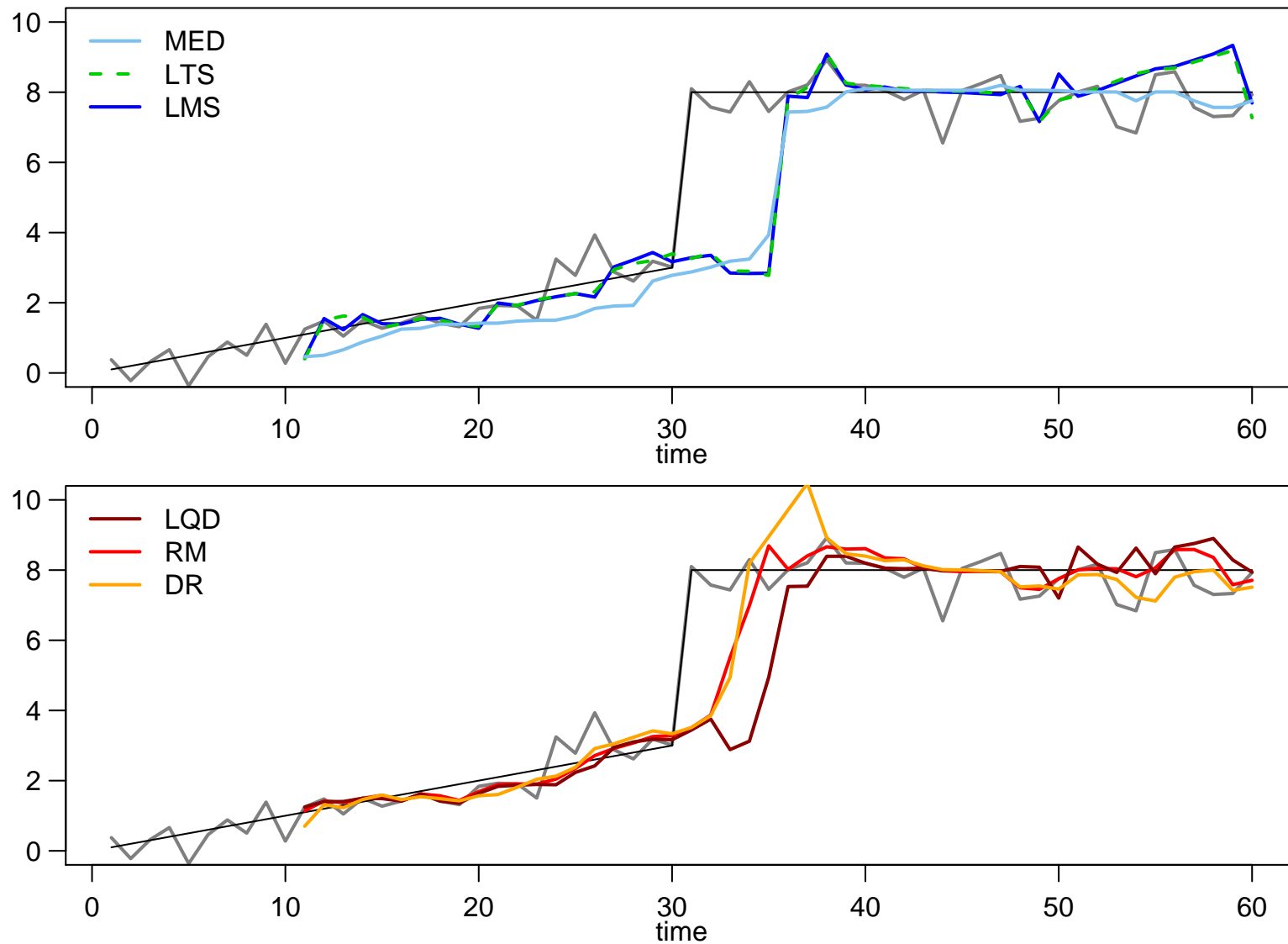
# robreg.filter – Example

## Filter Output with Delay



# robreg.filter – Example

## Online Filter Output

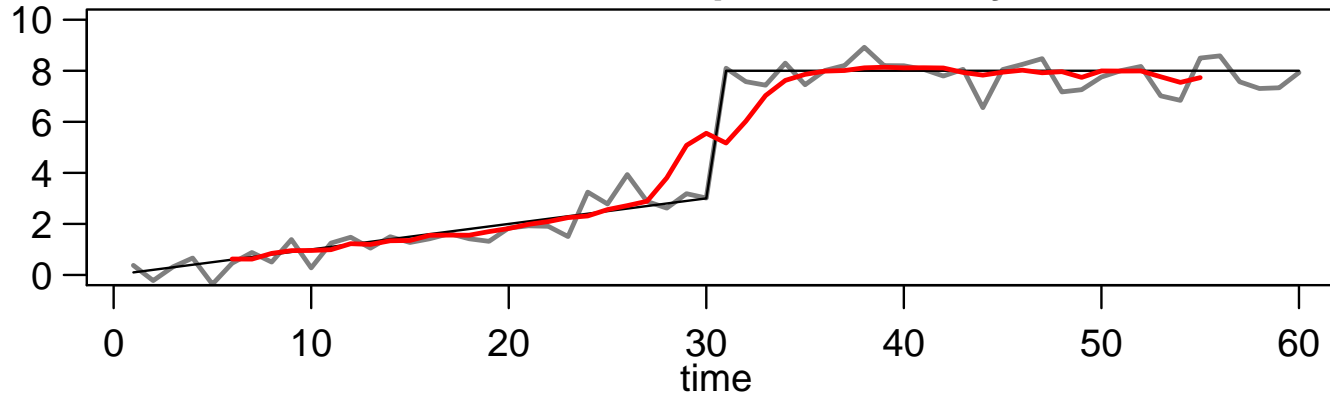




## robreg.filter – Option extrapolate

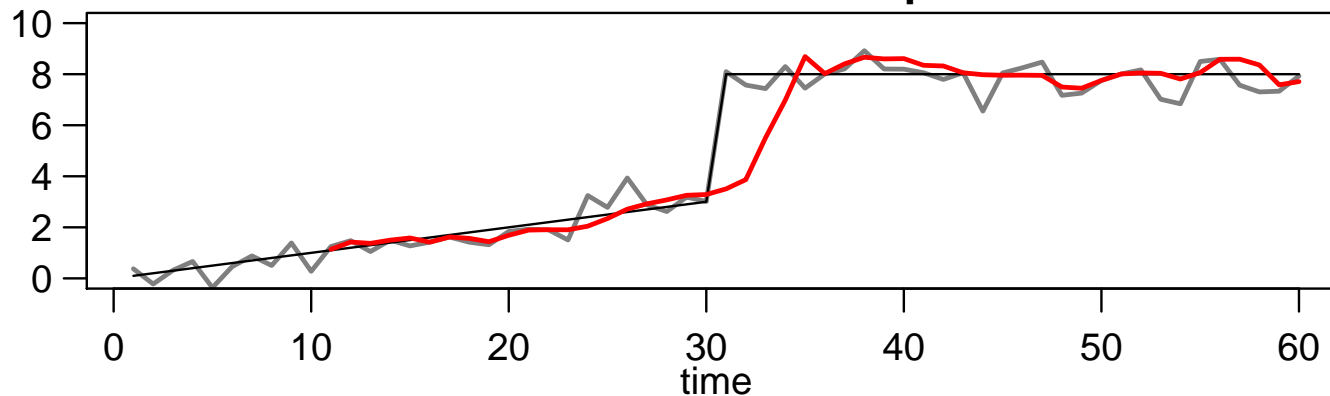
```
rm.filter(ts, width=11, extrapolate=FALSE, ... )
```

**RM-Filter Output with Delay**



online=FALSE

**Online RM-Filter Output**

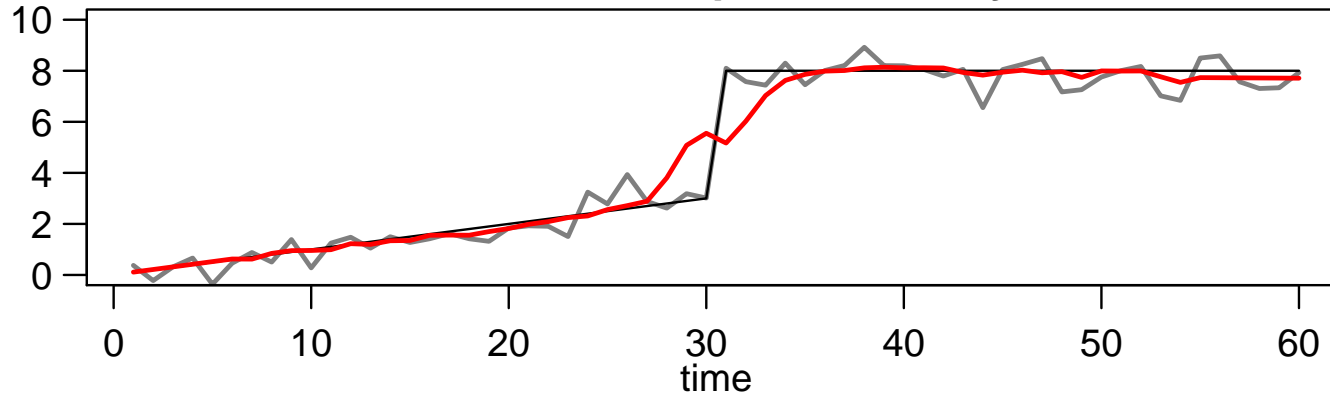


online=TRUE

## robreg.filter – Option extrapolate

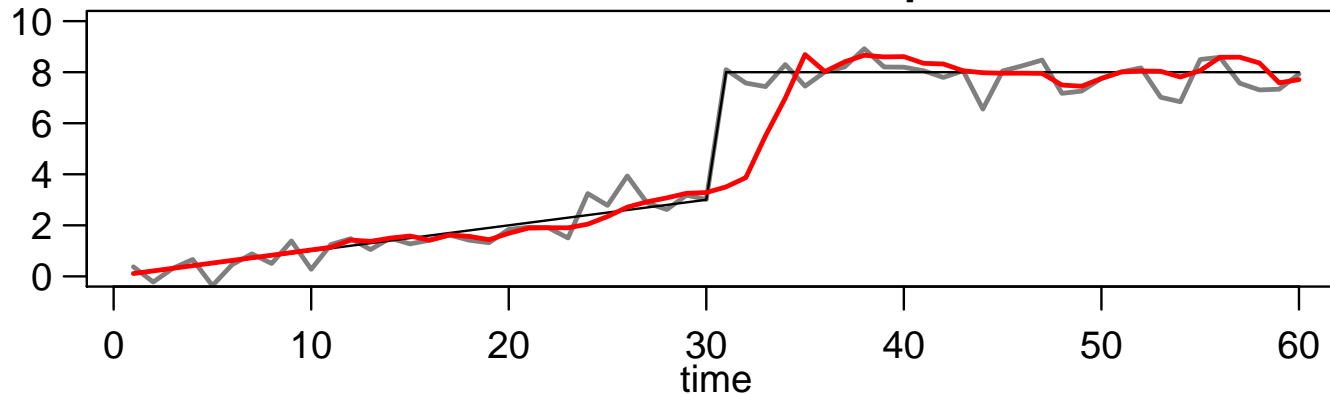
```
rm.filter(ts, width=11, extrapolate=TRUE , ... )
```

**RM-Filter Output with Delay**



online=FALSE

**Online RM-Filter Output**



online=TRUE

# Repeated Median Regression

(Siegel, 1982)

Sample in one time window  $\mathbf{y}_t = (y_{t+i})'$

$$\begin{aligned}\hat{\beta}_t^{\text{RM}} &= \text{med}_i \left\{ \text{med}_{j \neq i} \left\{ \frac{y_{t+i} - y_{t+j}}{i - j} \right\} \right\} \\ \hat{\mu}_t^{\text{RM}} &= \text{med}_i \left\{ y_{t+i} + \hat{\beta}_t^{\text{RM}} \cdot i \right\}\end{aligned}$$

with  $i = \begin{cases} -n + 1, \dots, -1, 0 & \text{(online)} \\ -m, \dots, m \text{ and } n = 2m + 1 & \text{(with delay)} \end{cases}$

## hybrid.filter

Application of 'subfilters' to a moving time window

$$\mathbf{y}_t = (y_{t-m}, \dots, y_{t-1}, y_t, y_{t+1}, \dots, y_{t+m})'$$

with  $m = \frac{\text{width}-1}{2}$

## hybrid.filter

Application of 'subfilters' to a moving time window

$$\mathbf{y}_t = (y_{t-m}, \dots, y_{t-1}, y_t, y_{t+1}, \dots, y_{t+m})'$$

with  $m = \frac{\text{width}-1}{2}$

$$\Phi_j(\mathbf{y}_t), \quad j = 1, \dots, k, \quad \text{subfilters}$$

**Filter output**  $\hat{\mu}_t = \text{med}\{\Phi_1(\mathbf{y}_t), \Phi_2(\mathbf{y}_t), \dots, \Phi_k(\mathbf{y}_t)\}$

## hybrid.filter

Application of 'subfilters' to a moving time window

$$\mathbf{y}_t = (y_{t-m}, \dots, y_{t-1}, y_t, y_{t+1}, \dots, y_{t+m})'$$

with  $m = \frac{\text{width}-1}{2}$

$$\Phi_j(\mathbf{y}_t), j = 1, \dots, k, \quad \text{subfilters}$$

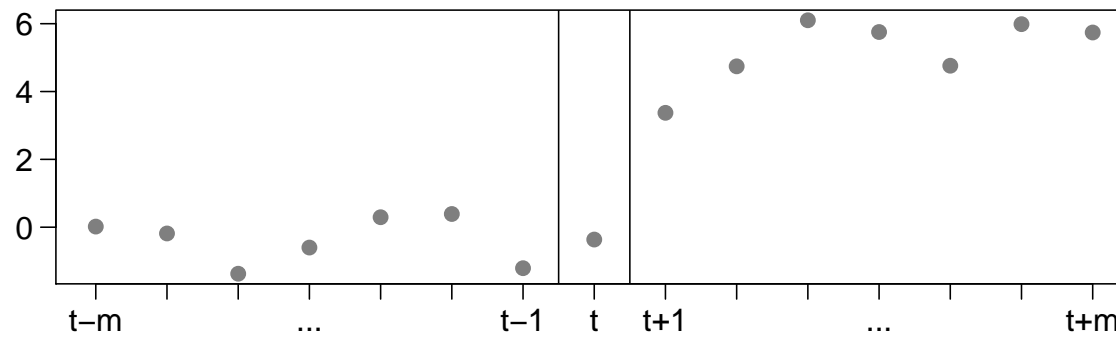
**Filter output**  $\hat{\mu}_t = \text{med}\{\Phi_1(\mathbf{y}_t), \Phi_2(\mathbf{y}_t), \dots, \Phi_k(\mathbf{y}_t)\}$

**Parameters options:**

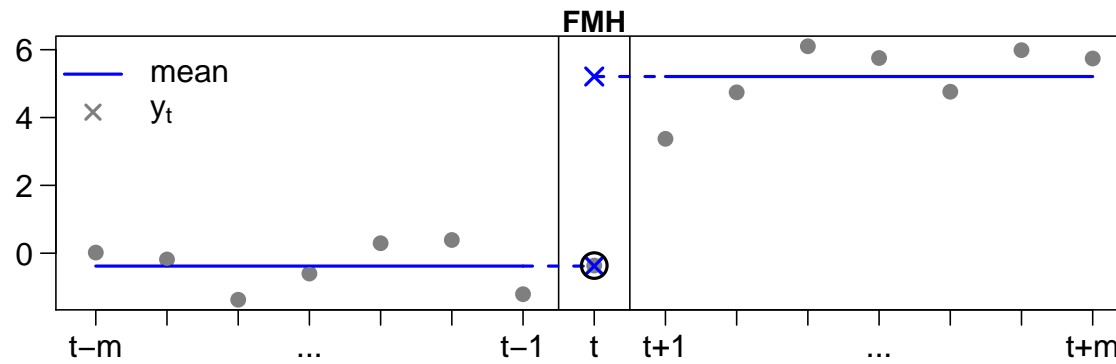
method, width, minNonNAs, and extrapolate

**Online estimation not possible!!!**

# hybrid.filter – Estimation in One Window

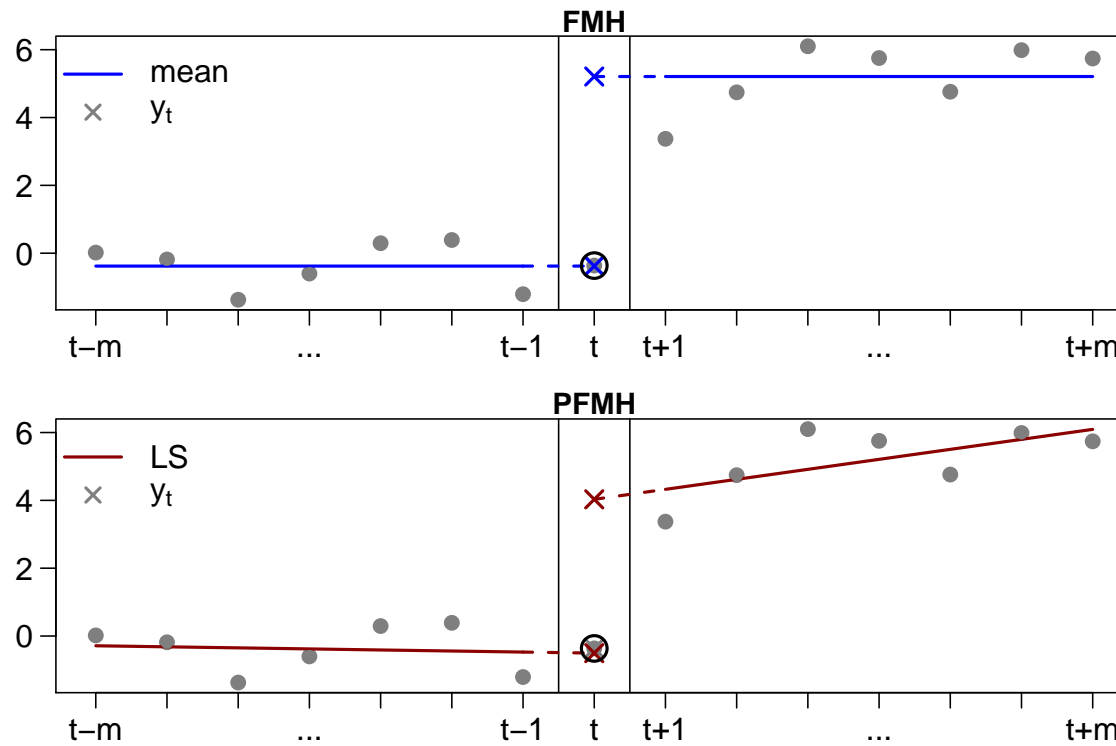


# hybrid.filter – Estimation in One Window

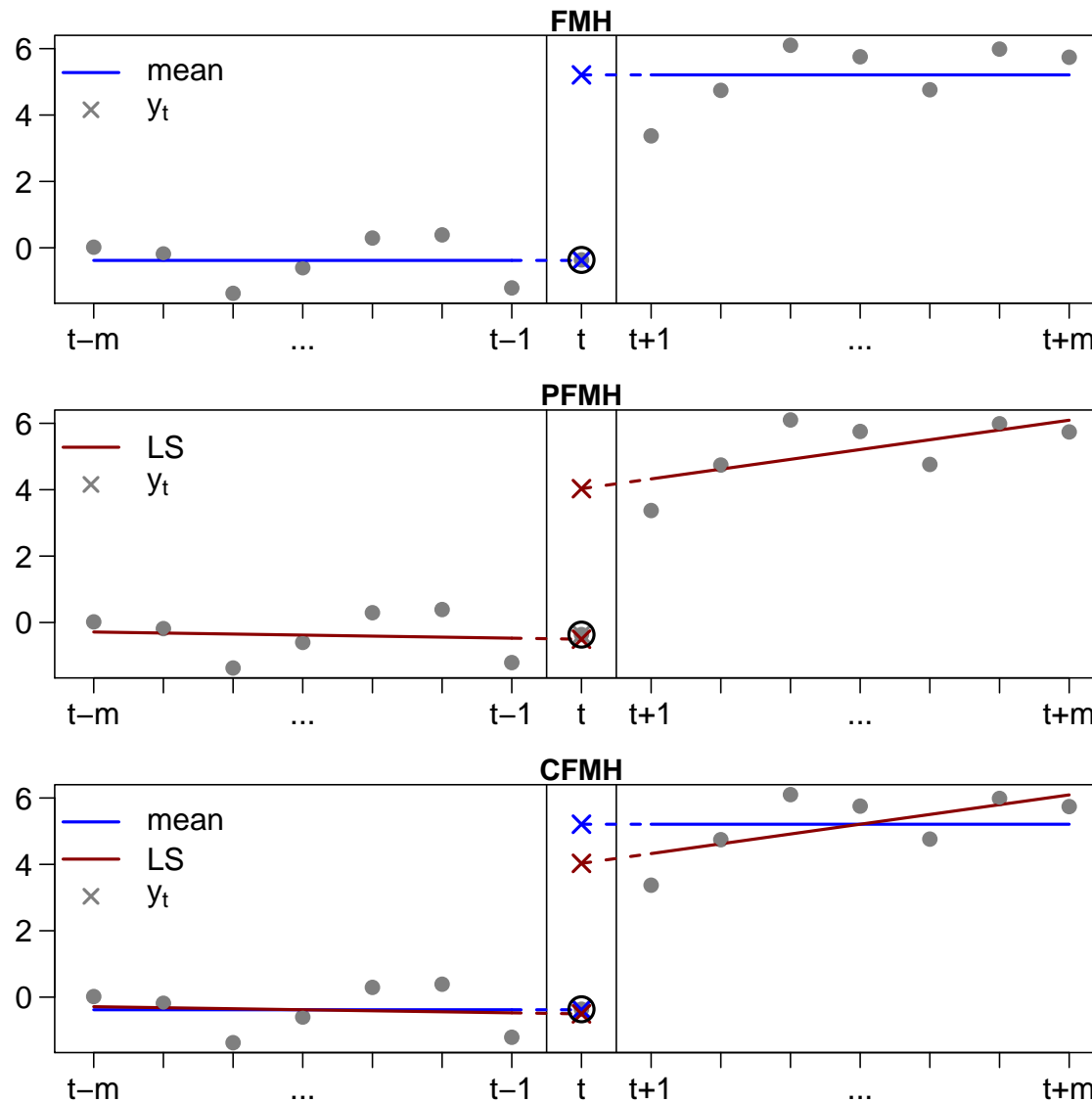




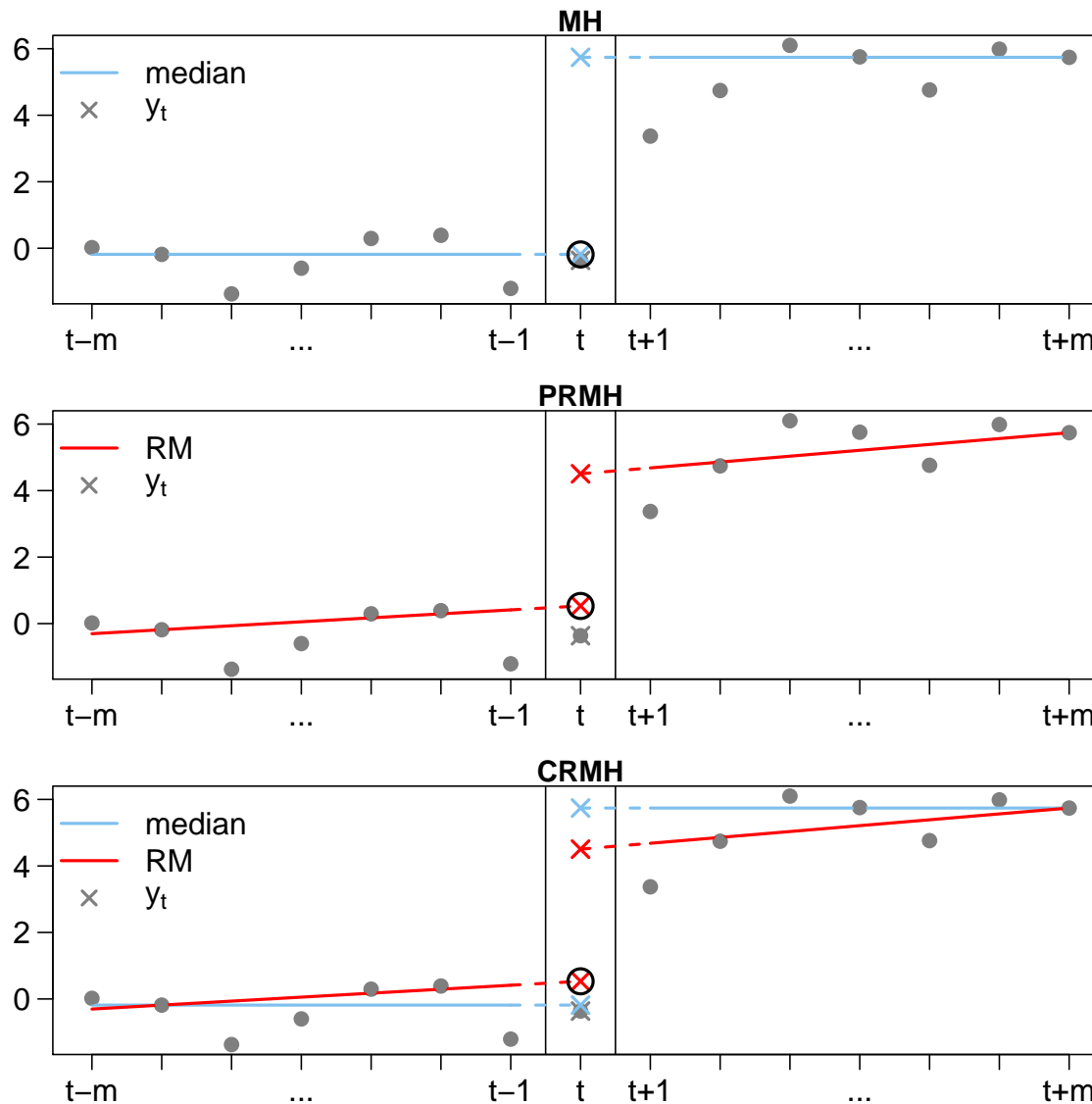
# hybrid.filter – Estimation in One Window



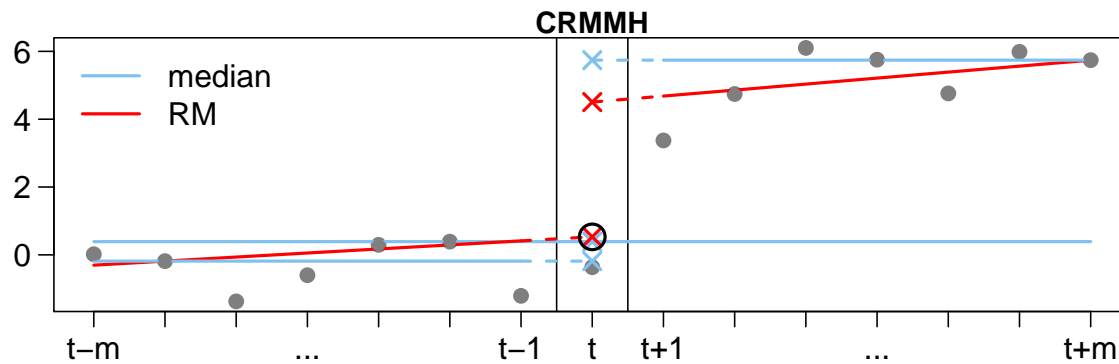
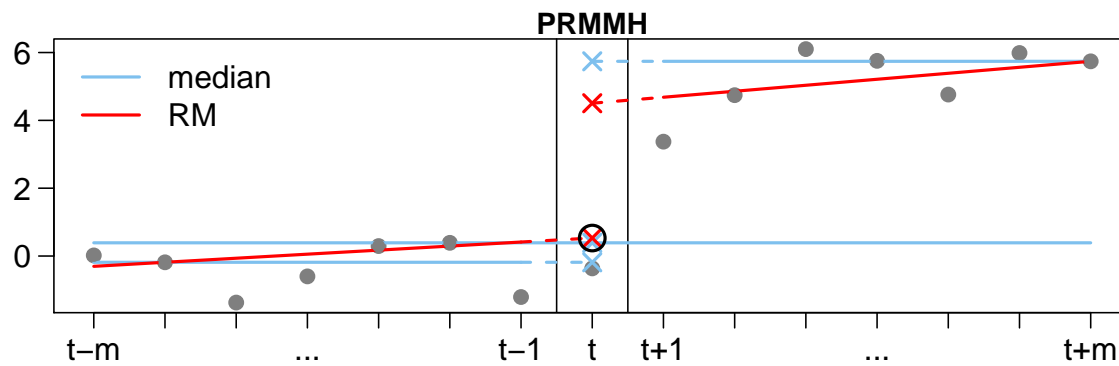
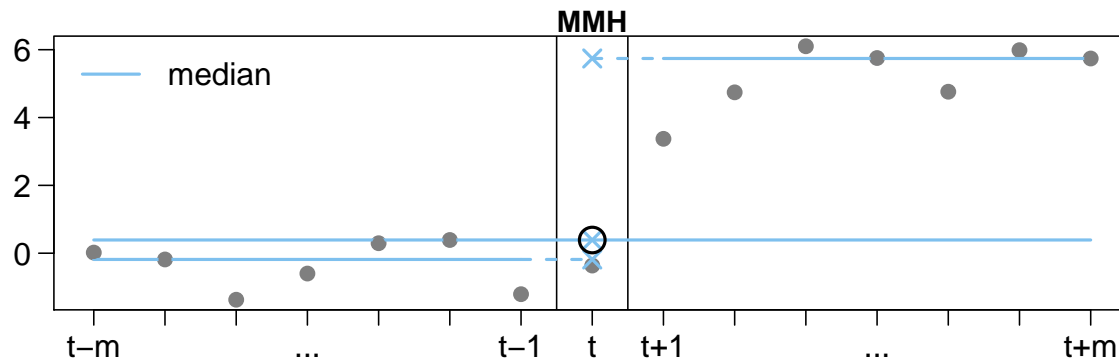
# hybrid.filter – Estimation in One Window



# hybrid.filter – Estimation in One Window



# hybrid.filter – Estimation in One Window

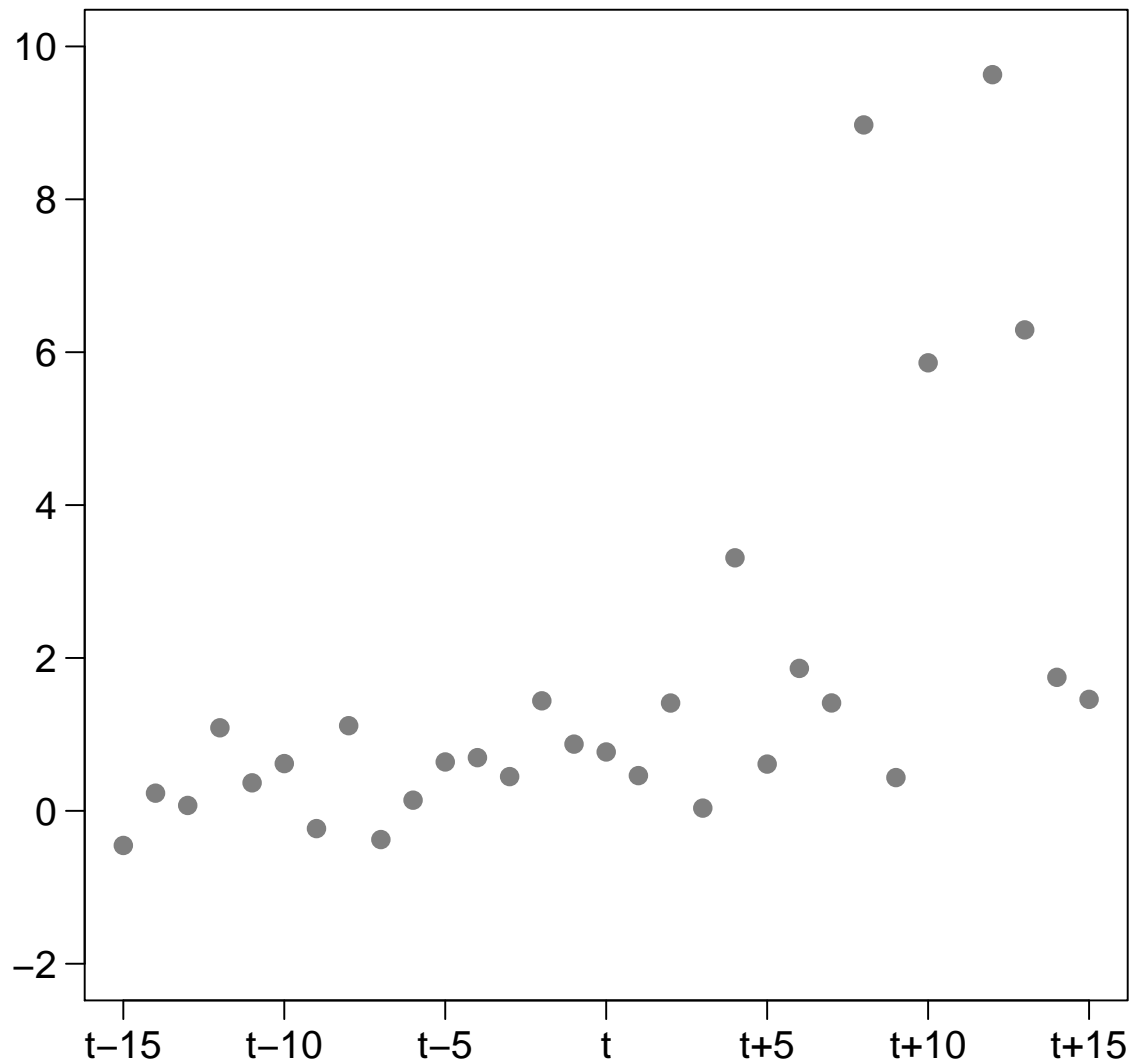


## dw.filter – Parameter Options

Double **w**indow filtering techniques with

<code>outer.width</code>	Outer window width ( $\in \mathbb{N}$ )
<code>inner.width</code>	Inner window width ( $\in \mathbb{N}, < \text{outer.width}$ )
<code>method</code>	Filter method(s)
<code>scale</code>	Scale estimation method
<code>d</code>	Trimming factor
<code>minNonNAs</code>	Minimum number of non-missing observations within one window
<code>online</code>	TRUE / FALSE for online / delayed estimation
<code>extrapolate</code>	TRUE / FALSE for extrapolation to the edges

## dw.filter – Estimation in One Window



---

method

---

MTM

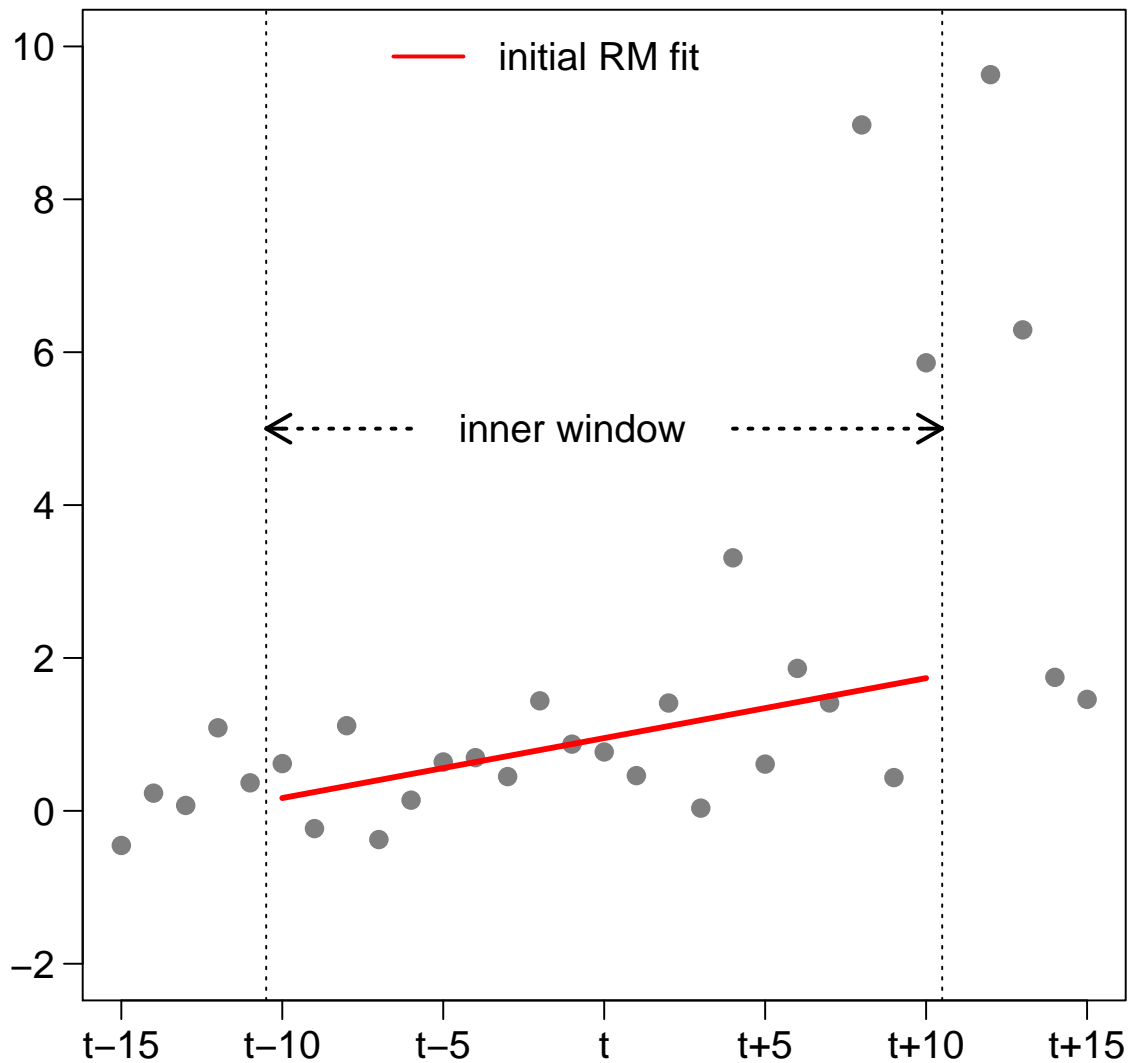
TRM

MRM

RM

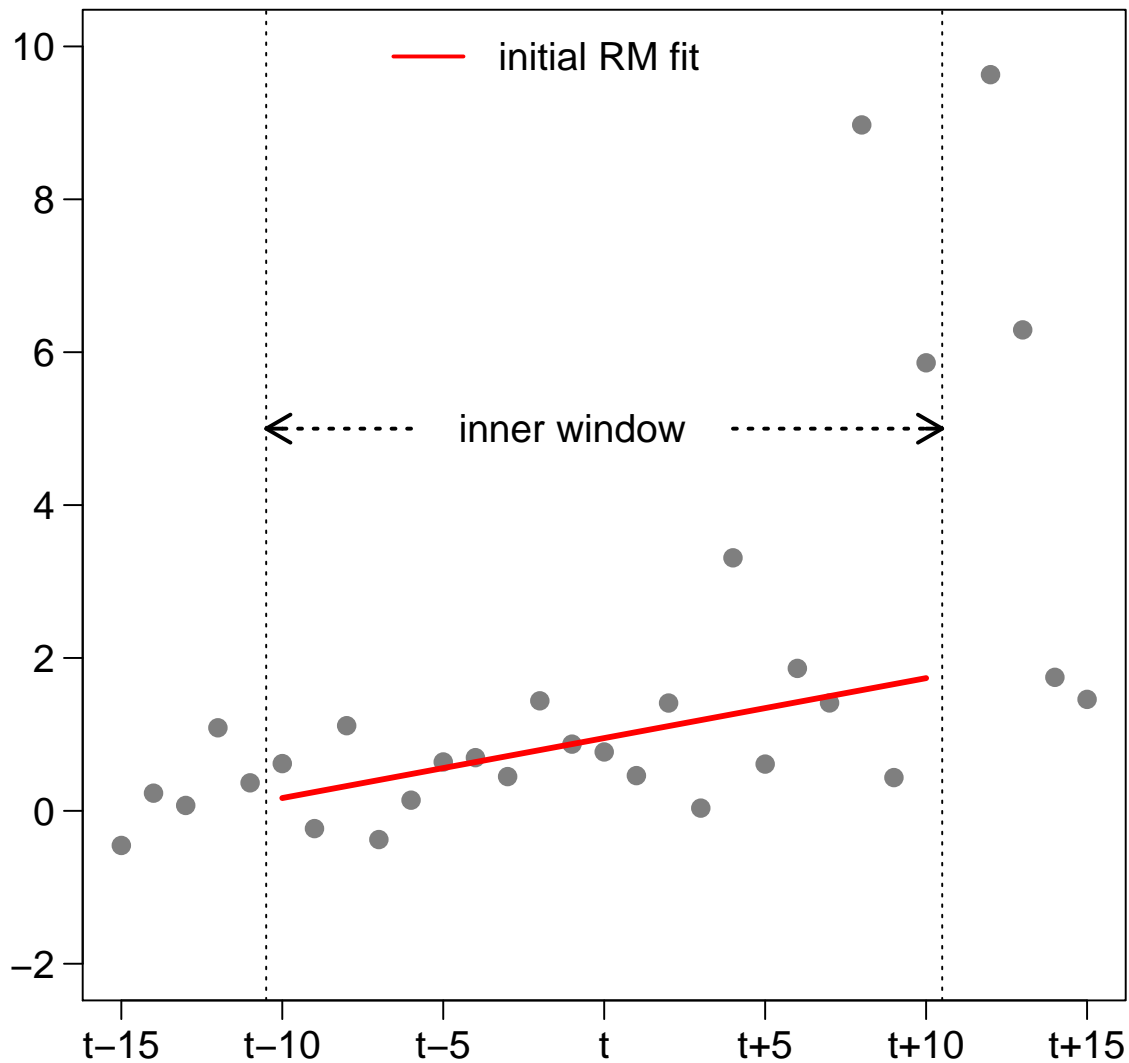
---

## dw.filter – Estimation in One Window



method	inner window
MTM	<b>MED</b>
TRM	<b>RM</b>
MRM	<b>RM</b>
RM	<b>RM</b> slope

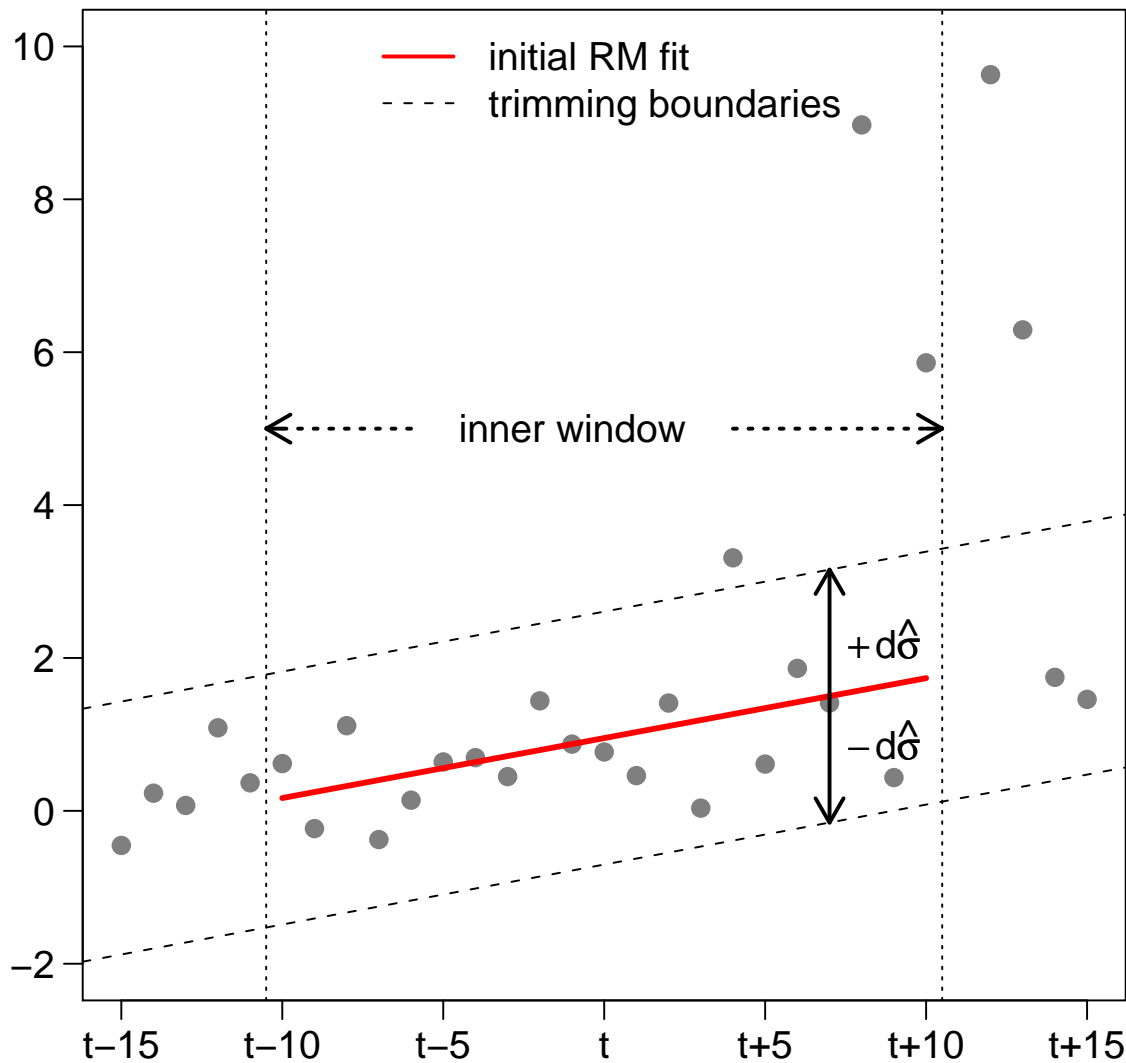
## dw.filter – Estimation in One Window



method	inner window
DWMTM	<b>MED</b>
DWTRM	<b>RM</b>
DWMRM	<b>RM</b>
DWRM	<b>RM</b> slope



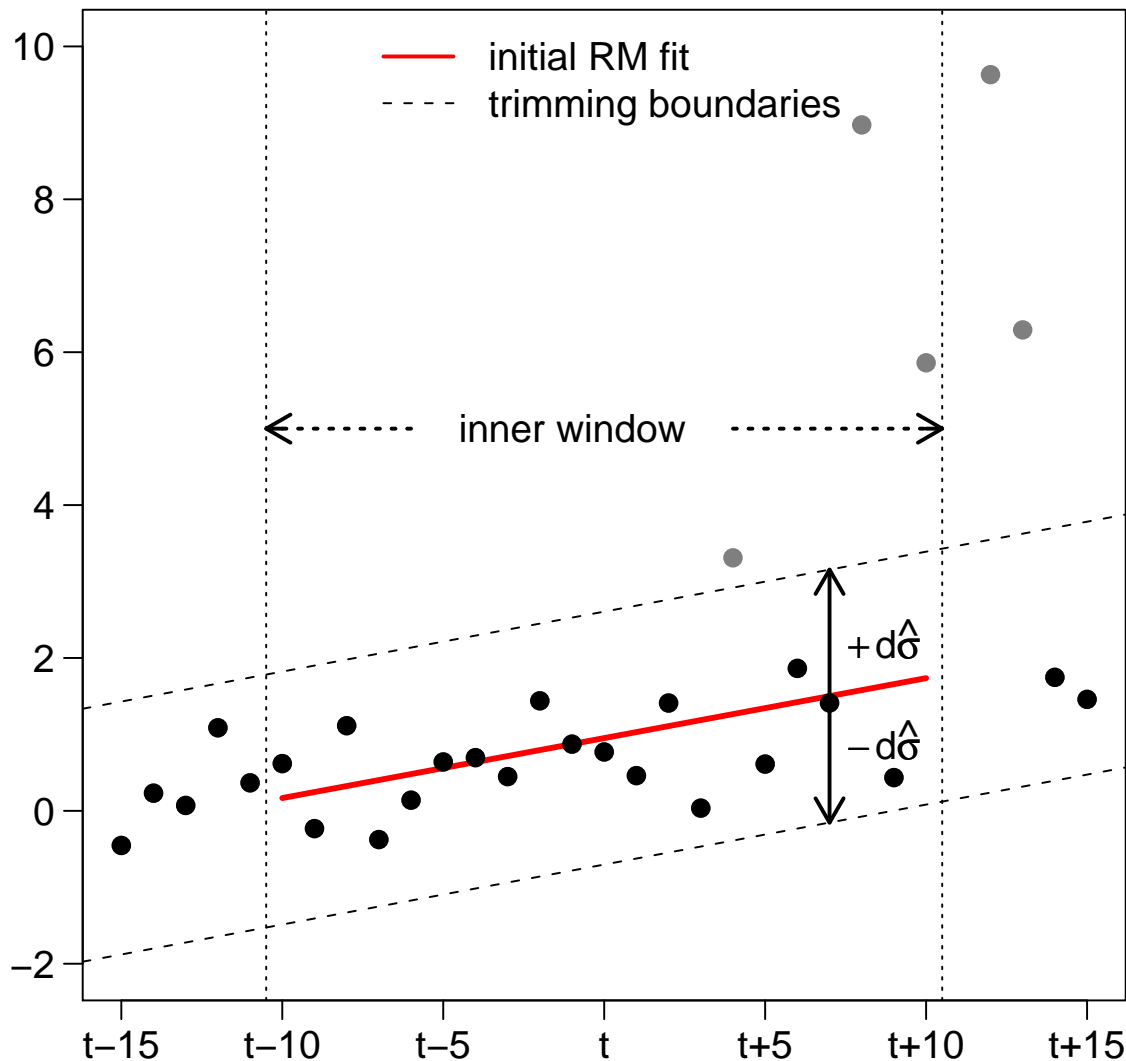
# dw.filter – Estimation in One Window



method	inner window
DWMTM	<b>MED</b>
DWTRM	<b>RM</b>
DWMRM	<b>RM</b>
DWRM	<b>RM</b> slope
scale	$\hat{\sigma}$
MAD	median absolute deviation
QN	$Q_n$ scale
SN	$S_n$ scale

(Rousseeuw, Croux, 1993)

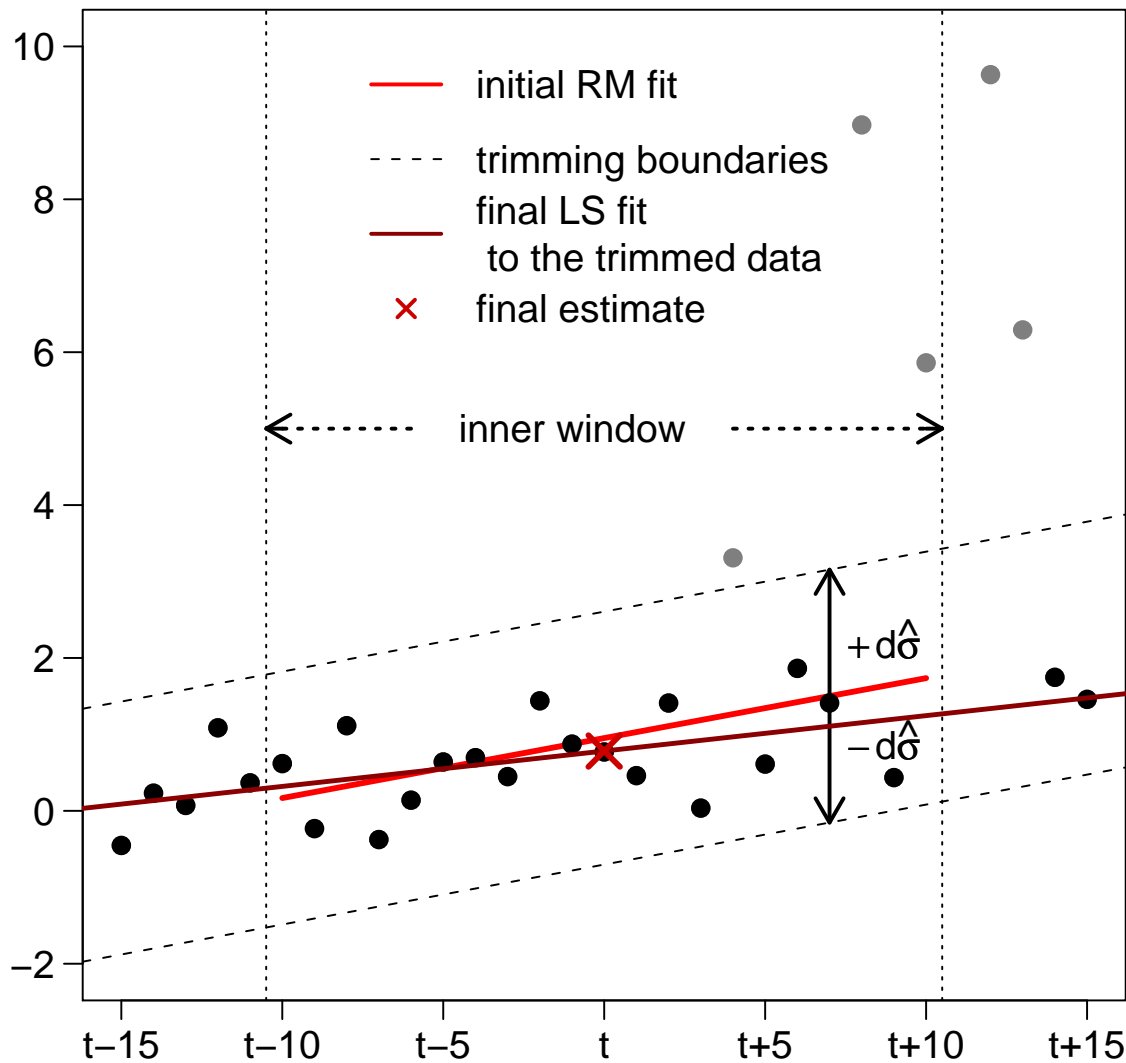
## dw.filter – Estimation in One Window



method	inner window
DWMTM	<b>MED</b>
DWTRM	<b>RM</b>
DWMRM	<b>RM</b>
DWRM	<b>RM</b> slope
scale	$\hat{\sigma}$
MAD	median absolute deviation
QN	$Q_n$ scale
SN	$S_n$ scale

(Rousseeuw, Croux, 1993)

# dw.filter – Estimation in One Window



method	inner window	outer window
DWMTM	<b>MED</b>	<b>MEAN</b>
DWTRM	<b>RM</b>	<b>LS</b>
DWMRM	<b>RM</b>	<b>RM</b>
DWRM	<b>RM</b> slope	<b>RM</b> location
scale	$\hat{\sigma}$	
MAD	Median Absolute Deviation	
QN	$Q_n$ scale	
SN	$S_n$ scale	
(Rousseeuw, Croux, 1993)		

## robust.filter – Parameter Options

width Window width  $\in \mathbb{N}$

trend "MED" Median  
"RM" Repeated Median  
"LMS" Least Median of Squares  
"LTS" Least Trimmed Squares

## robust.filter – Parameter Options

width		Window width $\in \mathbb{N}$
trend	"MED"	Median
	"RM"	Repeated Median
	"LMS"	Least Median of Squares
	"LTS"	Least Trimmed Squares
scale	"MAD"	Median Absolute Deviation
	"QN"	$Q_n$ scale
	"SN"	$S_n$ scale
	"LSH"	Length of the Shortest Half

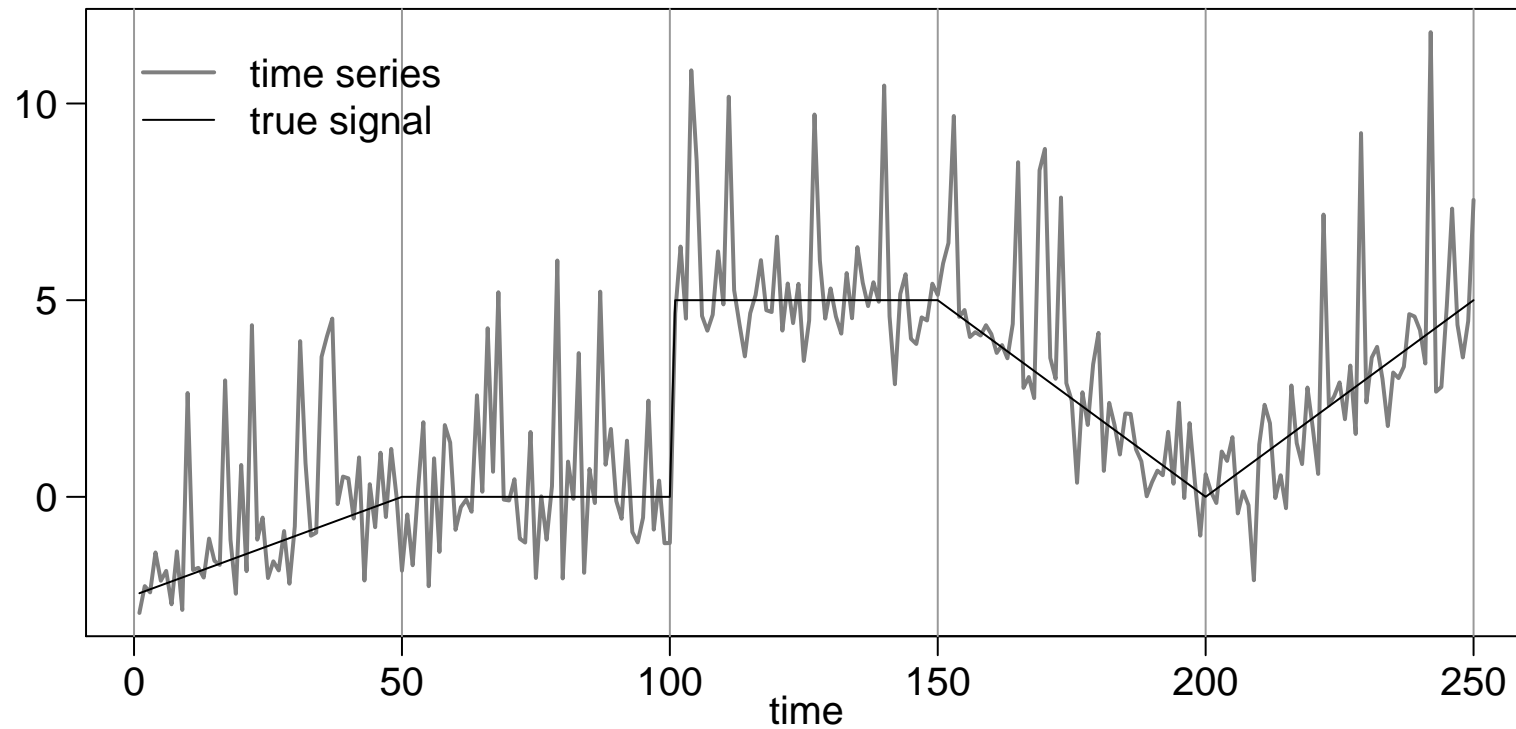
## robust.filter – Parameter Options

width	Window width $\in \mathbb{N}$
trend	"MED" Median
	"RM" Repeated Median
	"LMS" Least Median of Squares
	"LTS" Least Trimmed Squares
scale	"MAD" Median Absolute Deviation
	"QN" $Q_n$ scale
	"SN" $S_n$ scale
	"LSH" Length of the Shortest Half

➔ Options online and extrapolate

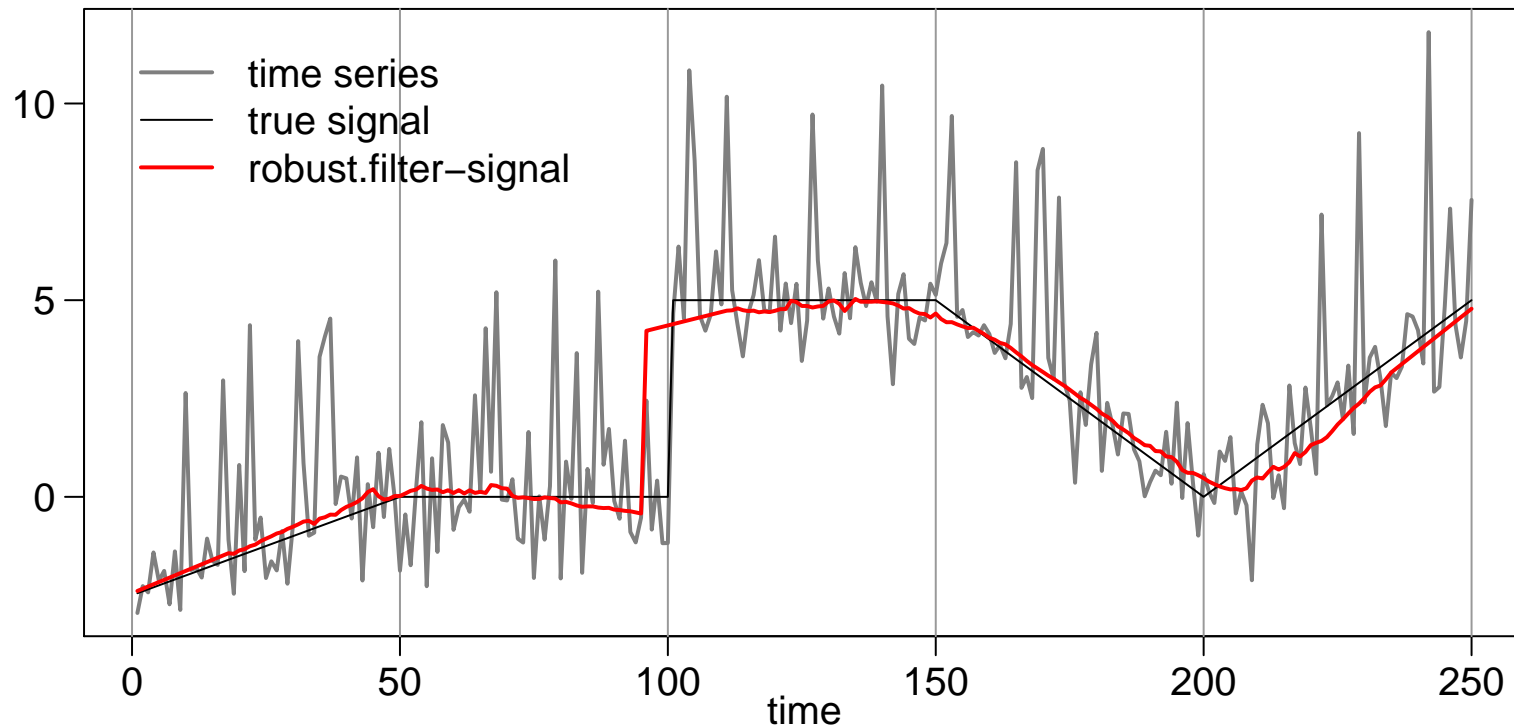
➔ Further arguments for level shift & outlier detection and outlier treatment

## robust.filter – Illustration



```
fit <- robust.filter(series, width=31)
```

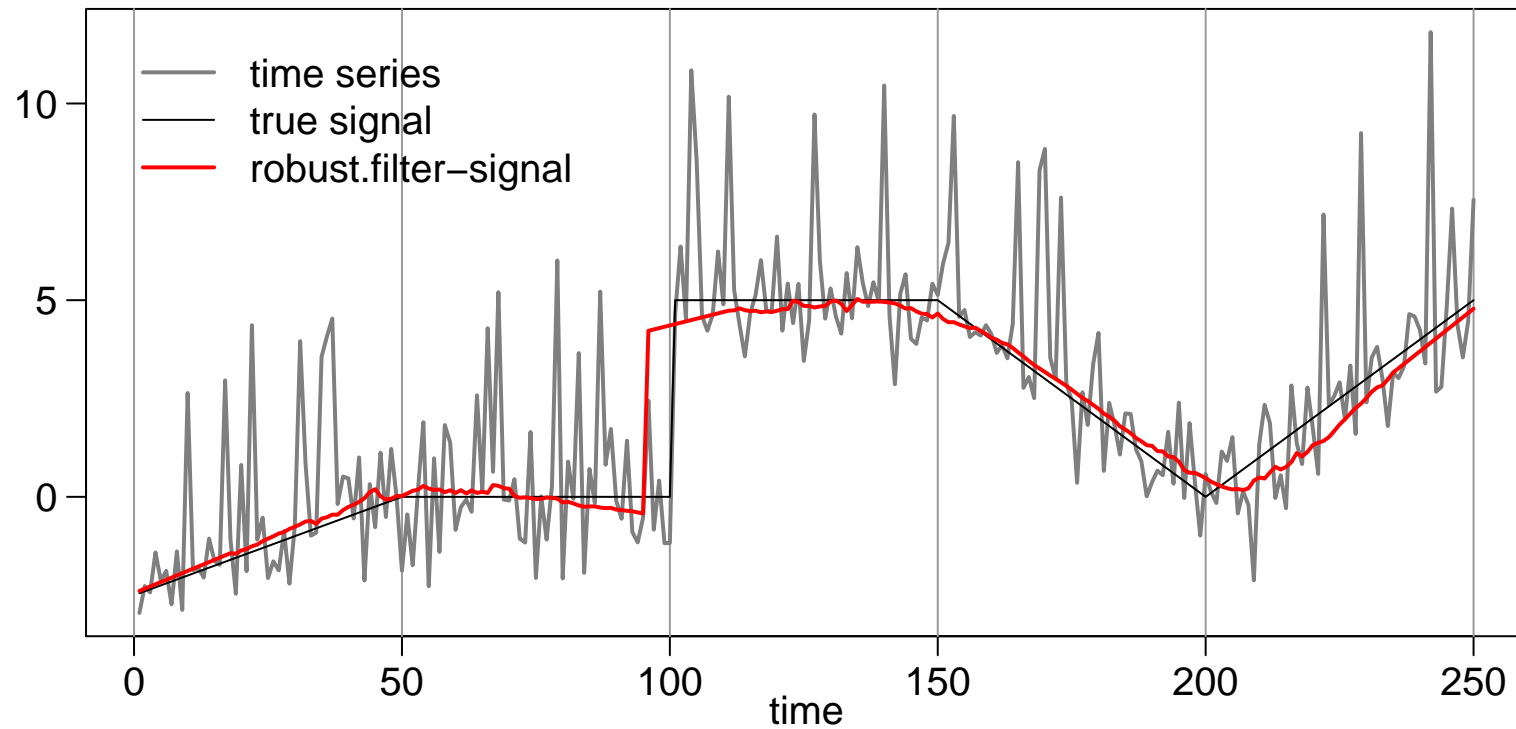
## robust.filter – Illustration



```
> fit $level
      [1] -2.391516 -2.335128 -2.278741 -2.222354 -2.165966 ...
      $slope
      [1] 0.056387 0.056387 0.056387 0.056387 0.056387 ...
      $sigma
      [1] 0.798726 0.798726 0.798726 0.798726 0.798726 ...
      245 observations omitted
```

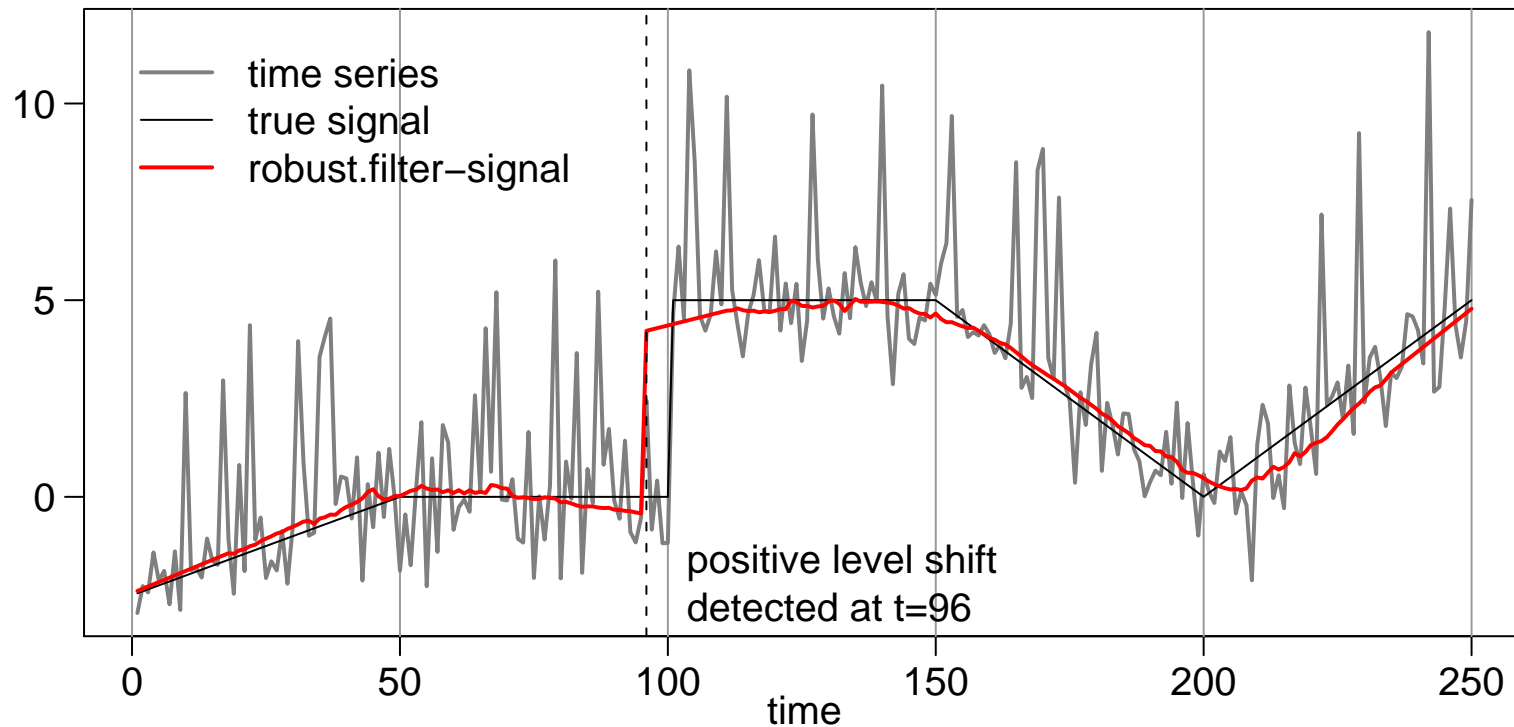


## robust.filter – Illustration



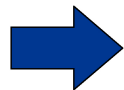
Level shifts?

## robust.filter – Illustration



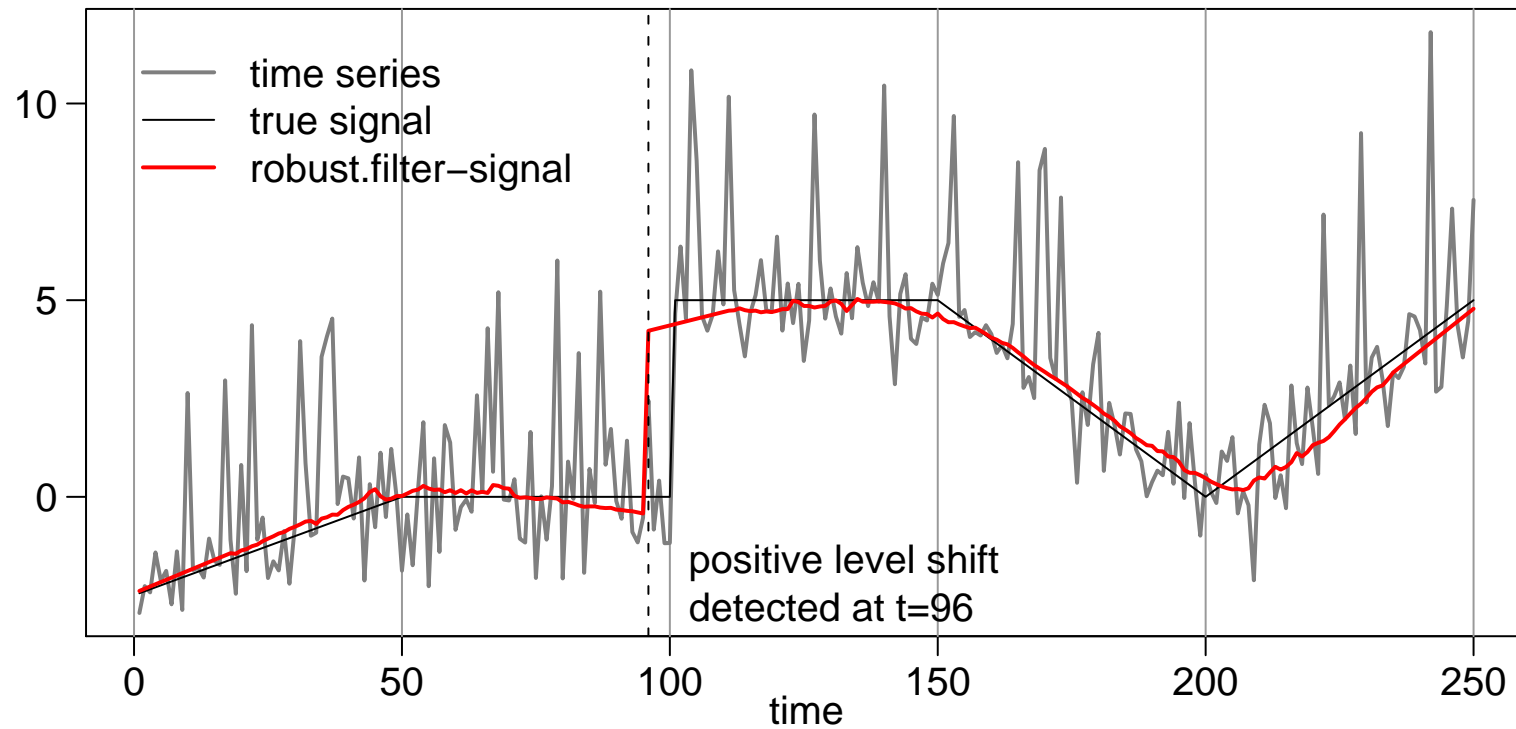
Level shifts?

```
> which(fit$level.shift!=0)
[1] 96
```



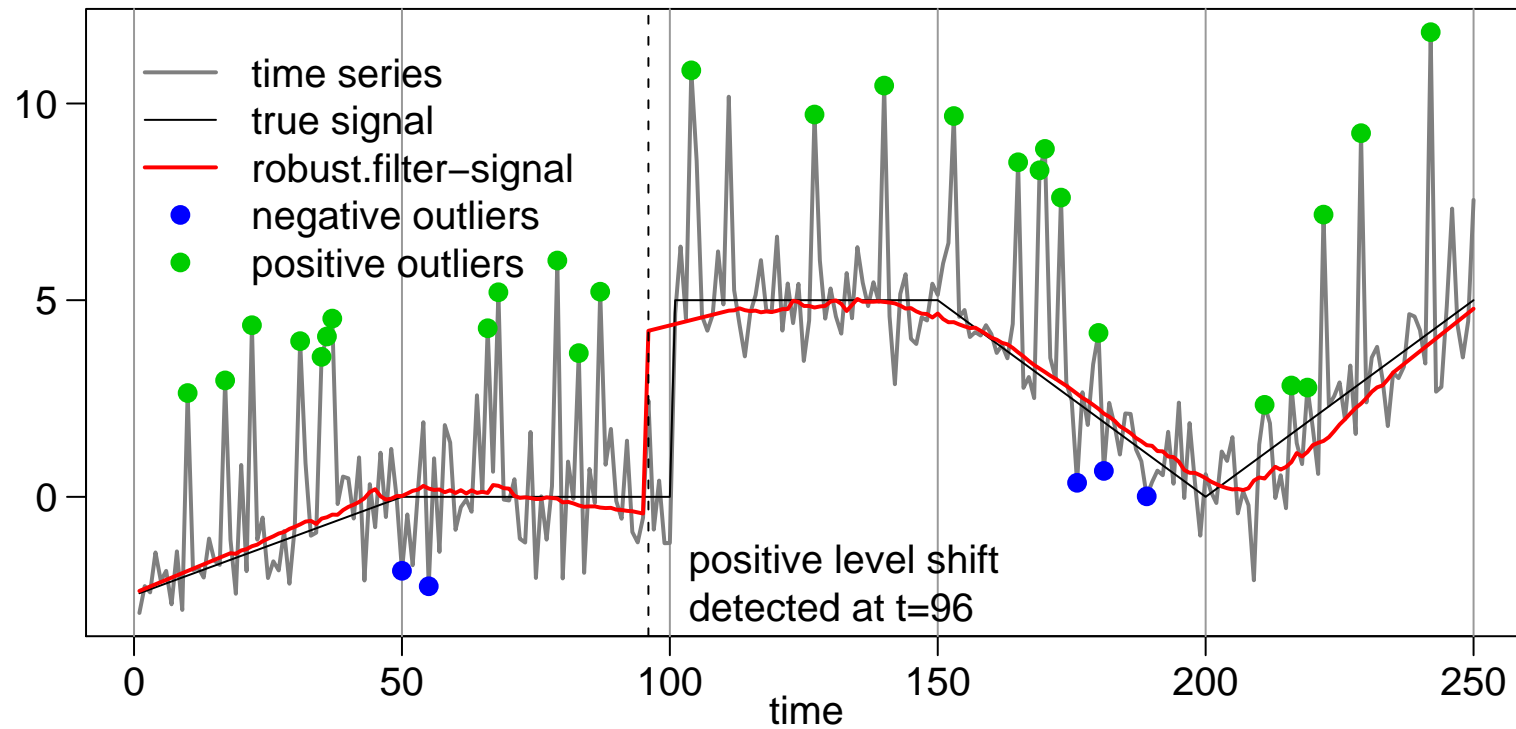
```
> fit$level.shift[which(fit$level.shift!=0)]
[1] 1
```

## robust.filter – Illustration



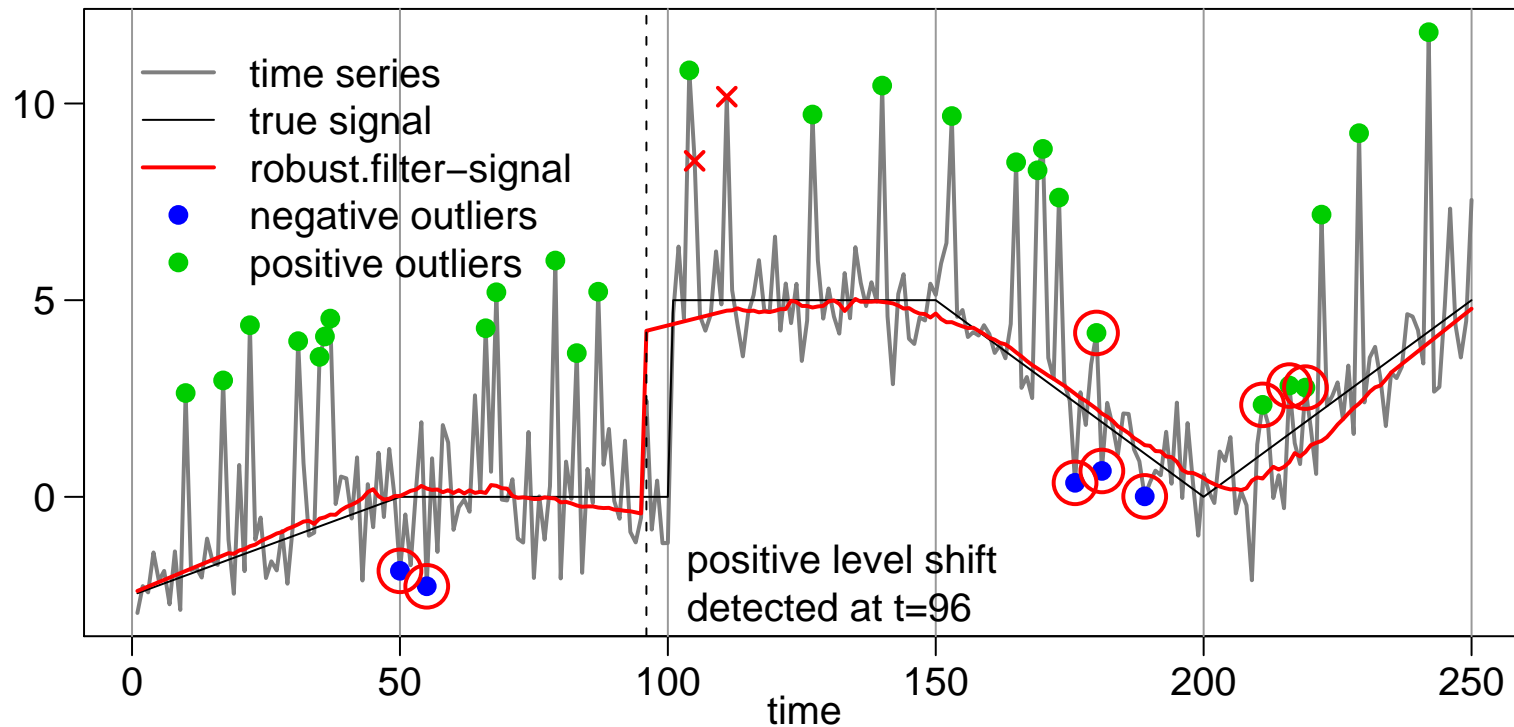
Outliers?

# robust.filter – Illustration

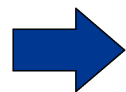


Outliers?

# robust.filter – Illustration



Outliers?



23 correctly detected  
9 falsely detected  
2 not detected

# Weighted Repeated Median Regression

(Fried, Einbeck, Gather, 2007)

Sample in one time window  $\mathbf{y}_t = (y_{t+i})'$

$$\begin{aligned}\hat{\beta}_t^{\text{RM}} &= \text{med}_i \left\{ \text{med}_{j \neq i} \left\{ \frac{y_{t+i} - y_{t+j}}{i - j} \right\} \right\} \\ \hat{\mu}_t^{\text{RM}} &= \text{med}_i \left\{ (y_{t+i} + \hat{\beta}_t^{\text{RM}} \cdot i) \right\}\end{aligned}$$

with  $i = \begin{cases} -n + 1, \dots, -1, 0 & \text{(online)} \\ -m, \dots, m \text{ and } n = 2m + 1 & \text{(with delay)} \end{cases}$

# Weighted Repeated Median Regression

(Fried, Einbeck, Gather, 2007)

Sample in one time window  $\mathbf{y}_t = (y_{t+i})'$

$$\hat{\beta}_t^{\text{RM}} = \text{med}_i w_i \diamond \left\{ \text{med}_{j \neq i} w_j \diamond \left\{ \frac{y_{t+i} - y_{t+j}}{i - j} \right\} \right\}$$
$$\hat{\mu}_t^{\text{RM}} = \text{med}_i \left\{ w_i \diamond (y_{t+i} + \hat{\beta}_t^{\text{RM}} \cdot i) \right\}$$

with  $i = \begin{cases} -n + 1, \dots, -1, 0 & \text{(online)} \\ -m, \dots, m \text{ and } n = 2m + 1 & \text{(with delay)} \end{cases}$

$w \diamond a$  denotes the set of  $w$  replications of  $a$ :  $3 \diamond a = \{a, a, a\}$ .

# Weighted Repeated Median Regression

(Fried, Einbeck, Gather, 2007)

**Generally:** Sample  $(\mathbf{x}, \mathbf{y}) = (x_i, y_i)'$ ,  $i = 1, \dots, n$

$$\hat{\beta}^{\text{RM}} = \text{med}_i w_i \diamond \left\{ \text{med}_{j \neq i} w_j \diamond \left\{ \frac{y_i - y_j}{x_i - x_j} \right\} \right\}$$
$$\hat{\mu}^{\text{RM}} = \text{med}_i \left\{ w_i \diamond (y_i + \hat{\beta}^{\text{RM}} \cdot x_i) \right\}$$

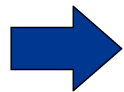


# Weighted Repeated Median Regression

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$$\hat{\mu}^{\text{RM}} = \text{med}_i \left\{ w_i \diamond (y_i + \hat{\beta}^{\text{RM}} \cdot x_i) \right\}$$



`wrm.smooth`:

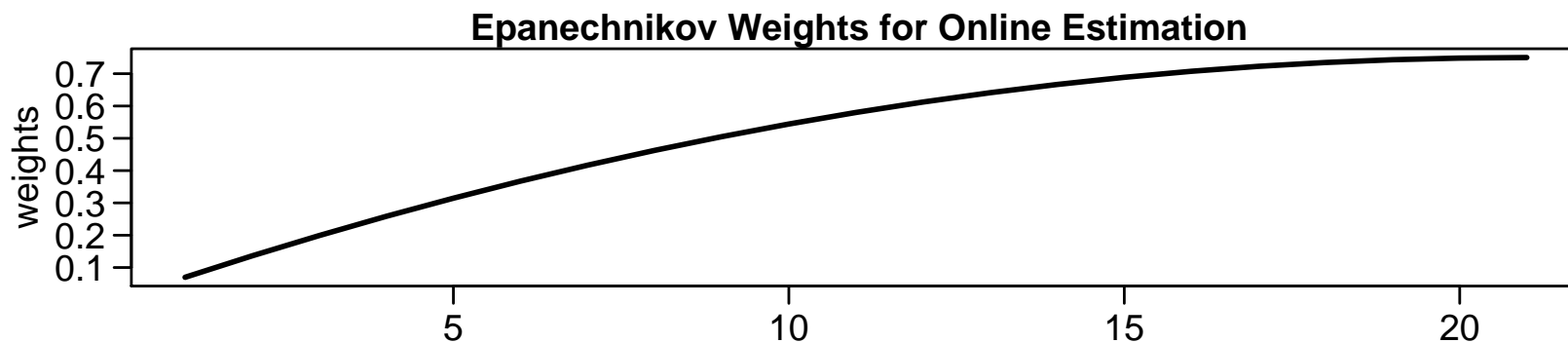
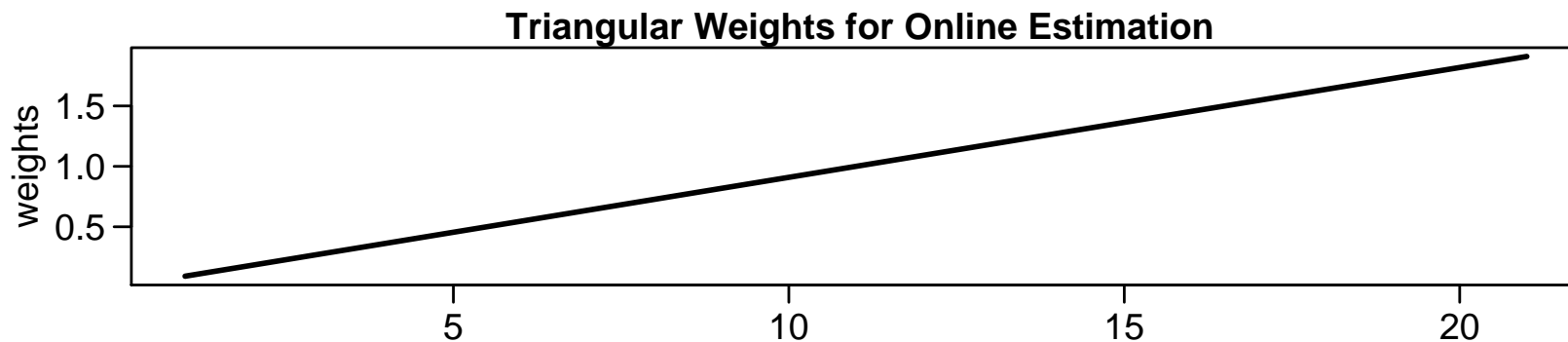
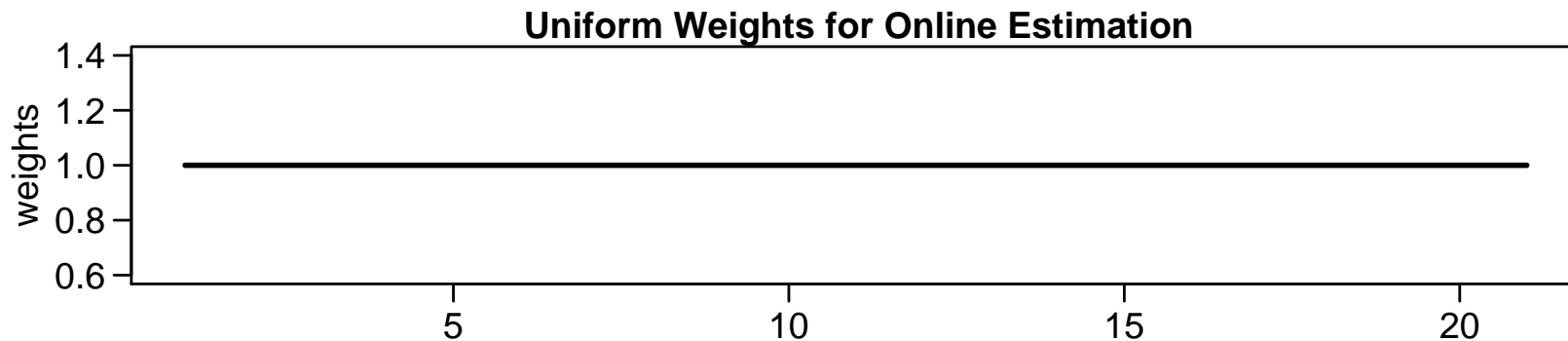
- Non-equidistant regressor variables
- Smoothing with kernel weight functions

## wrm.filter – Parameter Options

- `y` input time series (ts-object or vector)
- `width` window width ( $\in \mathbb{N}$ )
- `del` delay of the extracted signal: `del=0` means 'online', default is `del=floor(width/2)` (delayed)
- `weight` weight function:
- 0: equal weighting
  - 1: triangular weights
  - 2: Epanechnikov weights

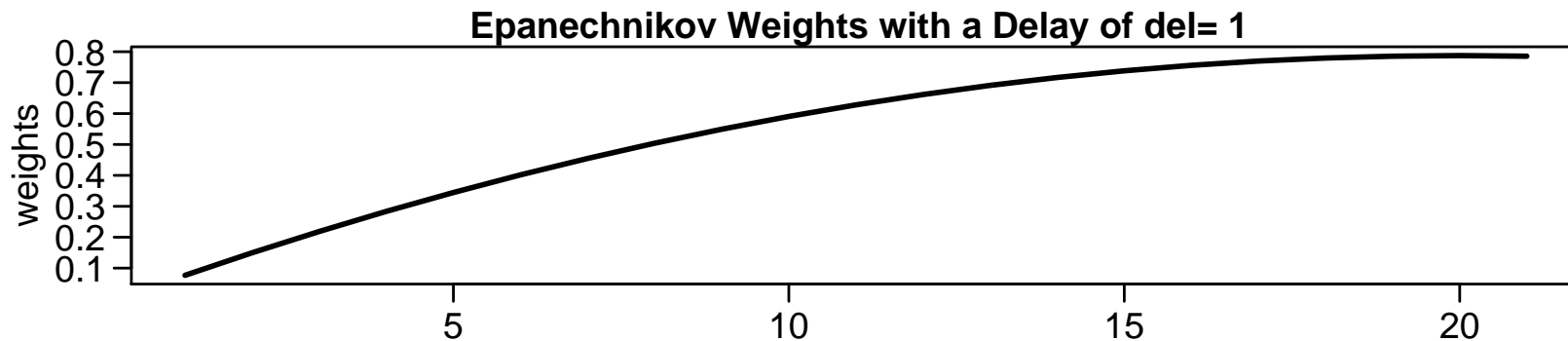
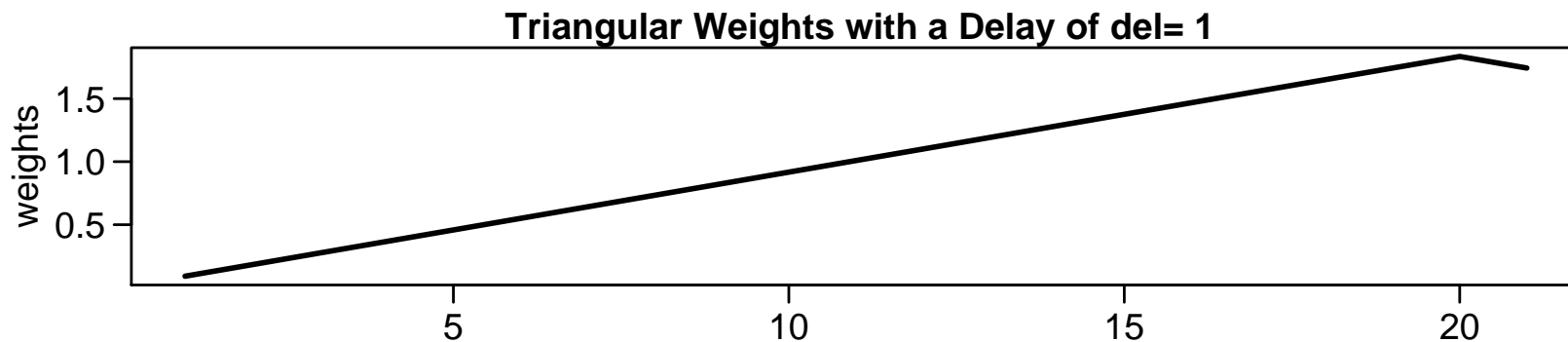
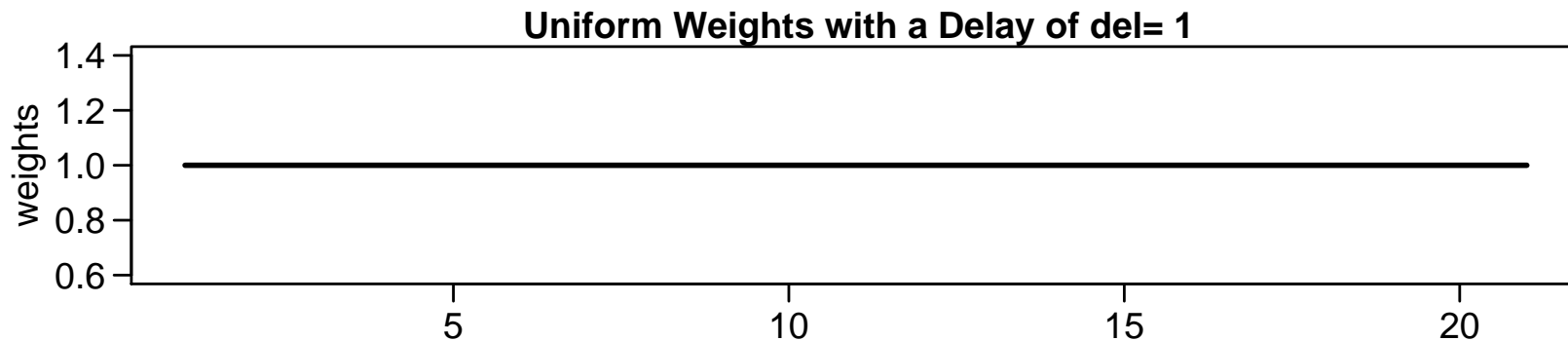
# wrm.filter – Weight Functions

Window width = 21



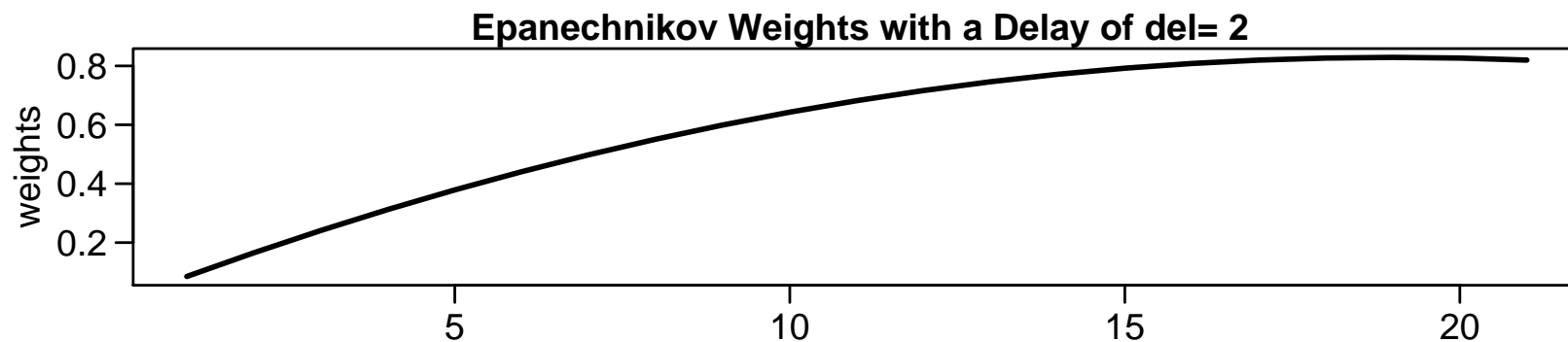
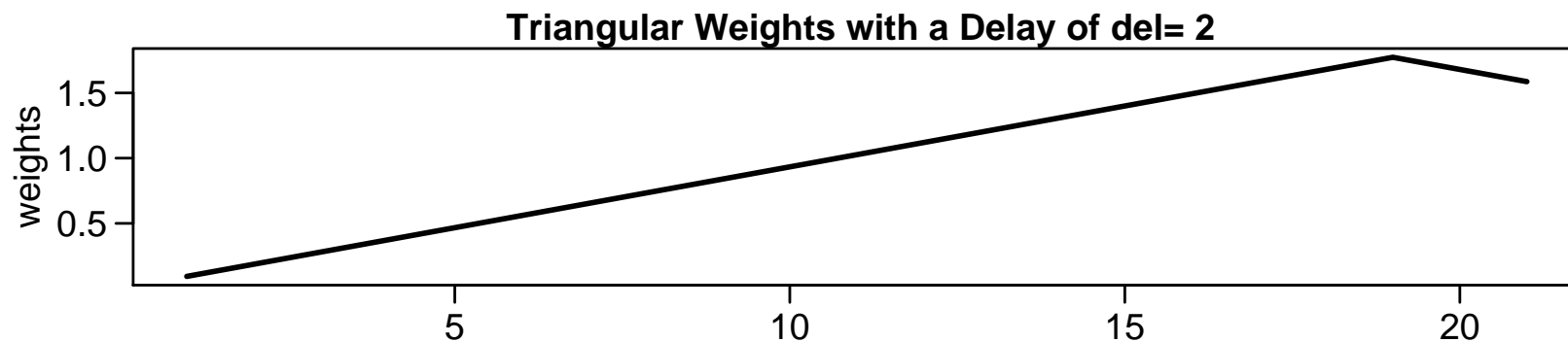
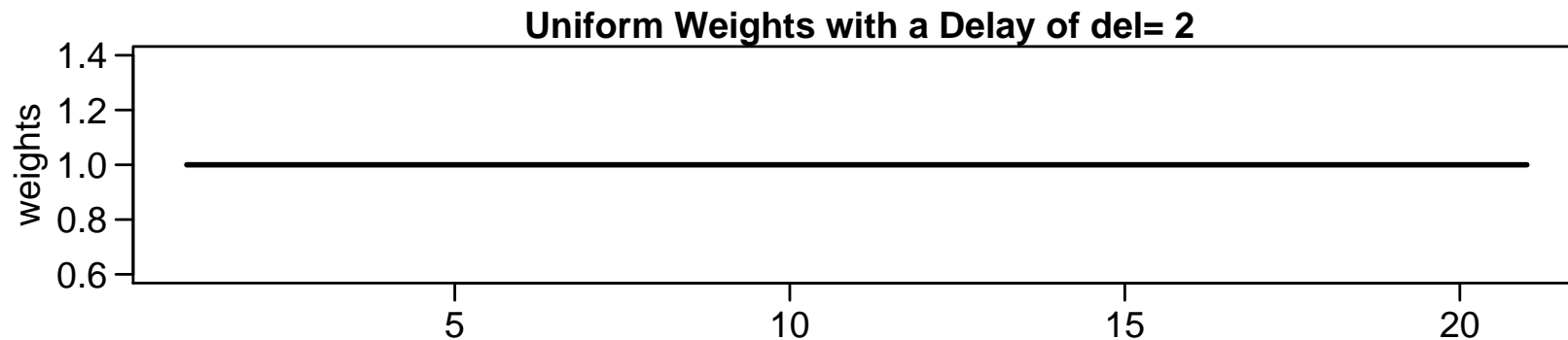
# wrm.filter – Weight Functions

Window width = 21



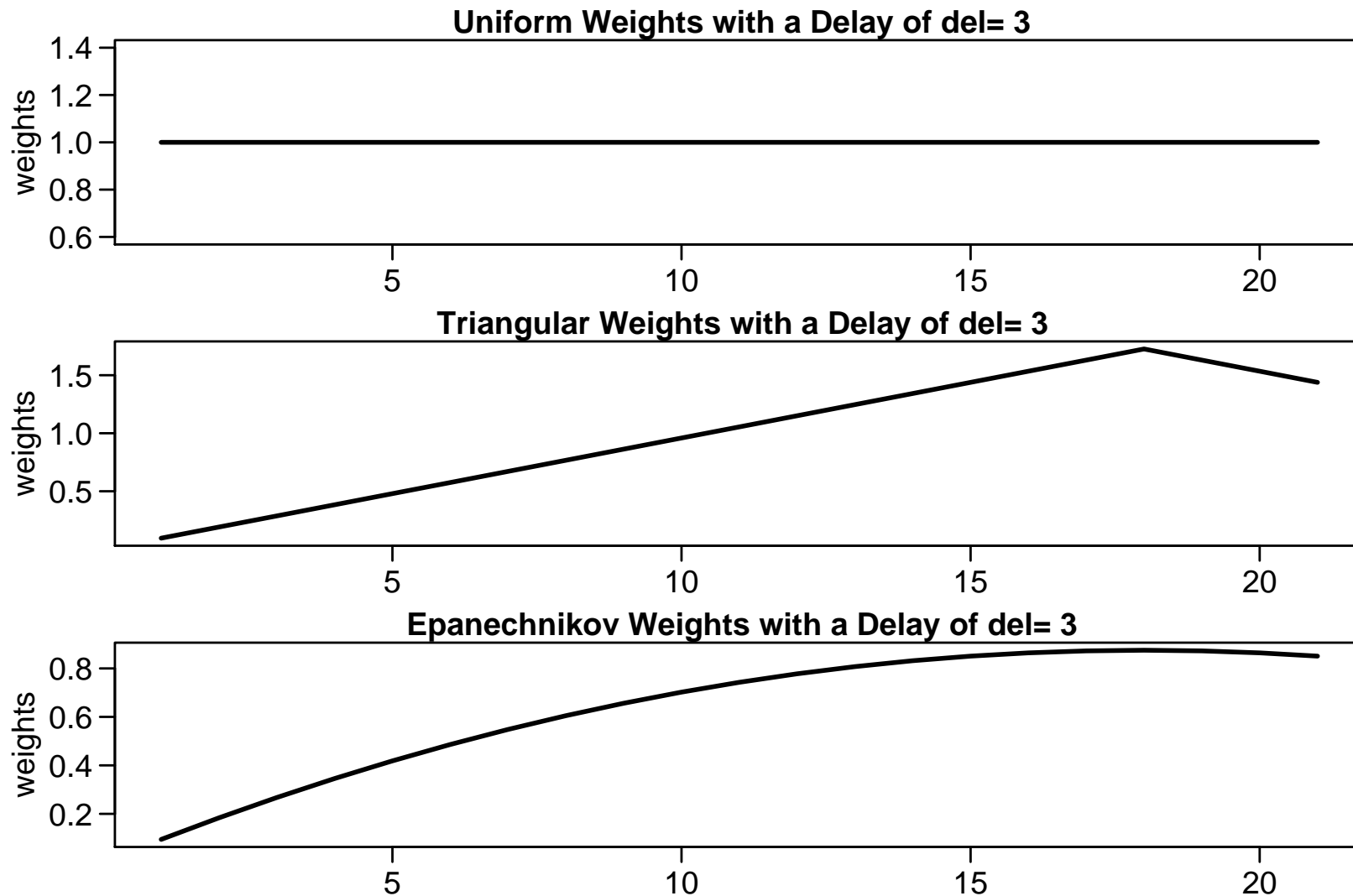
# wrm.filter – Weight Functions

Window width = 21



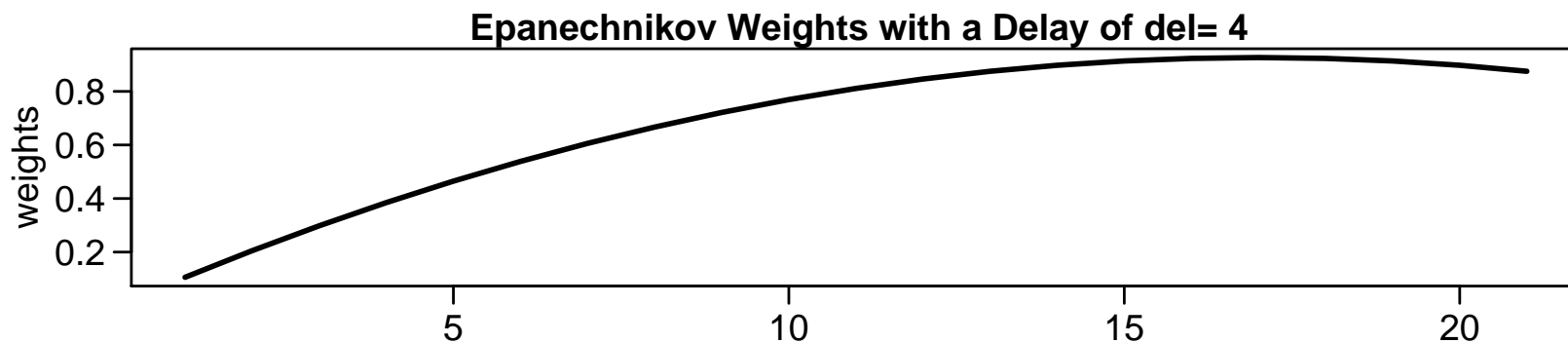
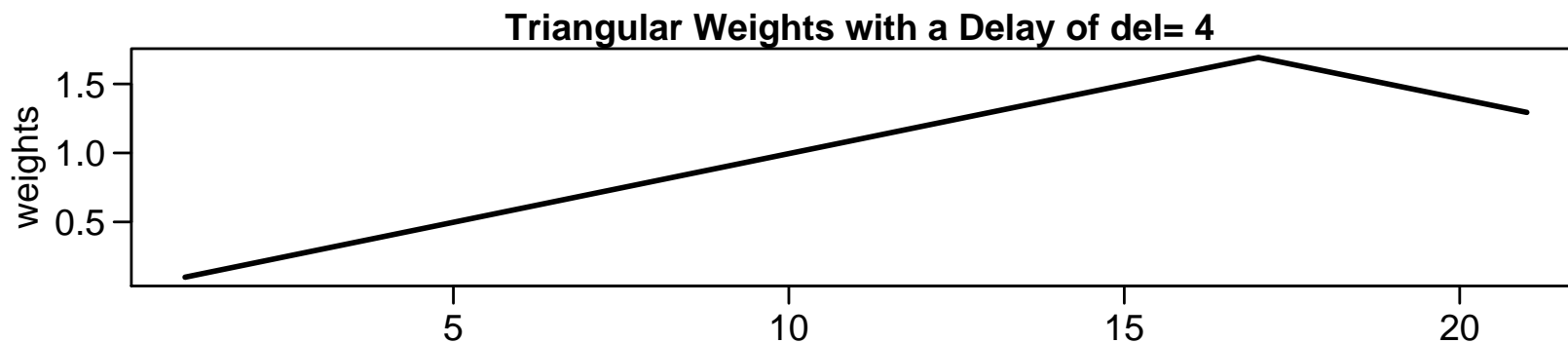
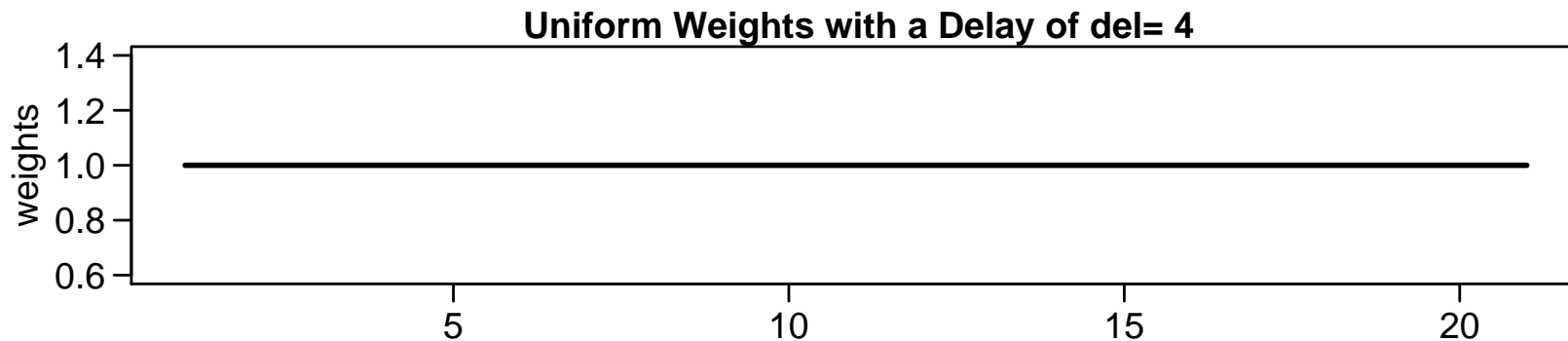
# wrm.filter – Weight Functions

Window width = 21



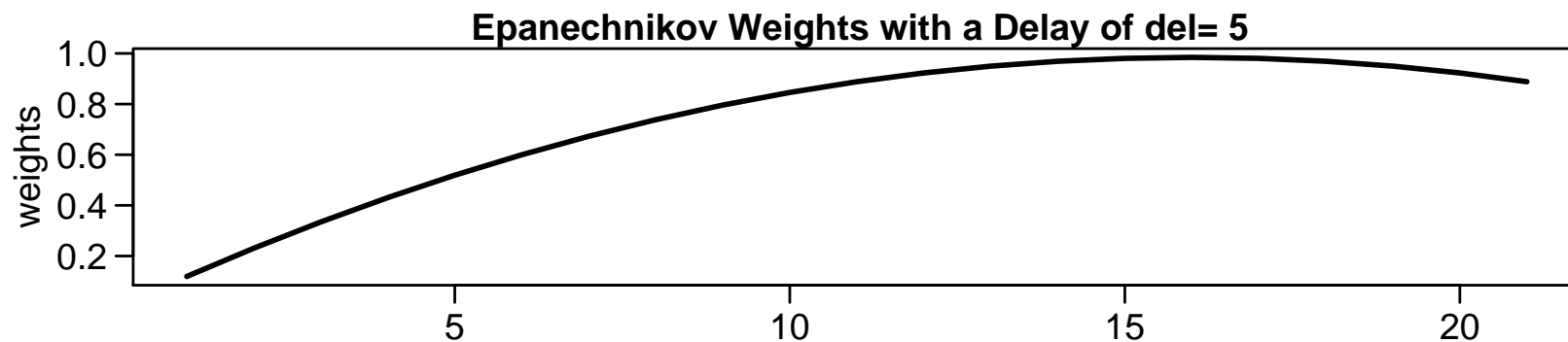
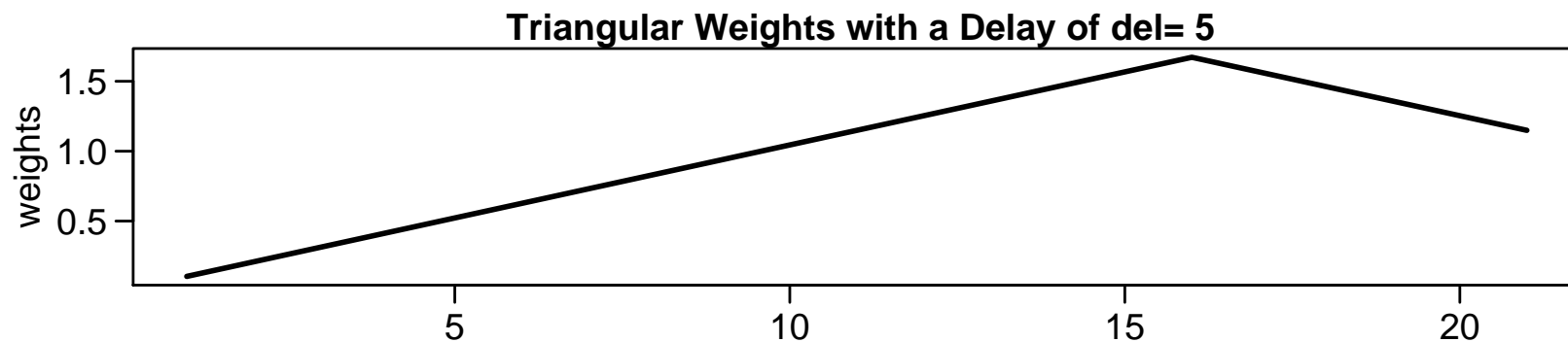
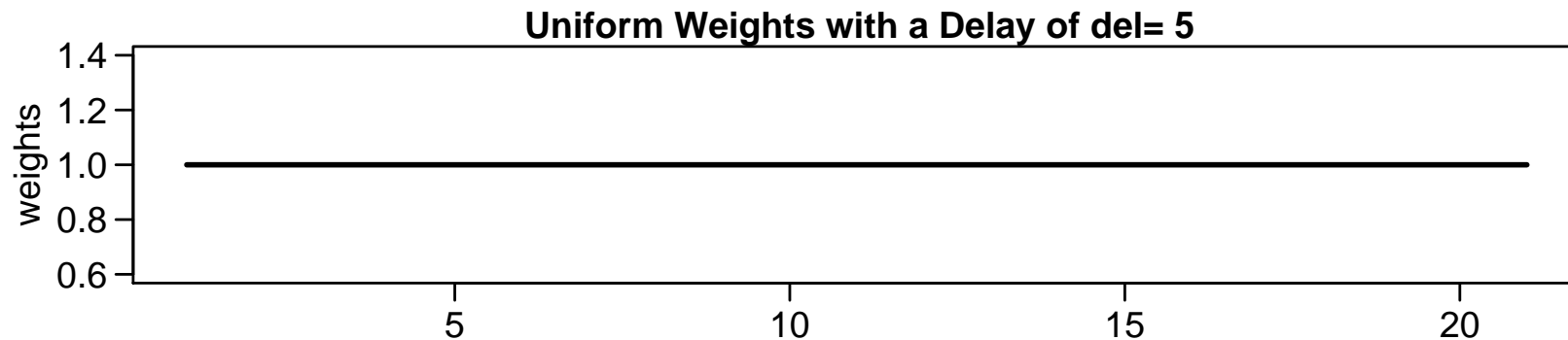
# wrm.filter – Weight Functions

Window width = 21



# wrm.filter – Weight Functions

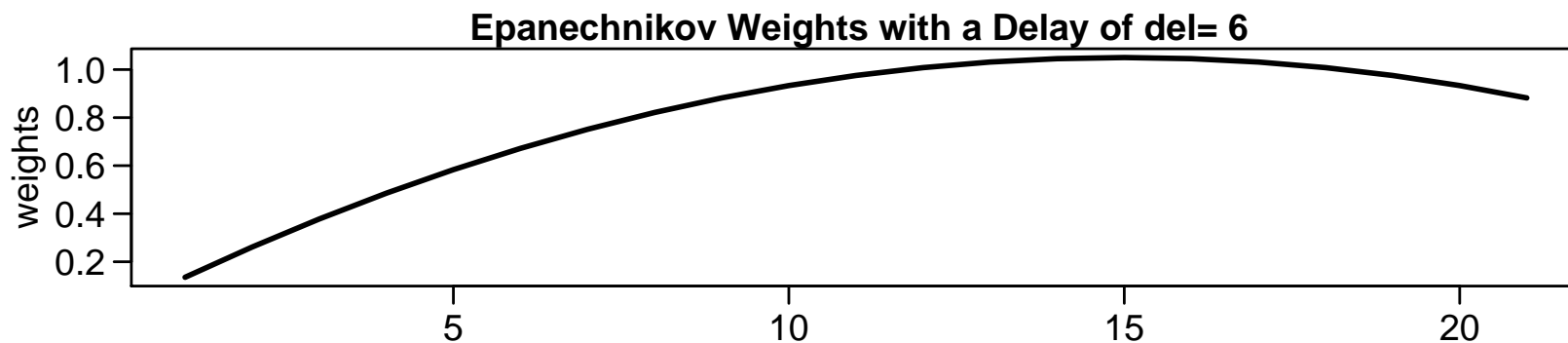
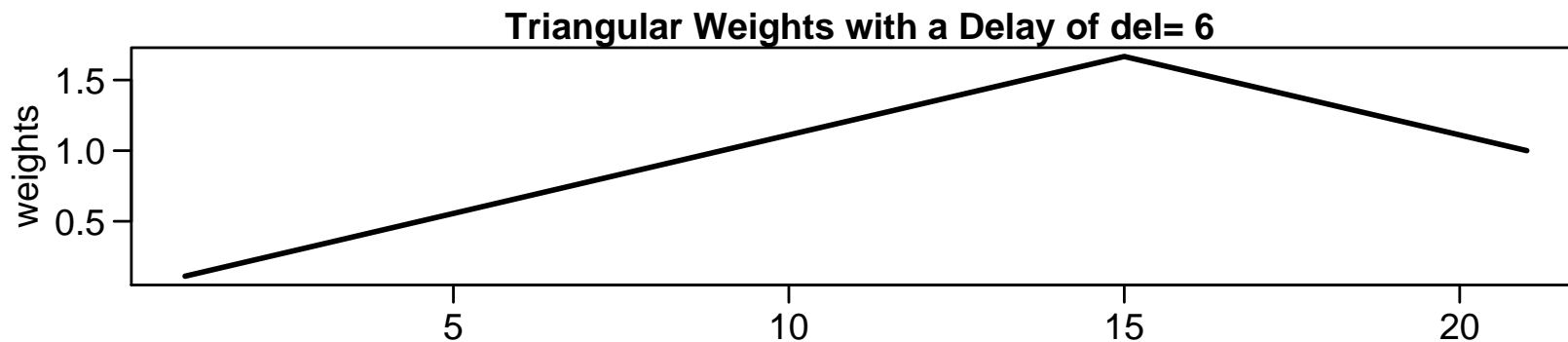
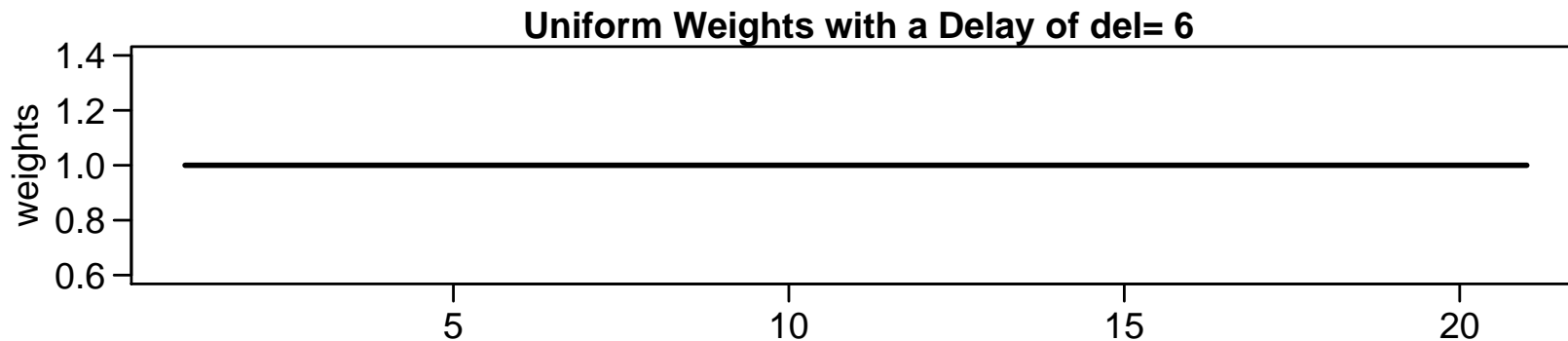
Window width = 21





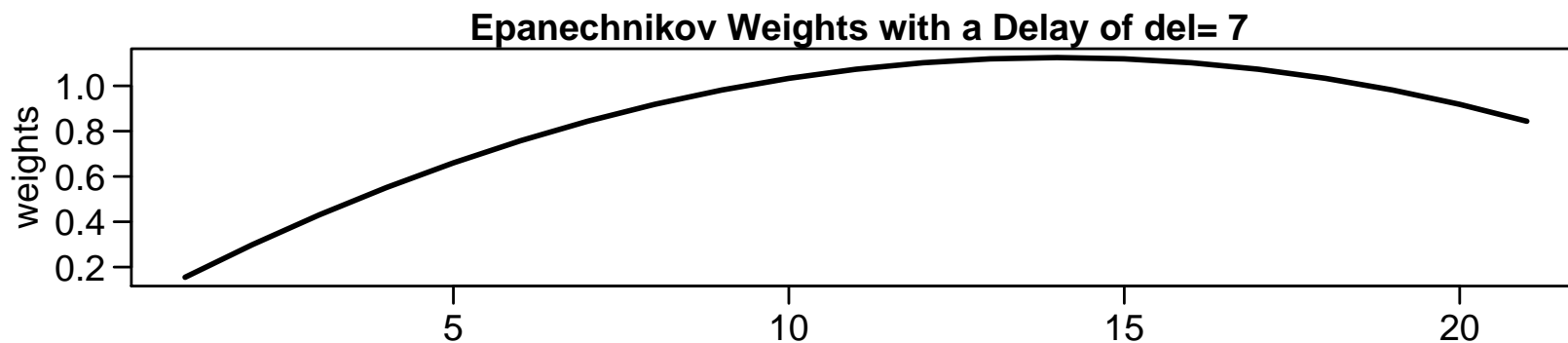
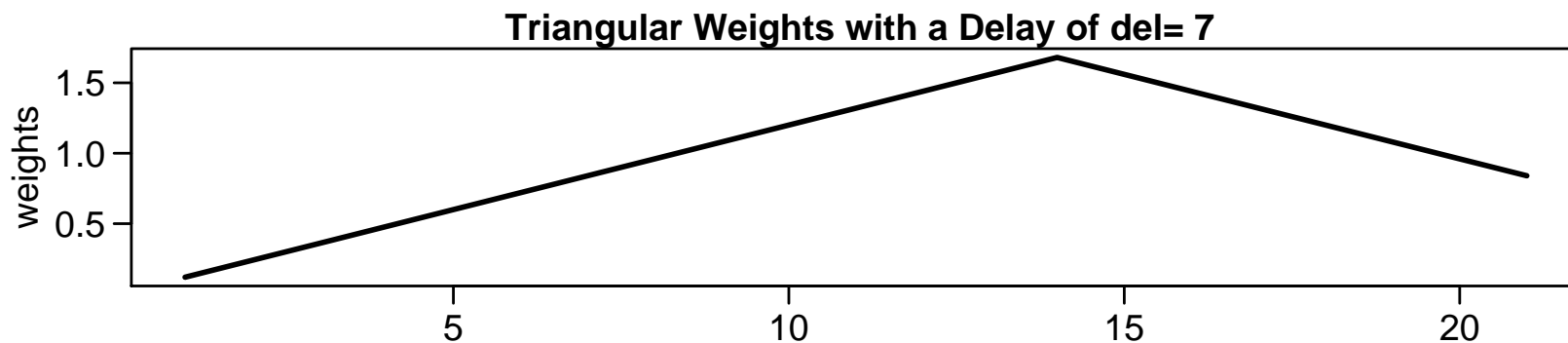
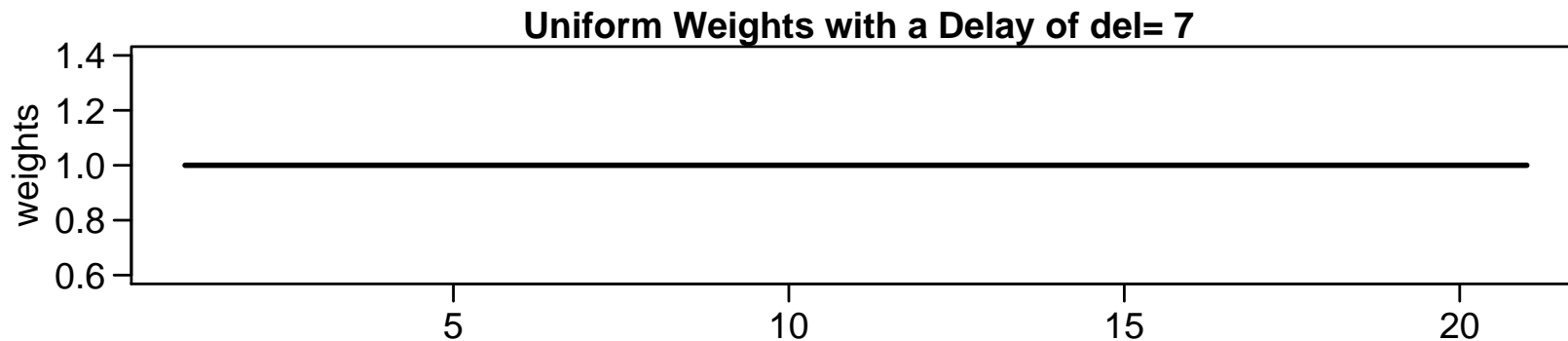
# wrm.filter – Weight Functions

Window width = 21



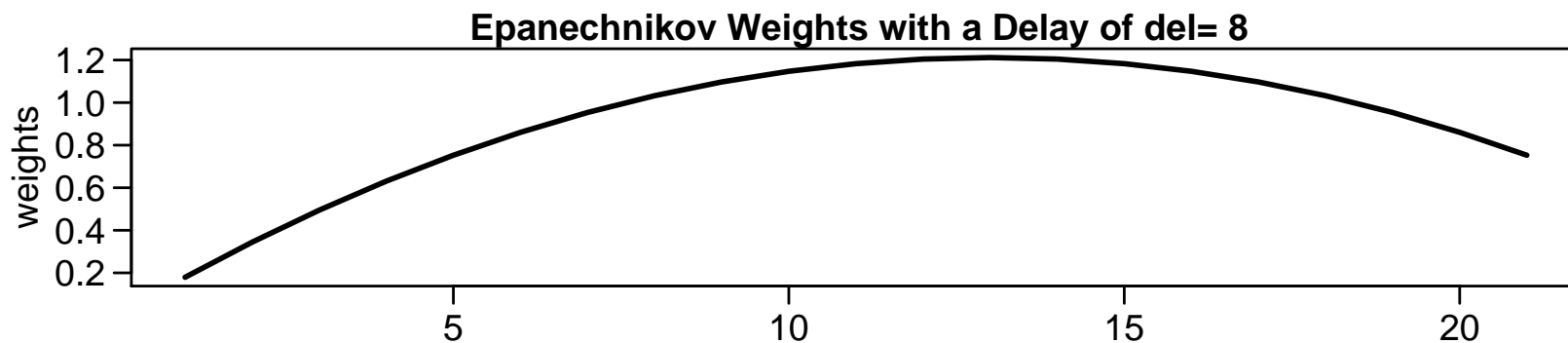
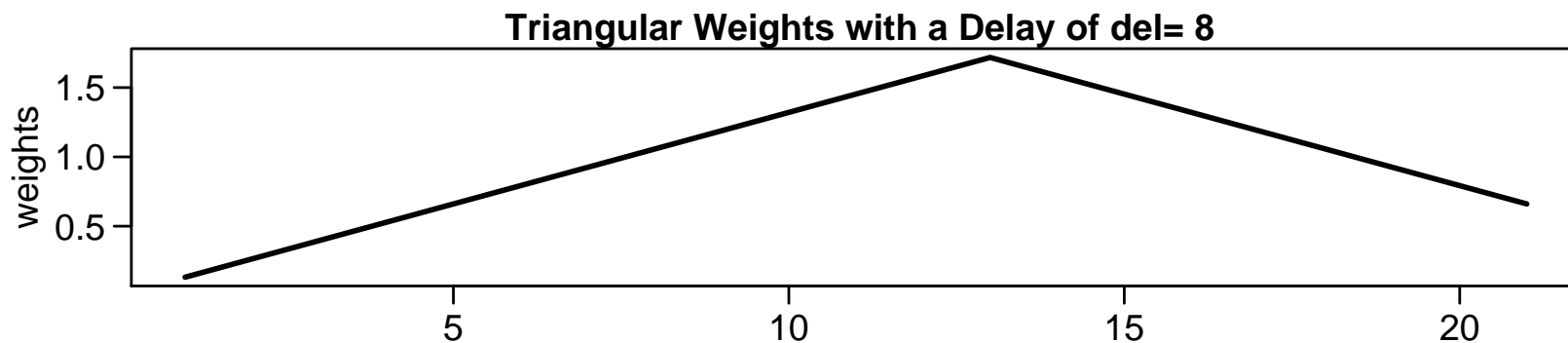
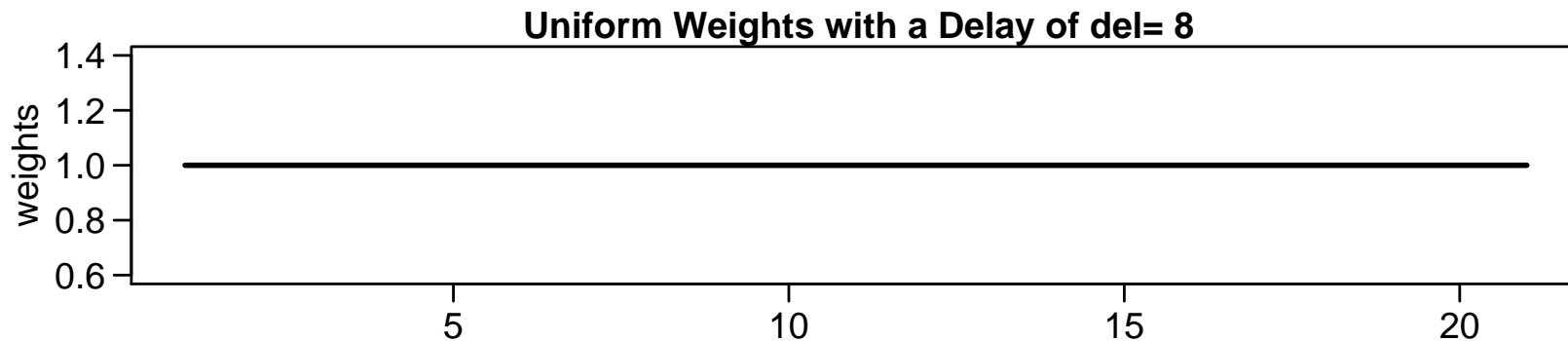
# wrm.filter – Weight Functions

Window width = 21



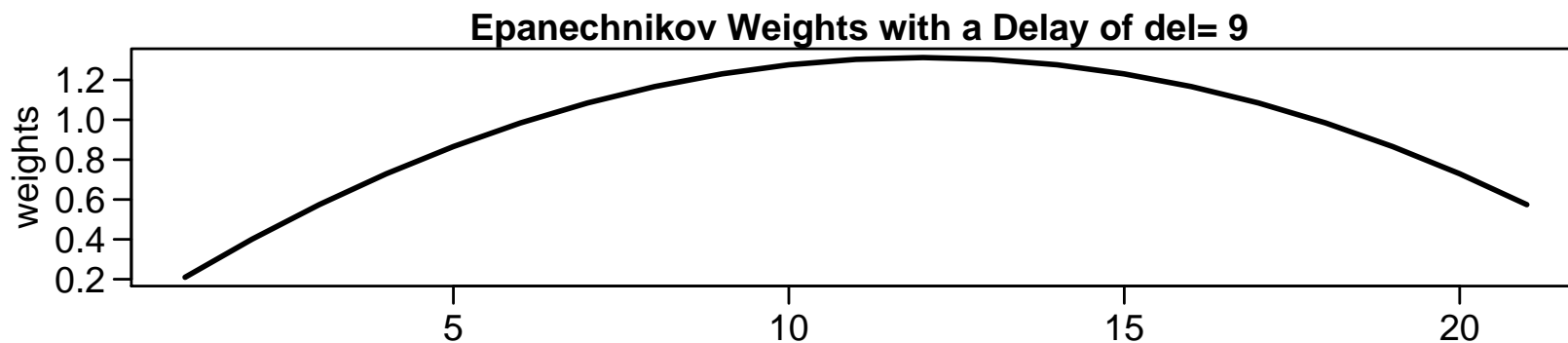
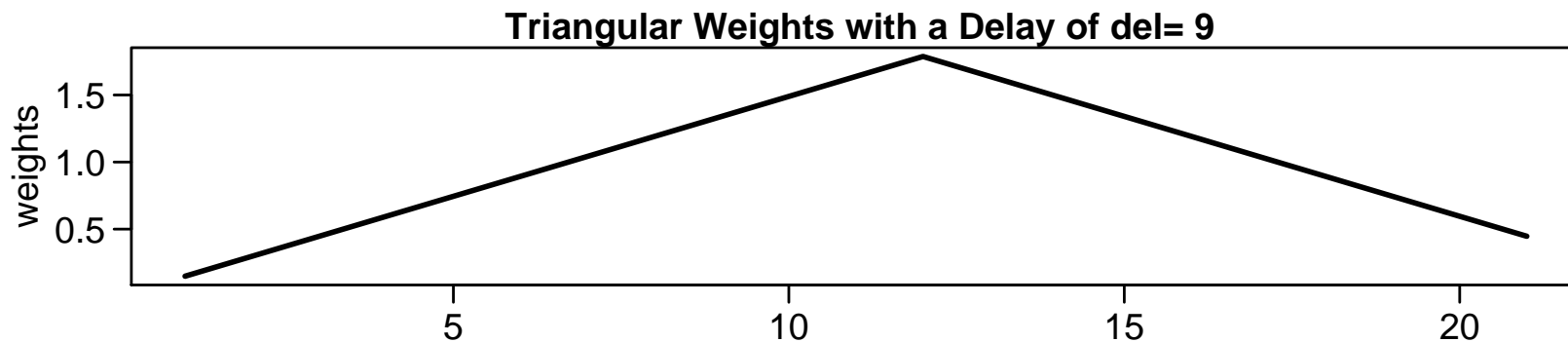
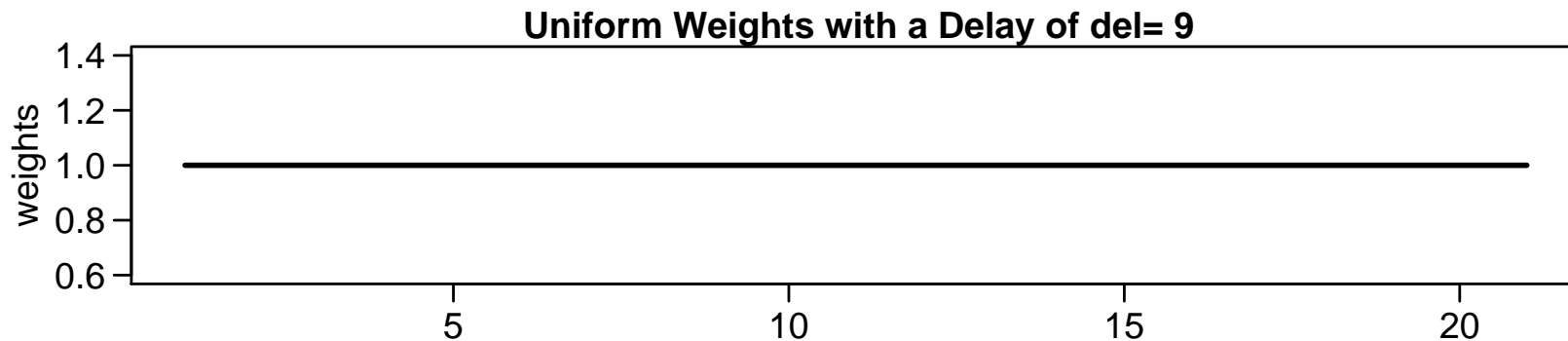
# wrm.filter – Weight Functions

Window width = 21



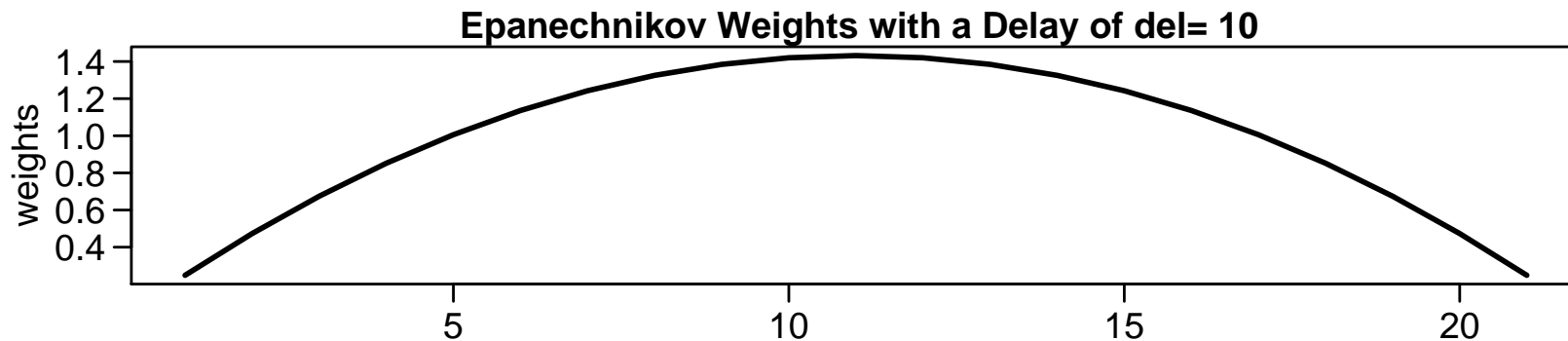
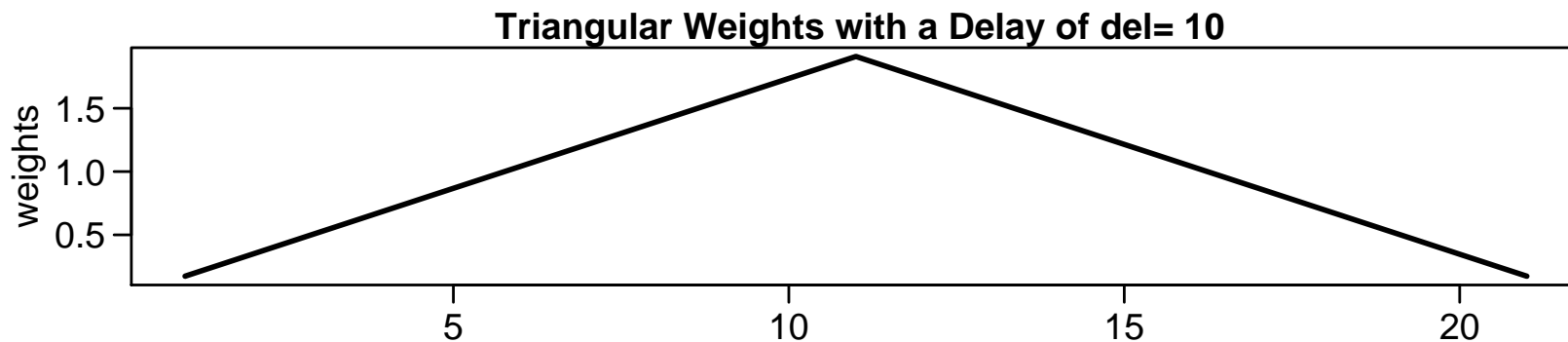
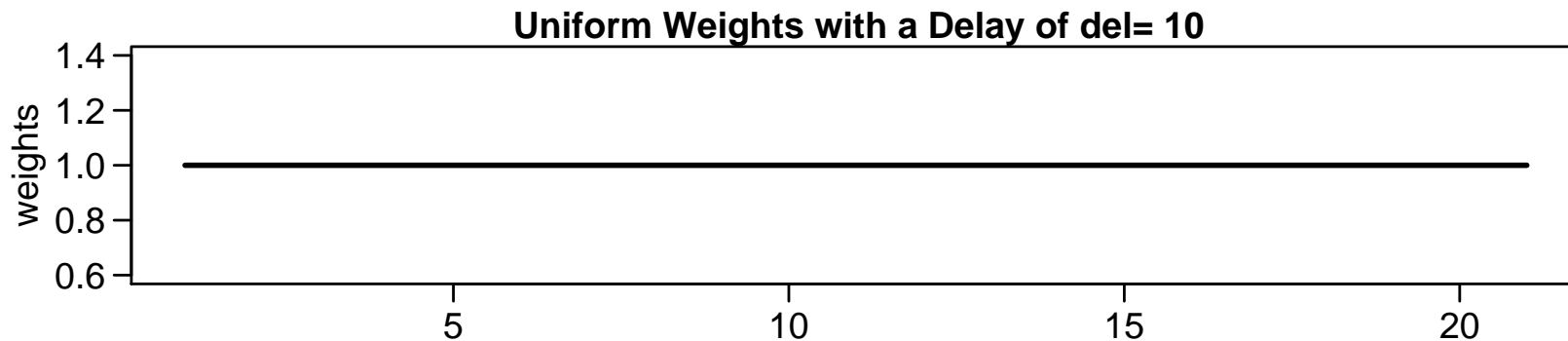
# wrm.filter – Weight Functions

Window width = 21

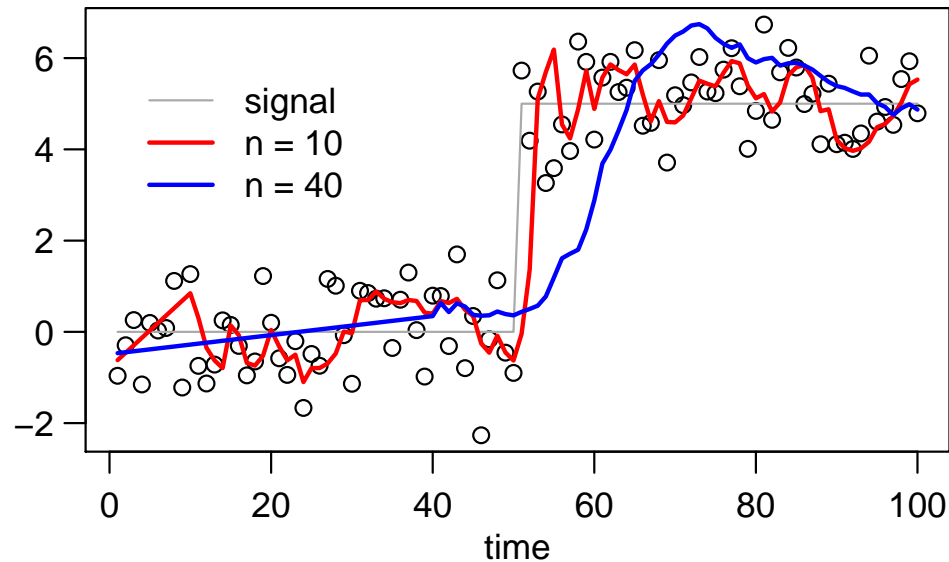


# wrm.filter – Weight Functions

Window width = 21



# Influence of the Window Width



*n* small

- + small bias
- + adapts quickly to changes
- + short computation time



*n* large

- + small variance
- + smooth
- + robust

➔ **Data adaptive choice of window width**

## adore.filter

(Schettlinger, Fried, Gather, 2008)

**adaptive online repeated median filtering:**

Window width adaptation by a test

using the 'balance' of the residual signs

$$\left( \sum_{i=1}^n \text{sign}(r_{t,i}) = 0 \right)$$

RM approximation in current time window



**Test:** Selection of positive & negative residuals balanced?

# adore.filter

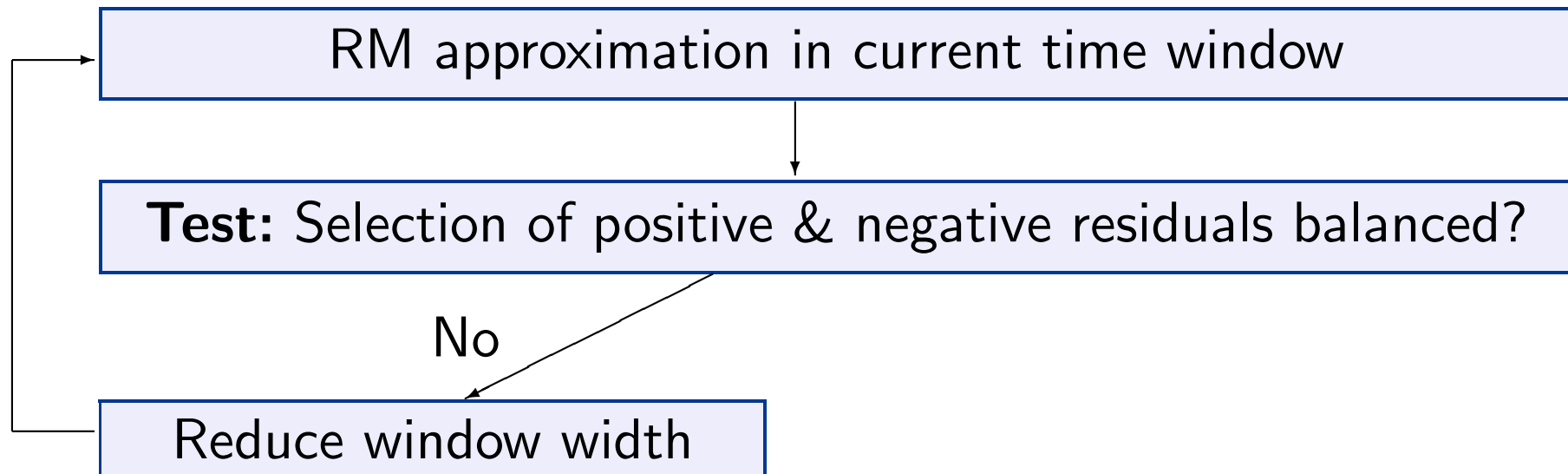
(Schettlinger, Fried, Gather, 2008)

**adaptive online repeated median filtering:**

Window width adaptation by a test

using the 'balance' of the residual signs

$$\left( \sum_{i=1}^n \text{sign}(r_{t,i}) = 0 \right)$$





# adore.filter

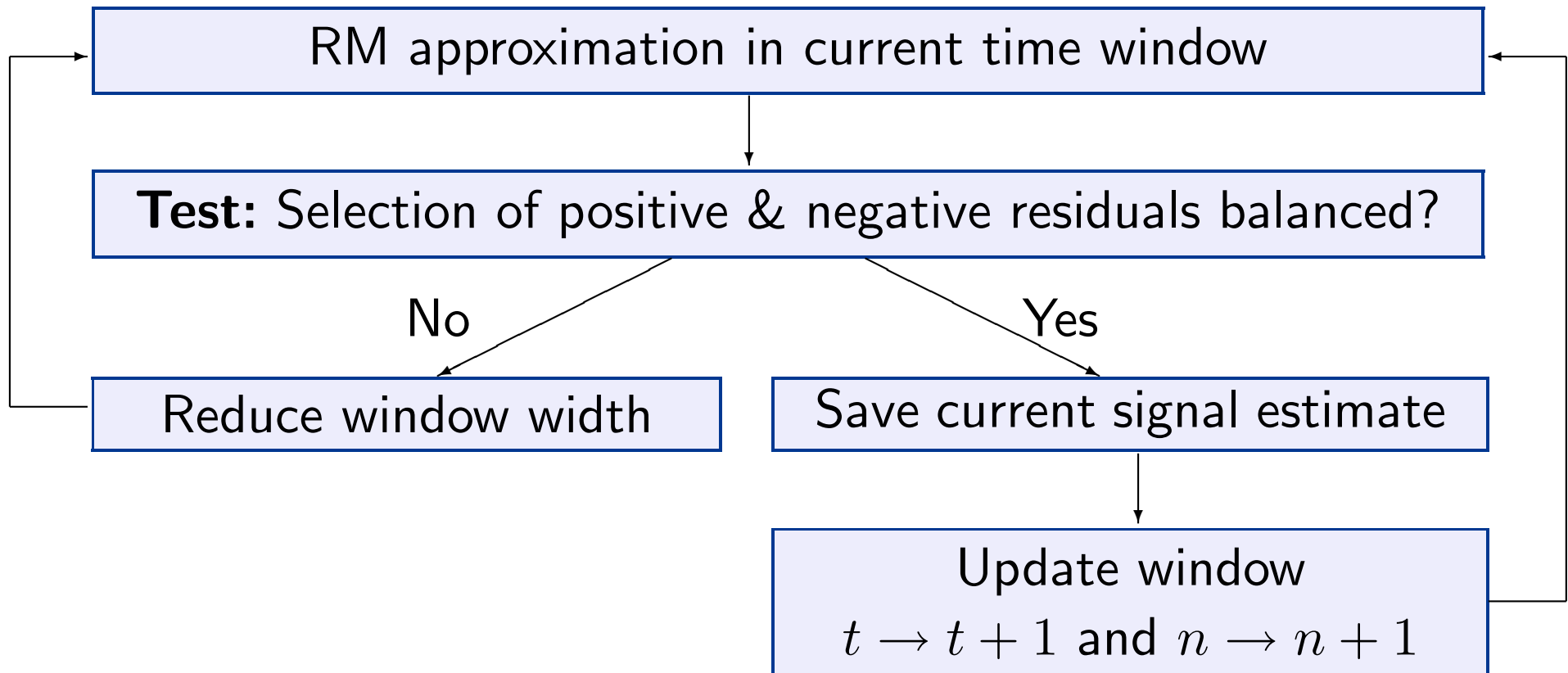
(Schettlinger, Fried, Gather, 2008)

**adaptive online repeated median filtering:**

Window width adaptation by a test

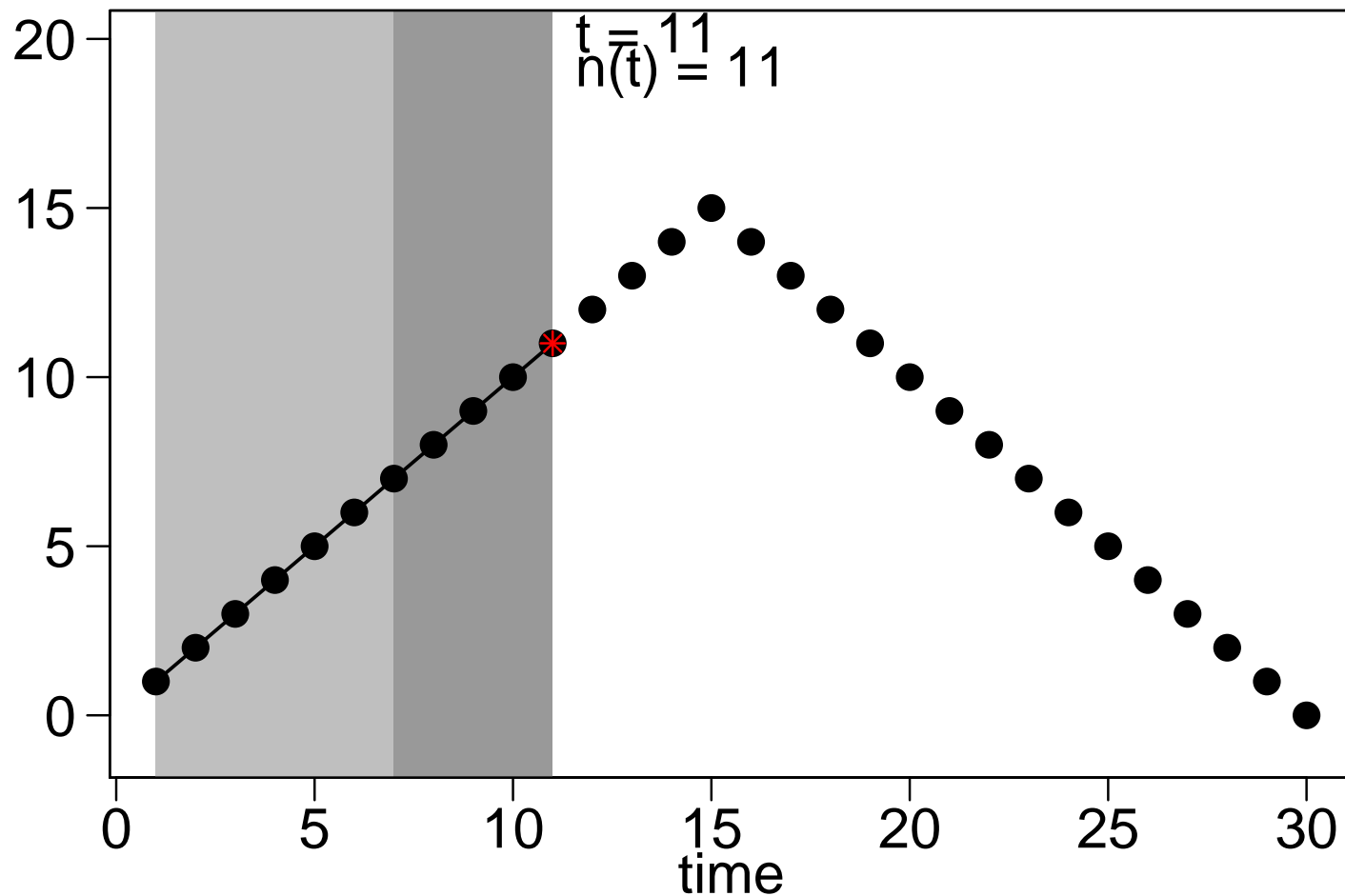
using the 'balance' of the residual signs

$$\left( \sum_{i=1}^n \text{sign}(r_{t,i}) = 0 \right)$$



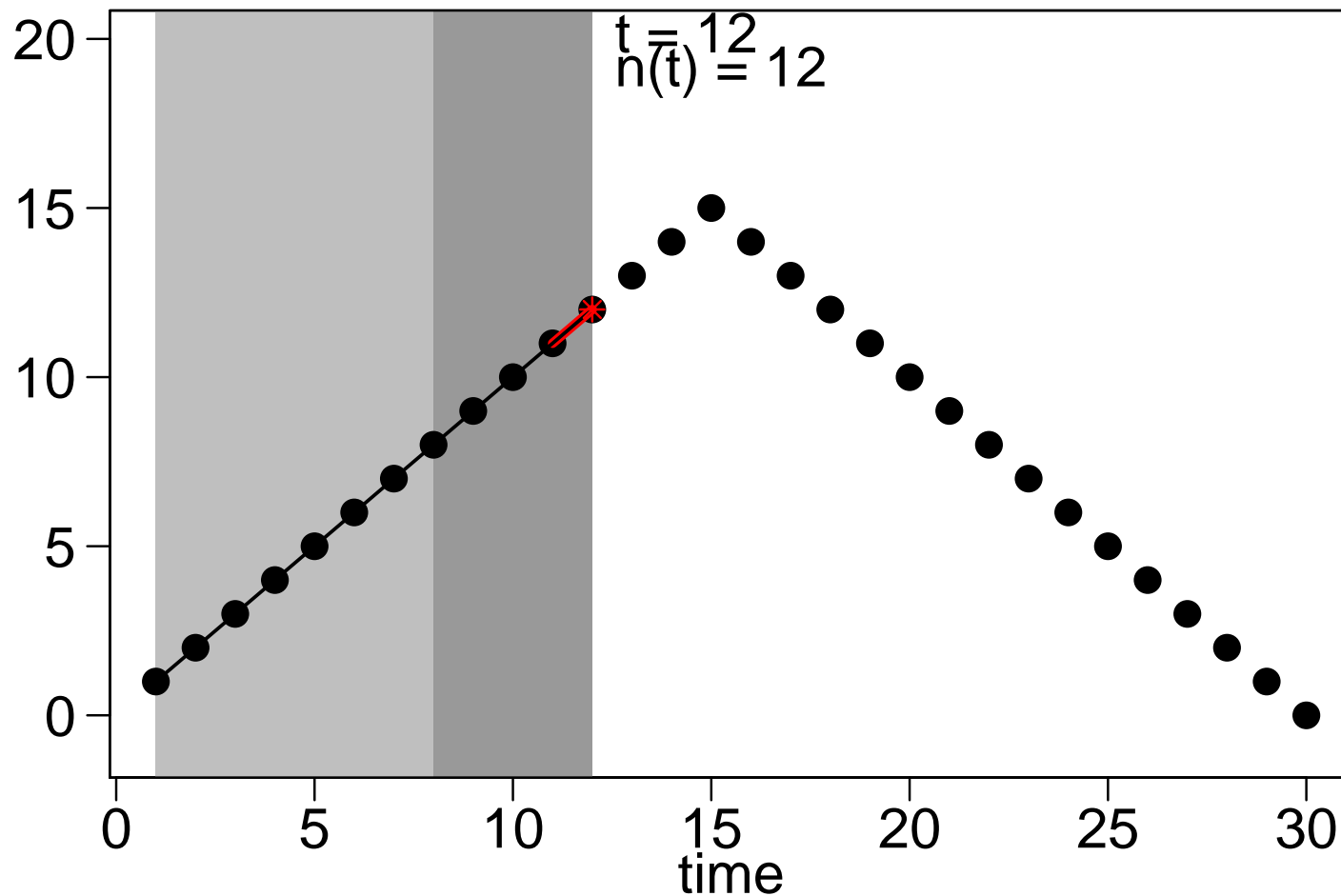
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



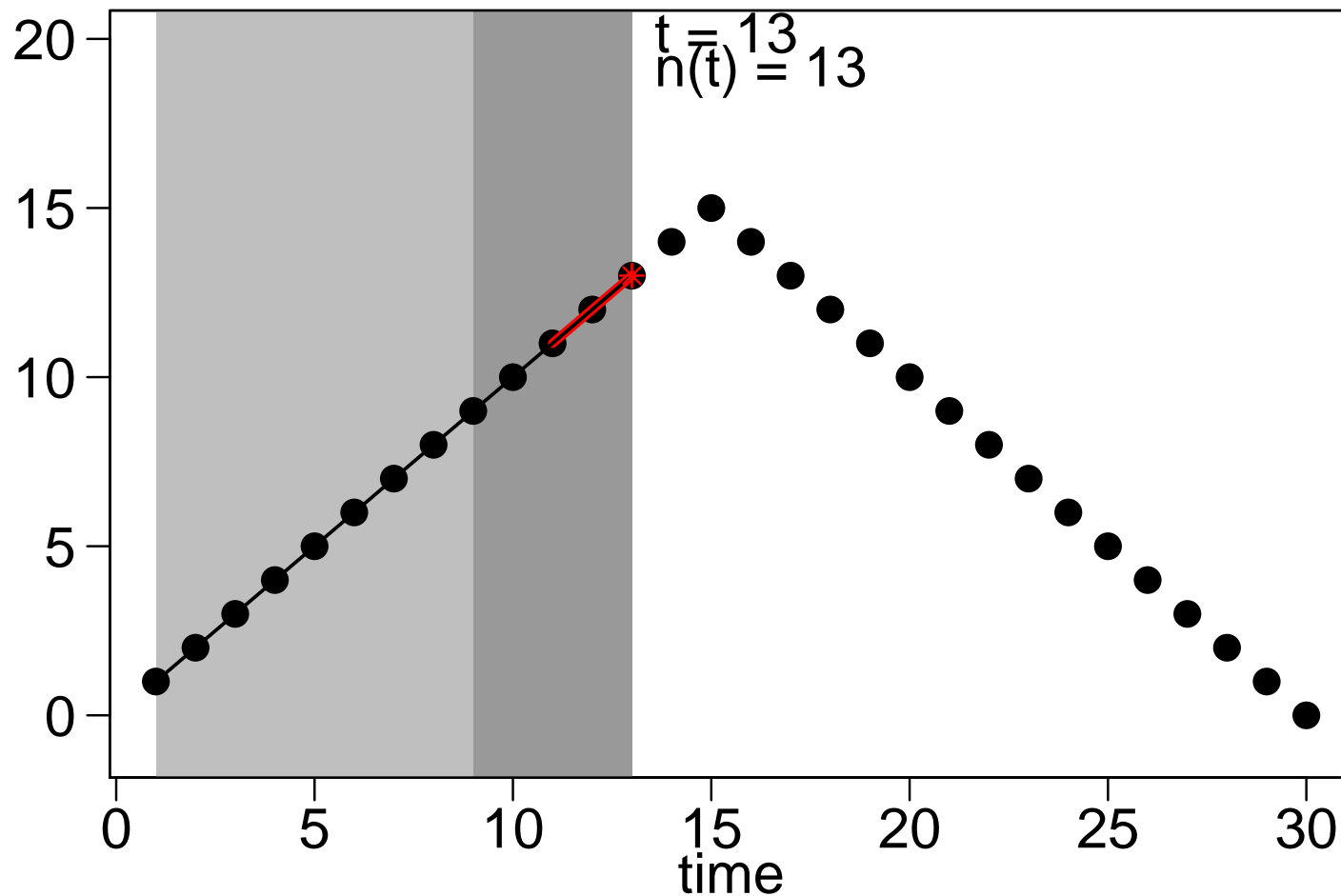
## adore.filter – Illustration

`min.width=11`    `p.test=5`    `max.width=100`



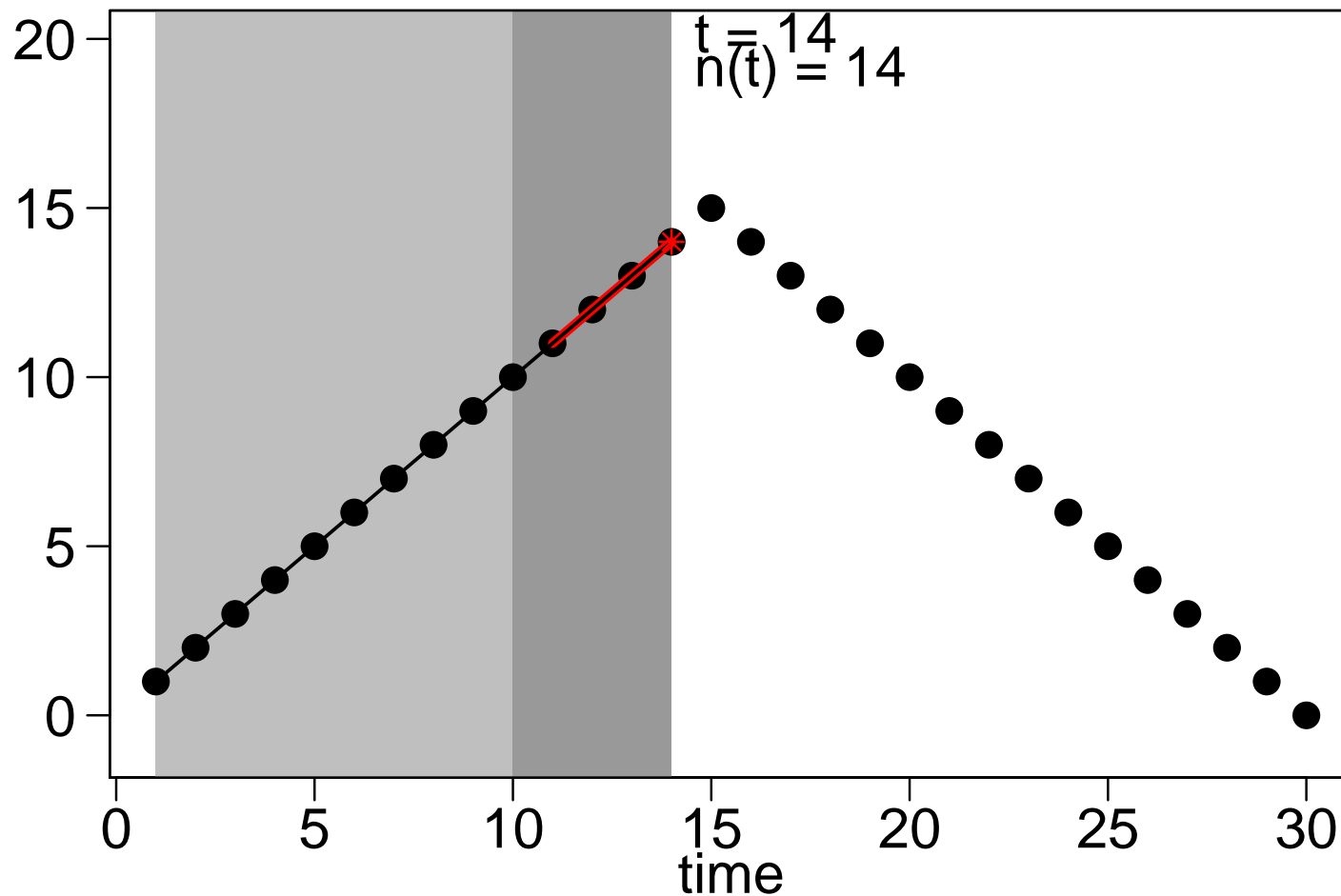
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



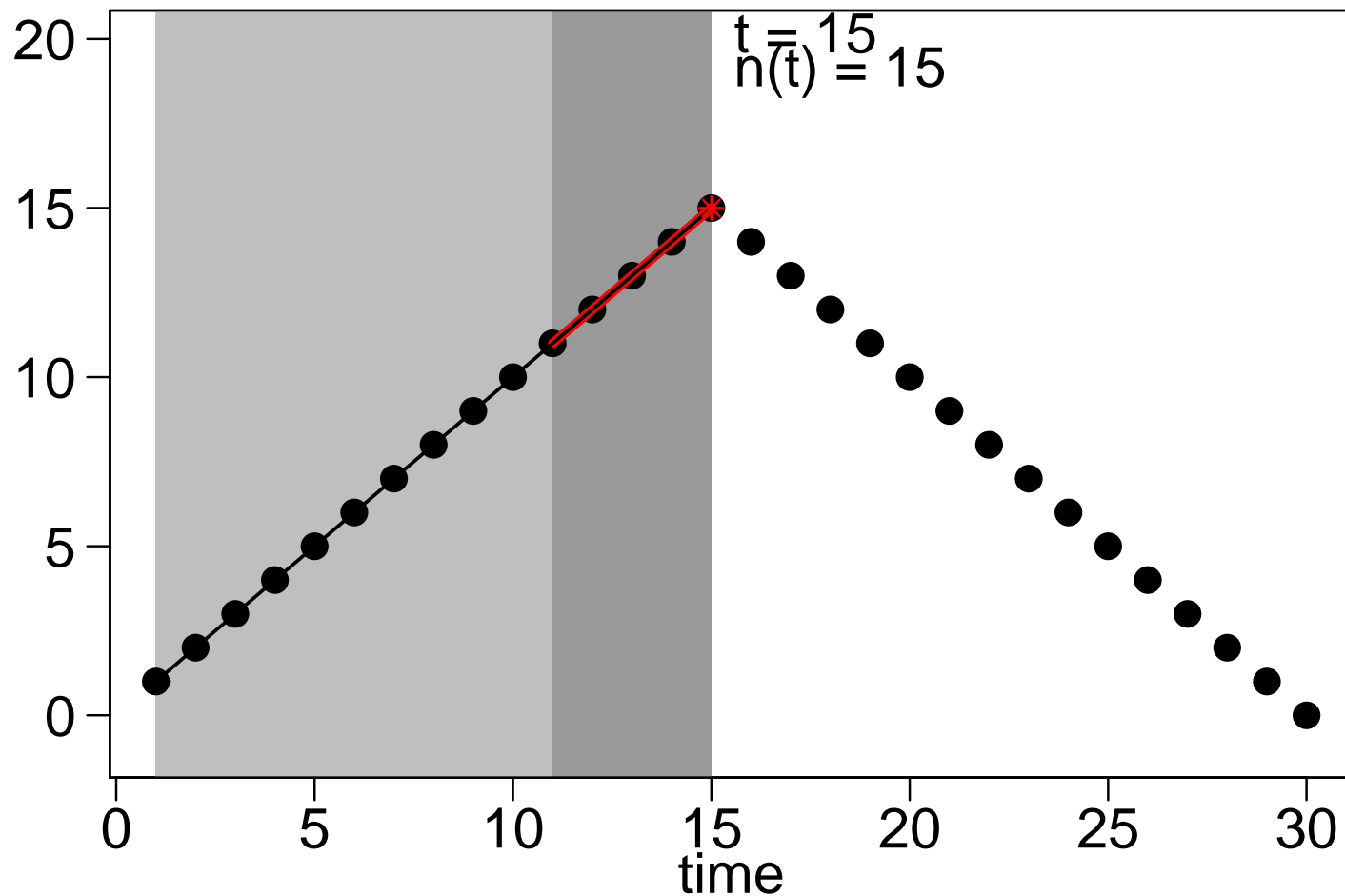
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



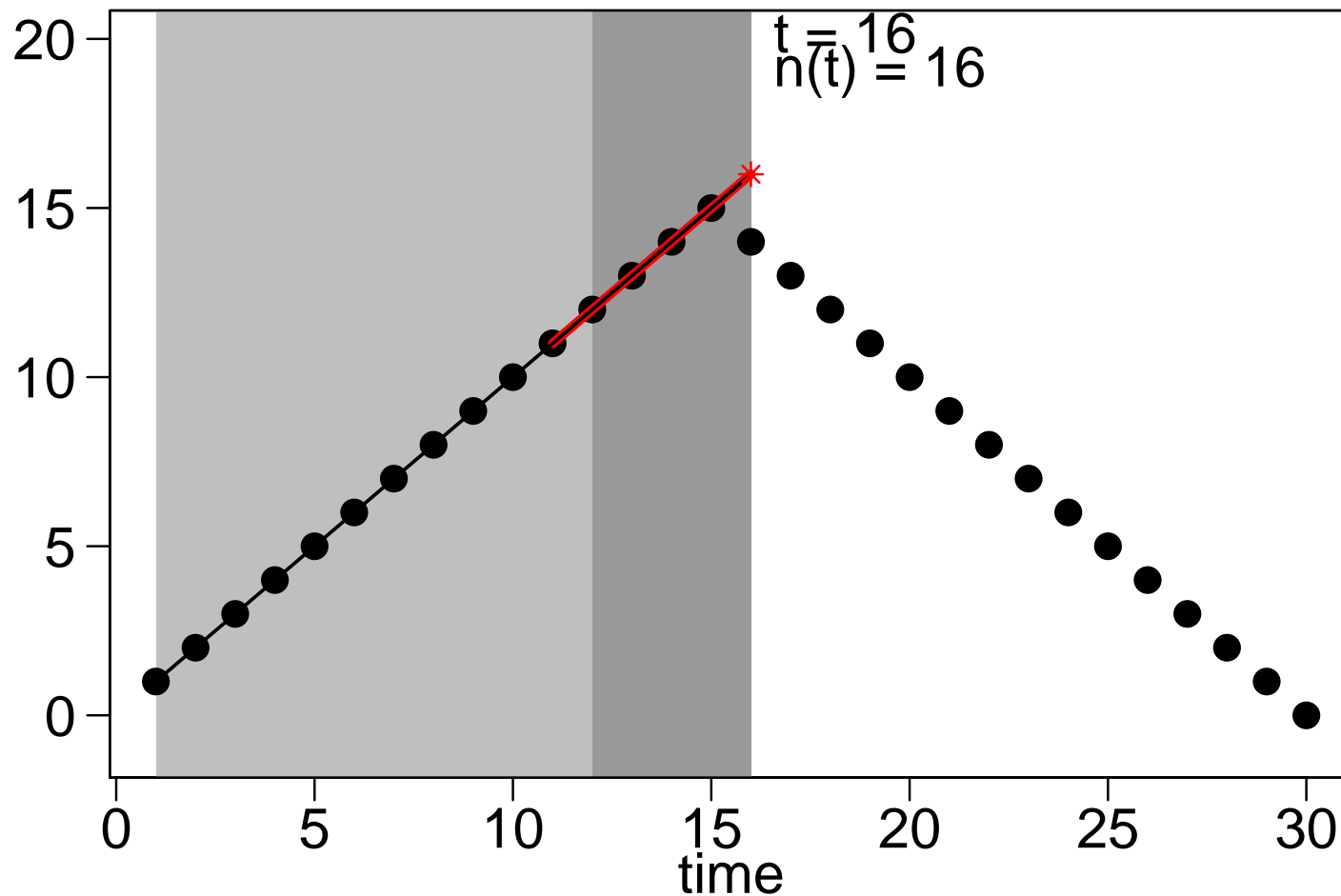
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



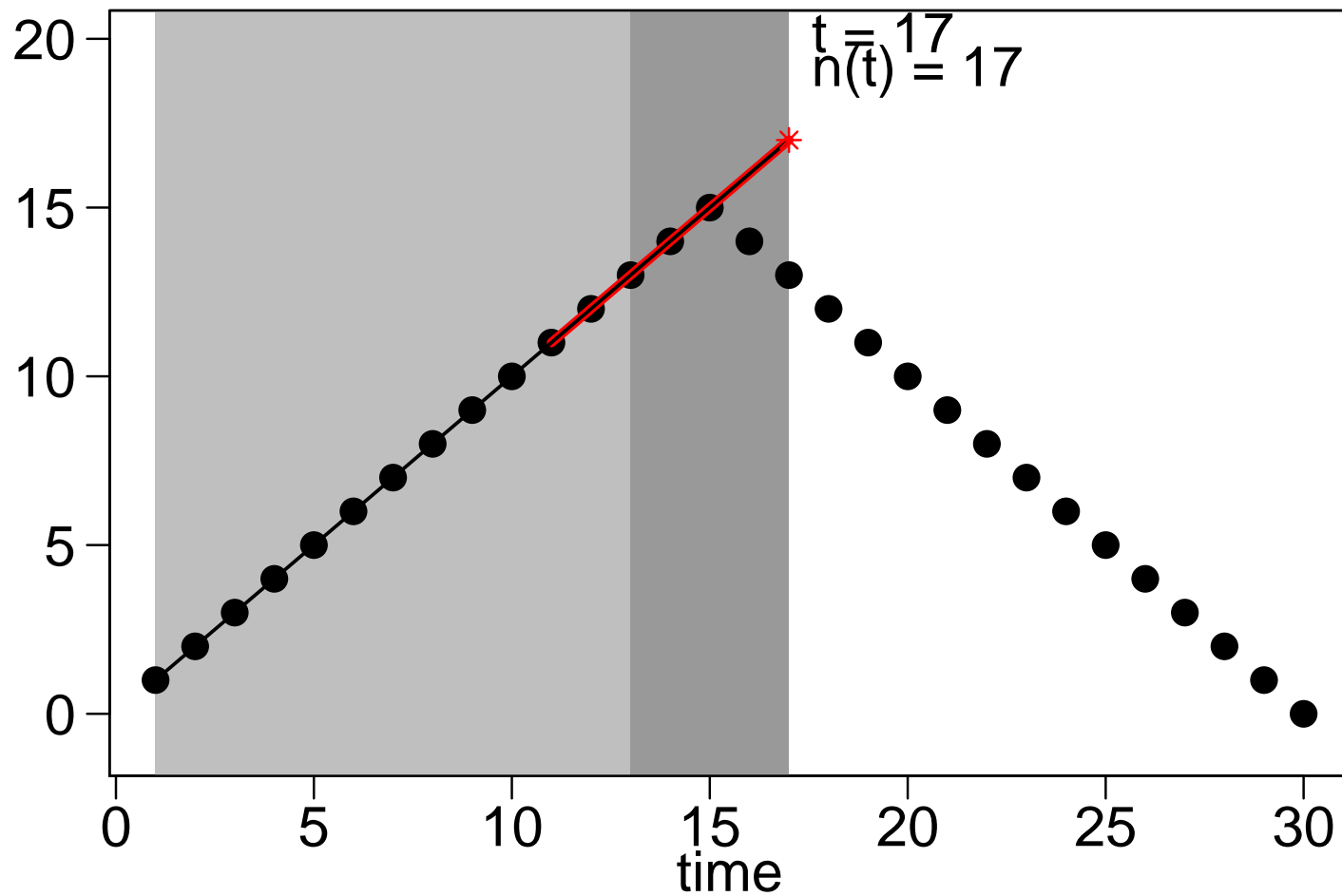
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



## adore.filter – Illustration

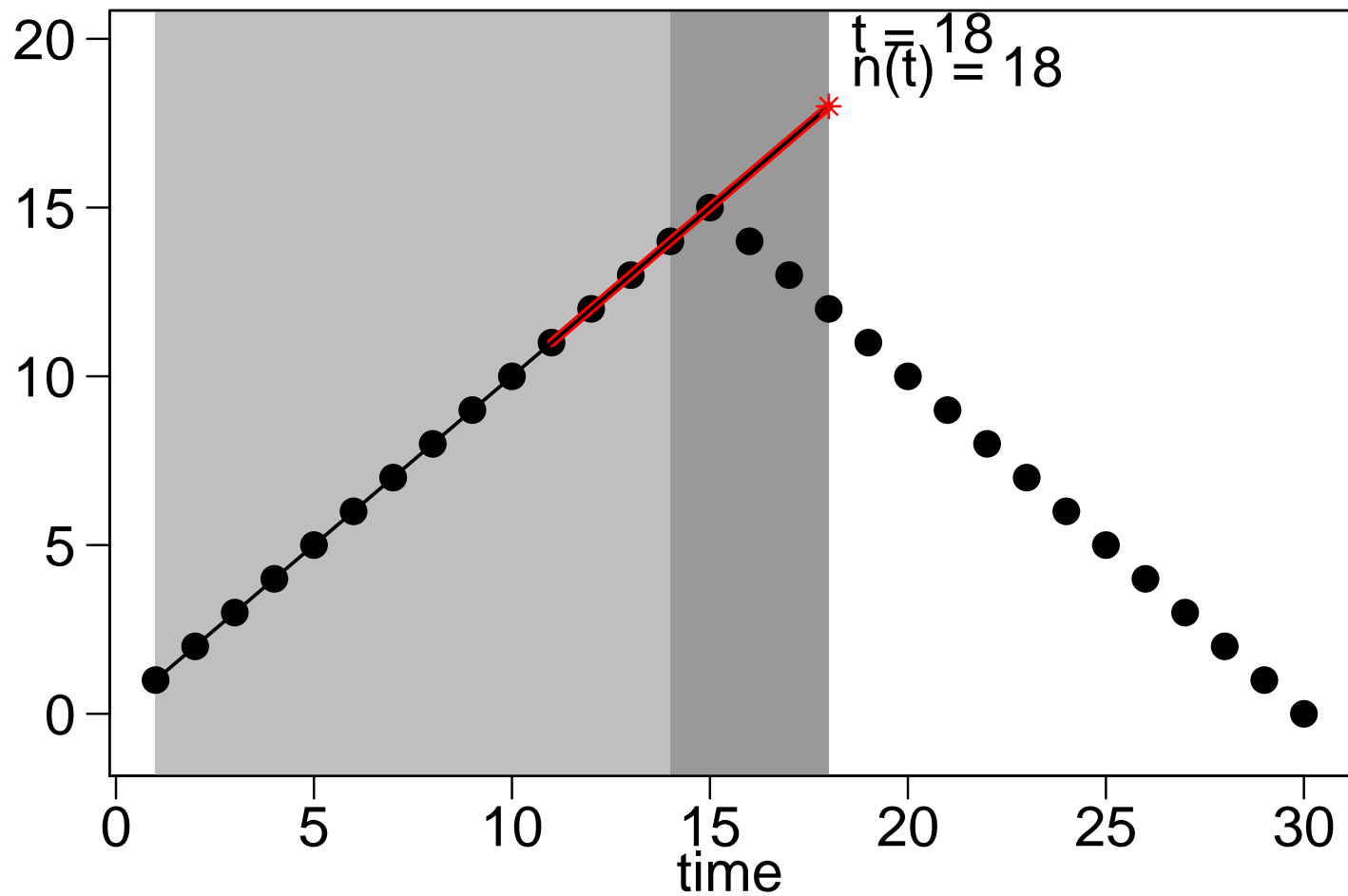
min.width=11    p.test=5    max.width=100





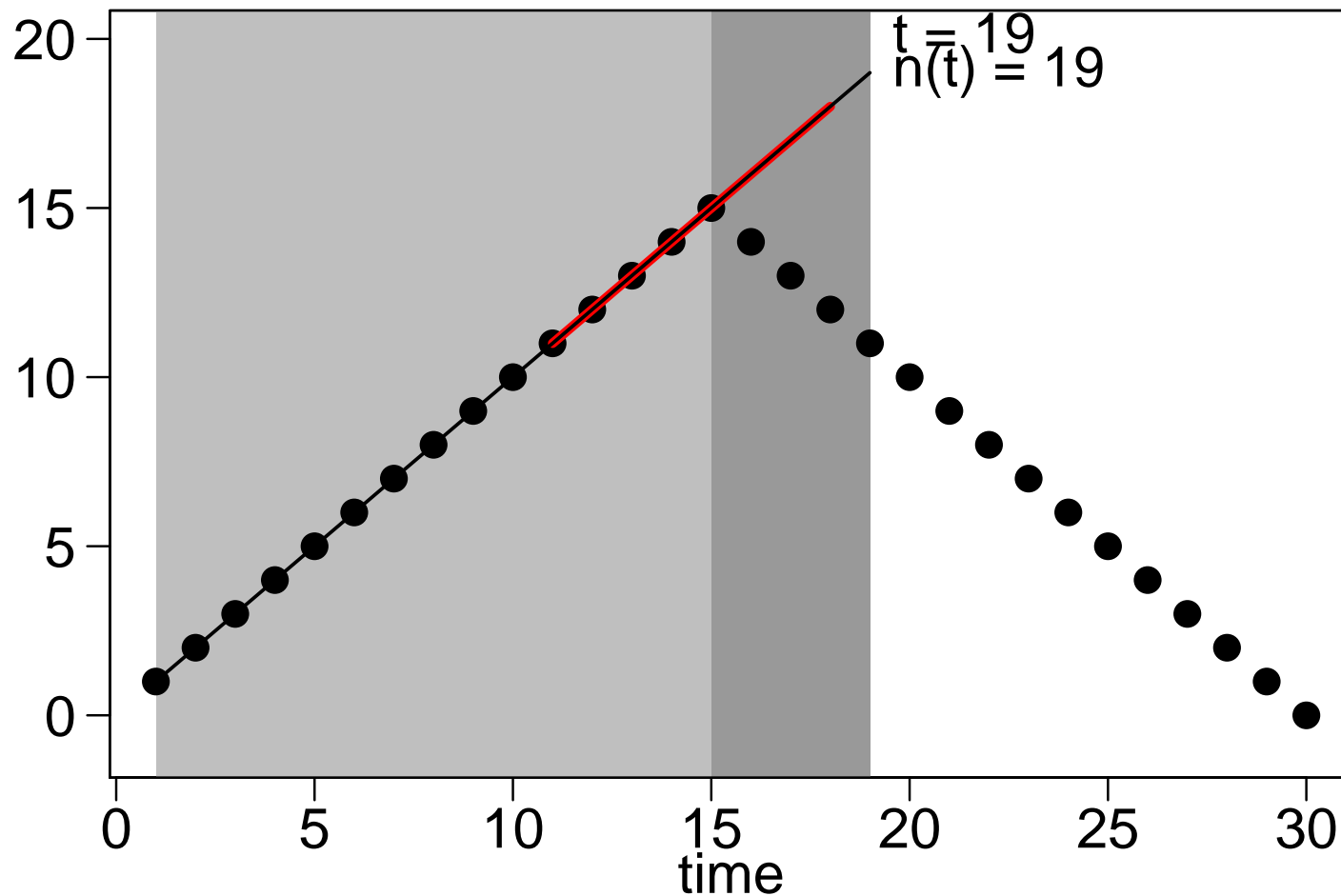
## adore.filter – Illustration

`min.width=11`    `p.test=5`    `max.width=100`



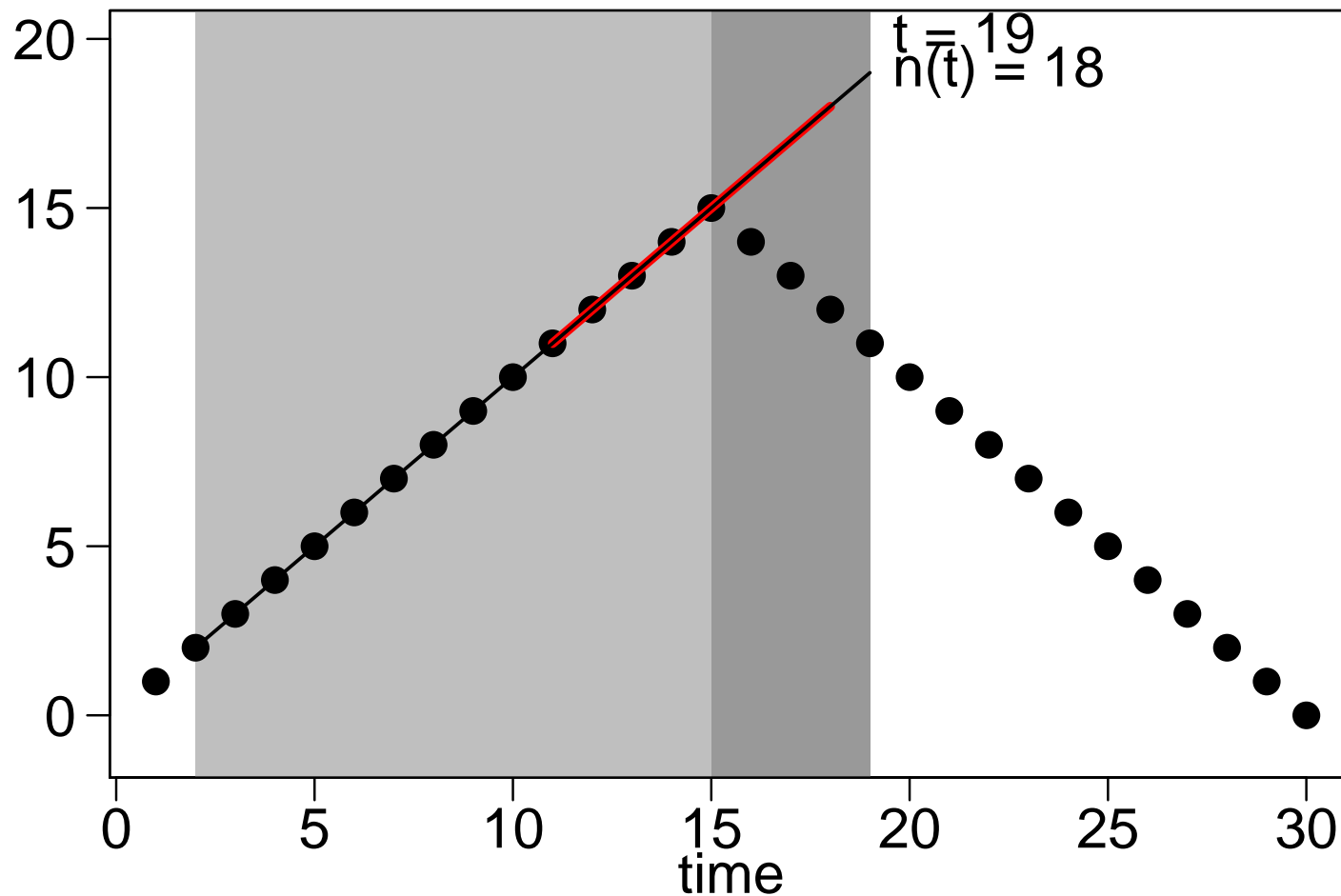
## adore.filter – Illustration

`min.width=11`    `p.test=5`    `max.width=100`



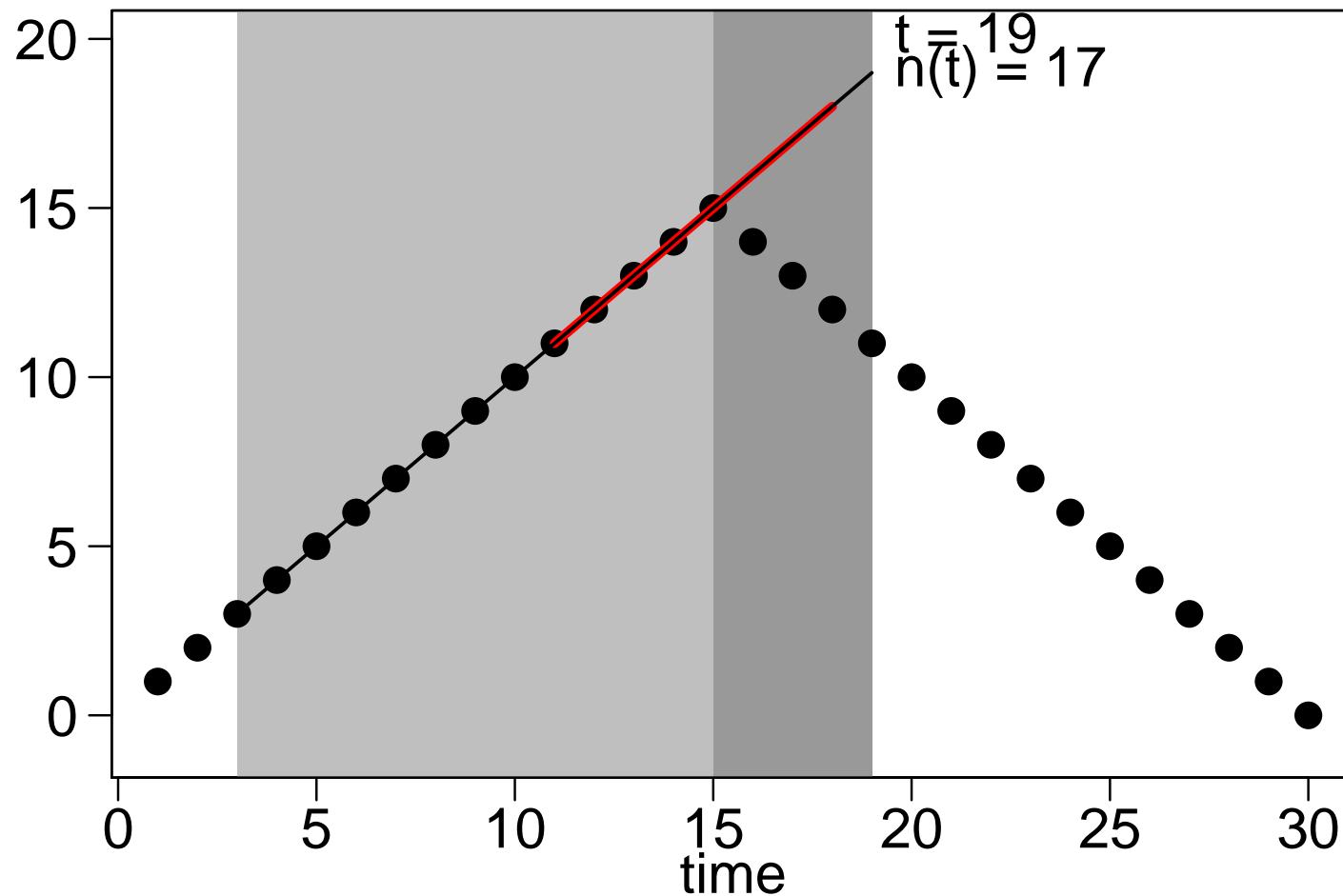
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



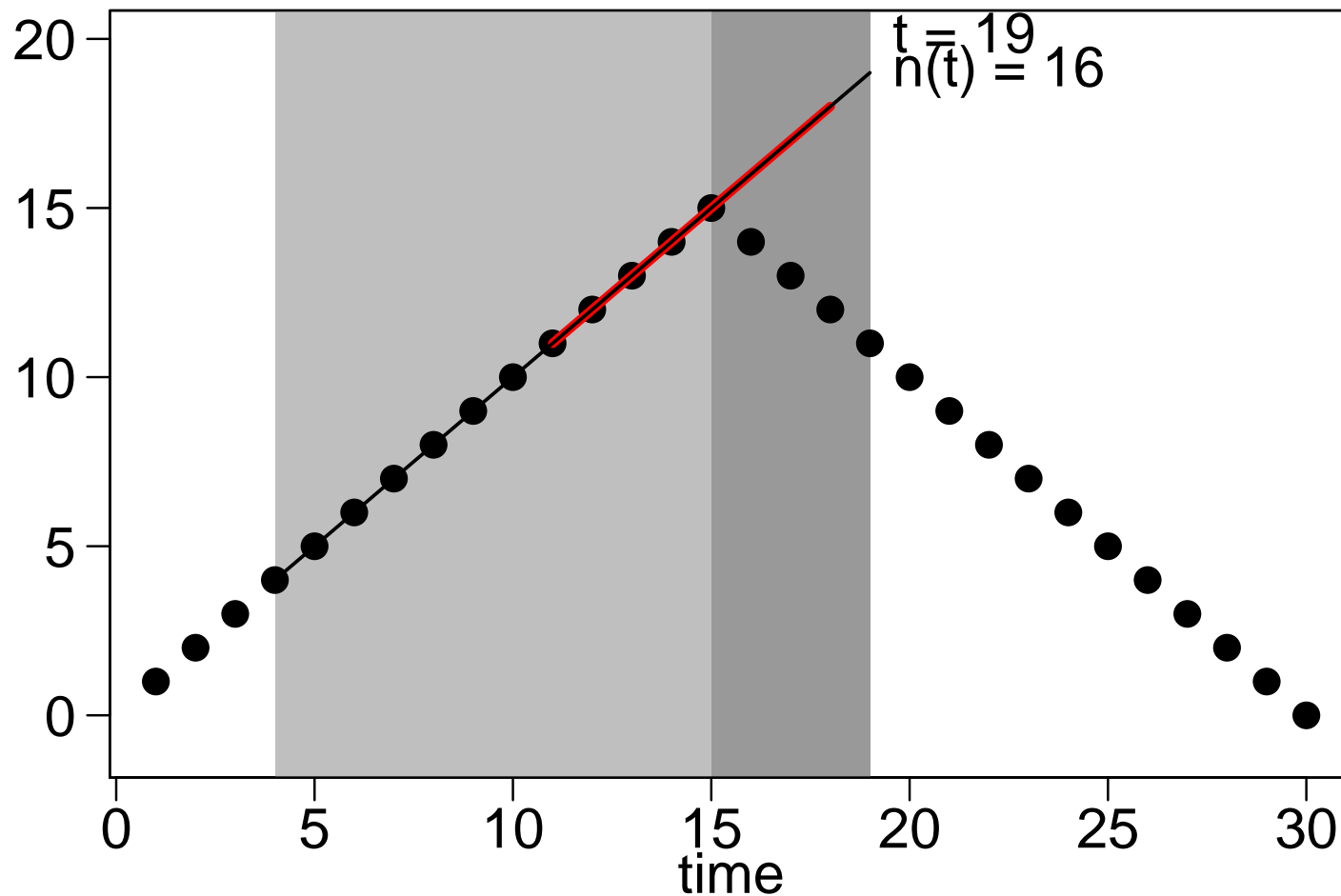
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



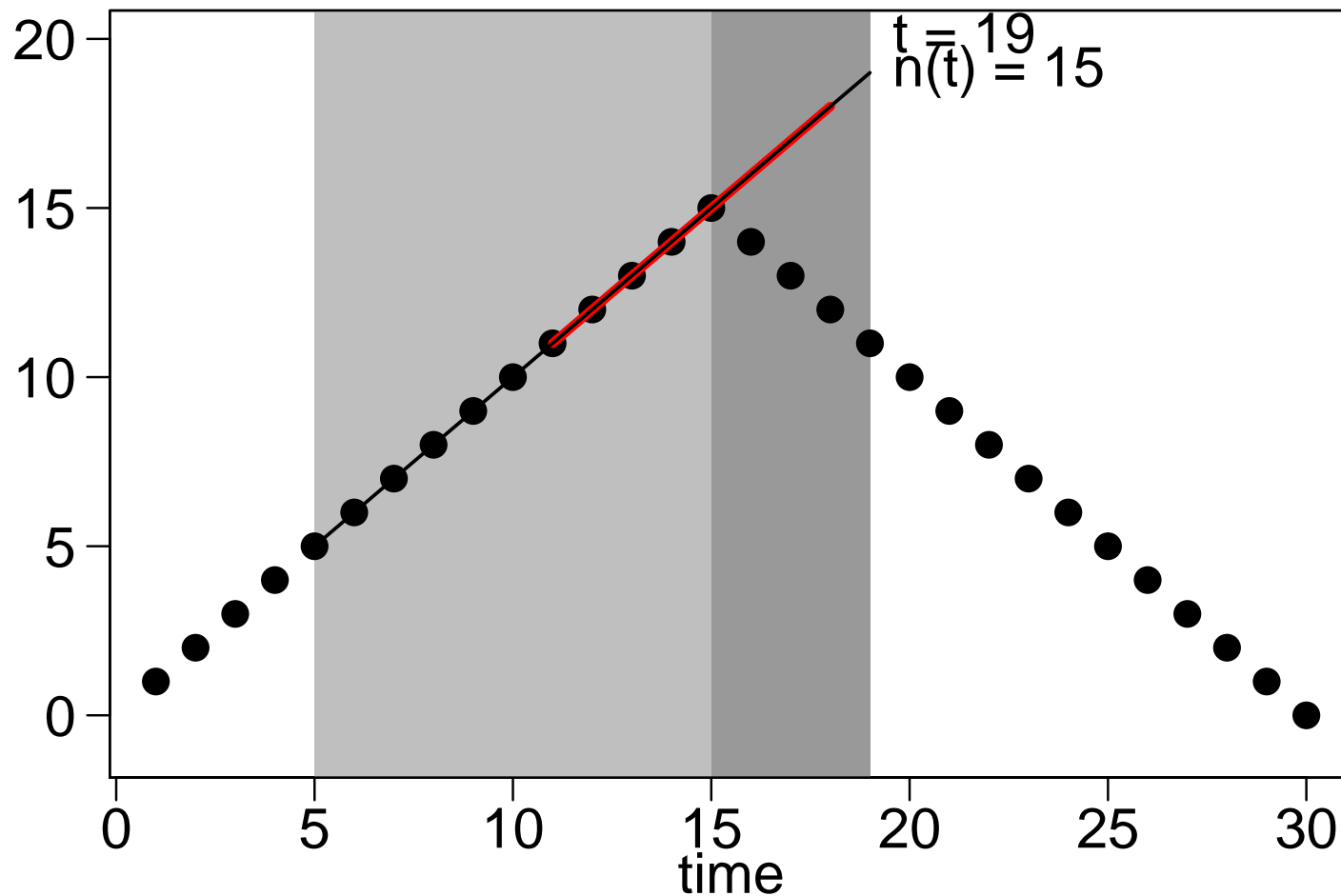
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



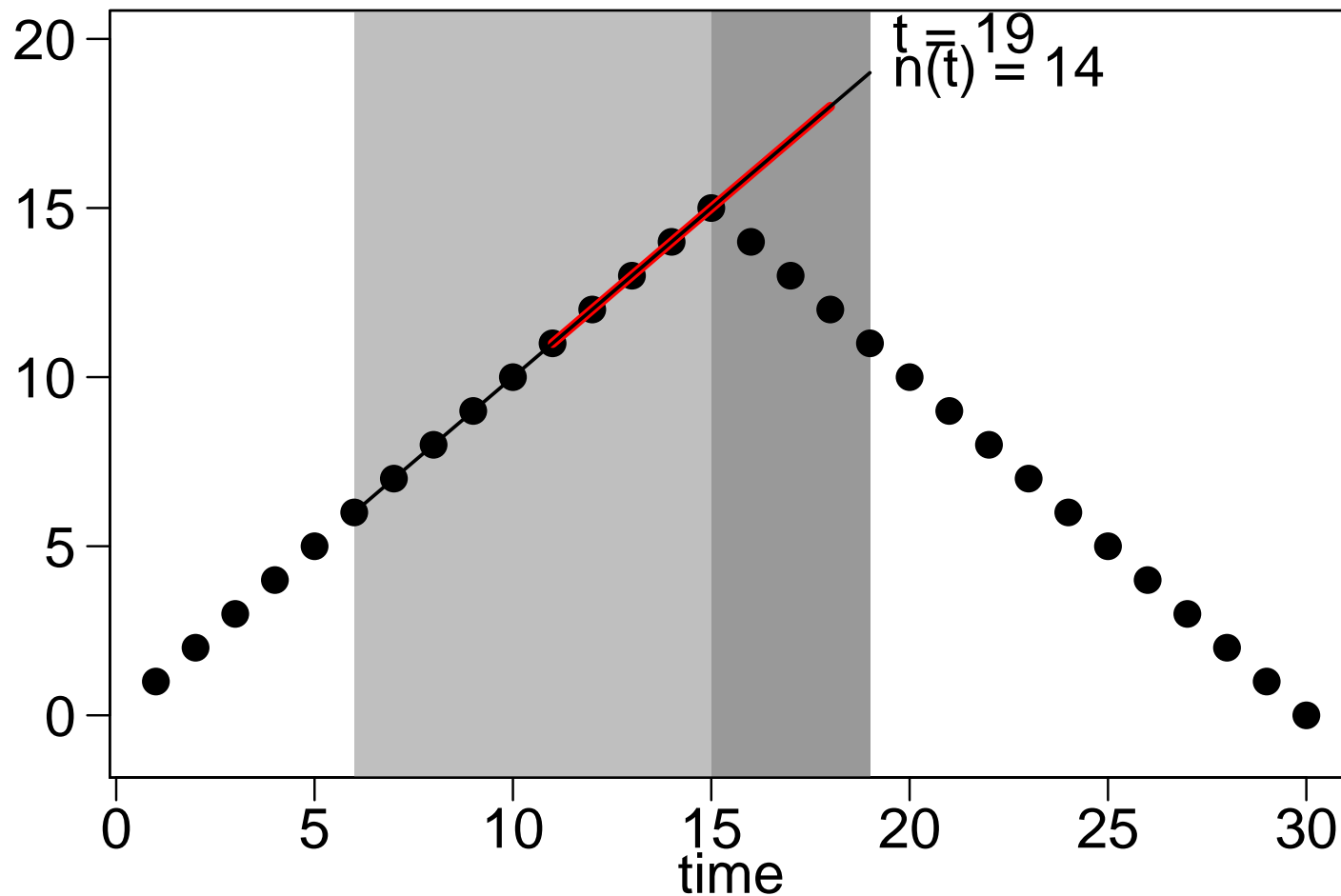
## adore.filter – Illustration

`min.width=11`    `p.test=5`    `max.width=100`



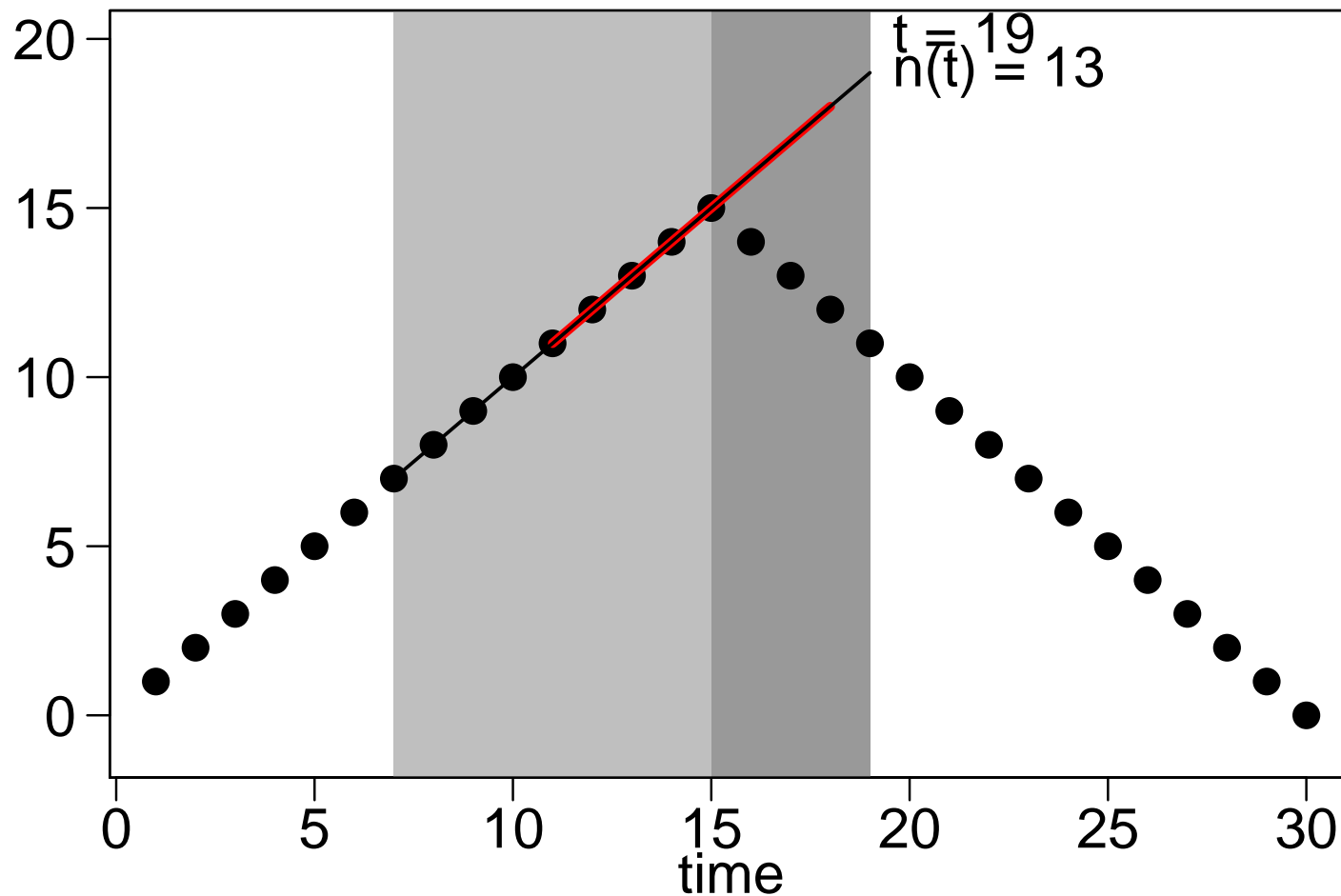
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



## adore.filter – Illustration

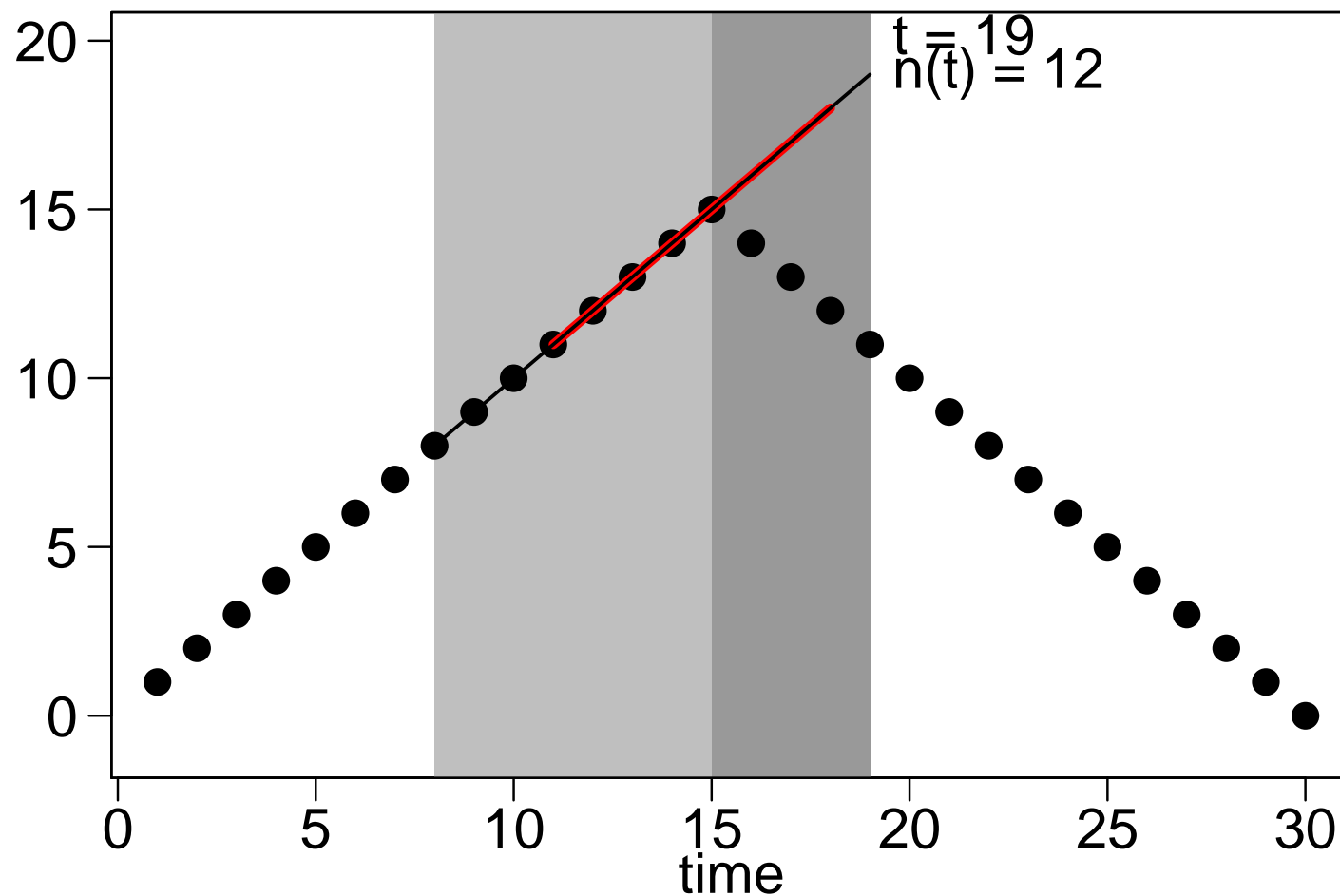
min.width=11    p.test=5    max.width=100





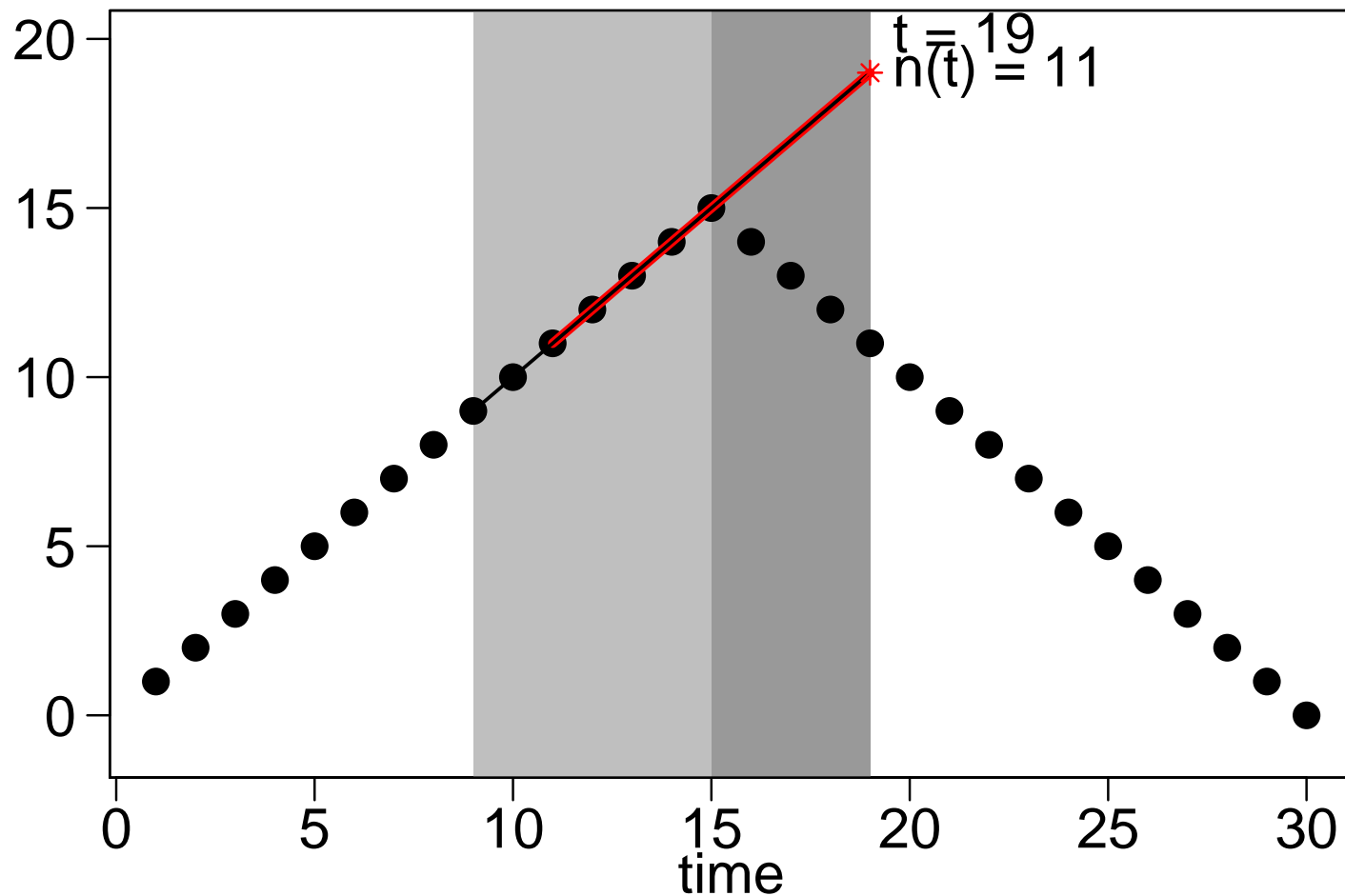
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



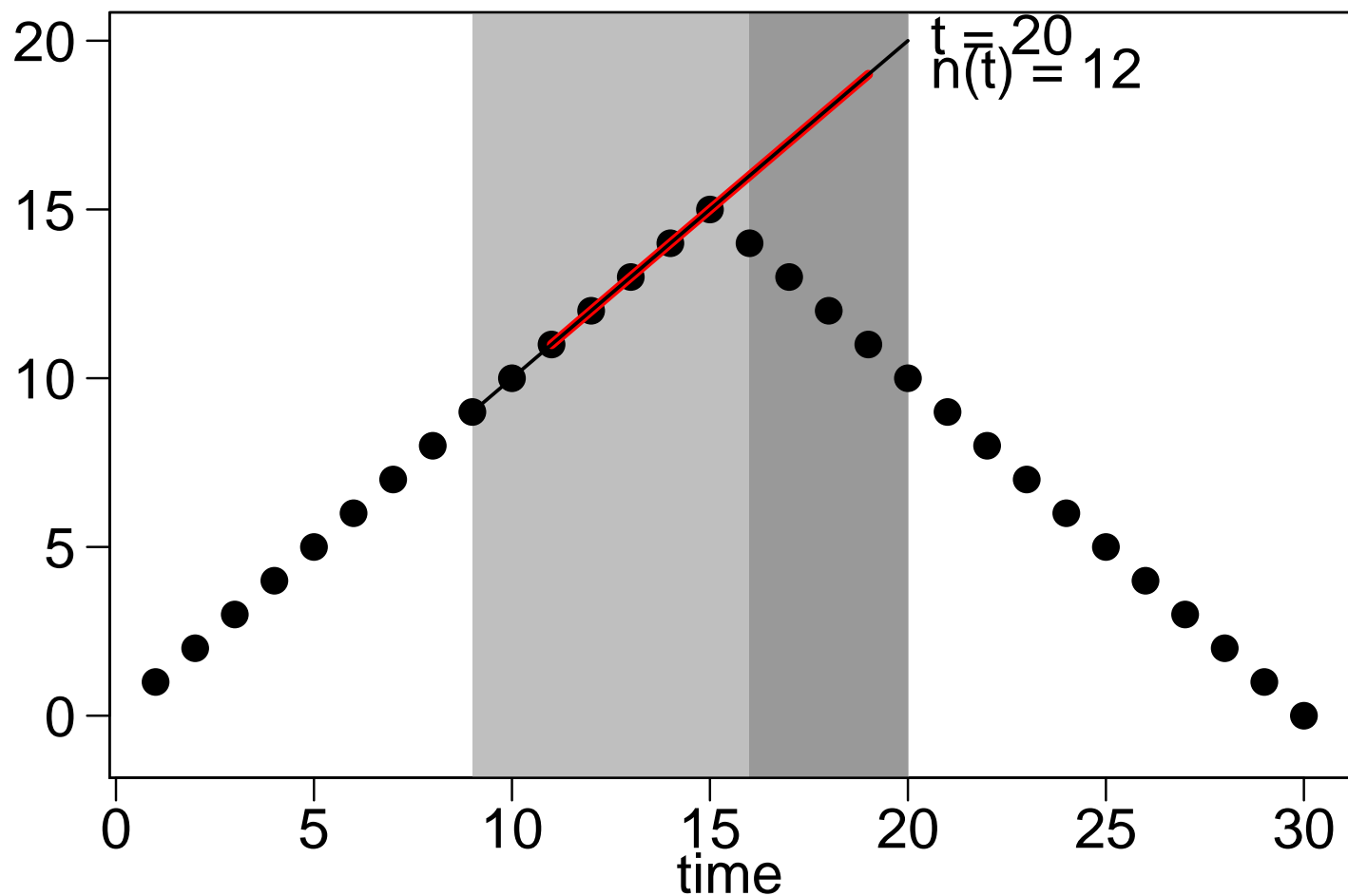
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



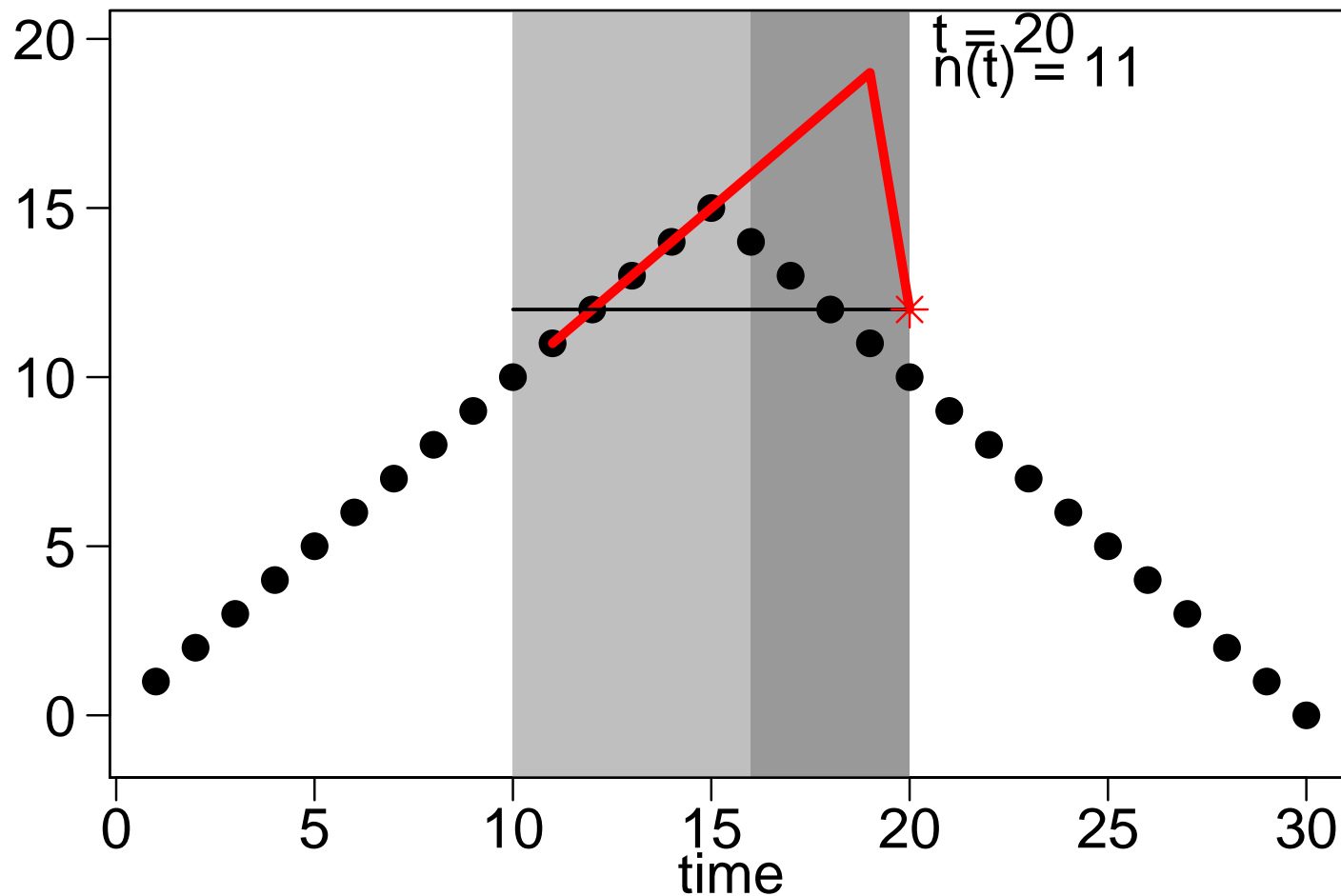
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



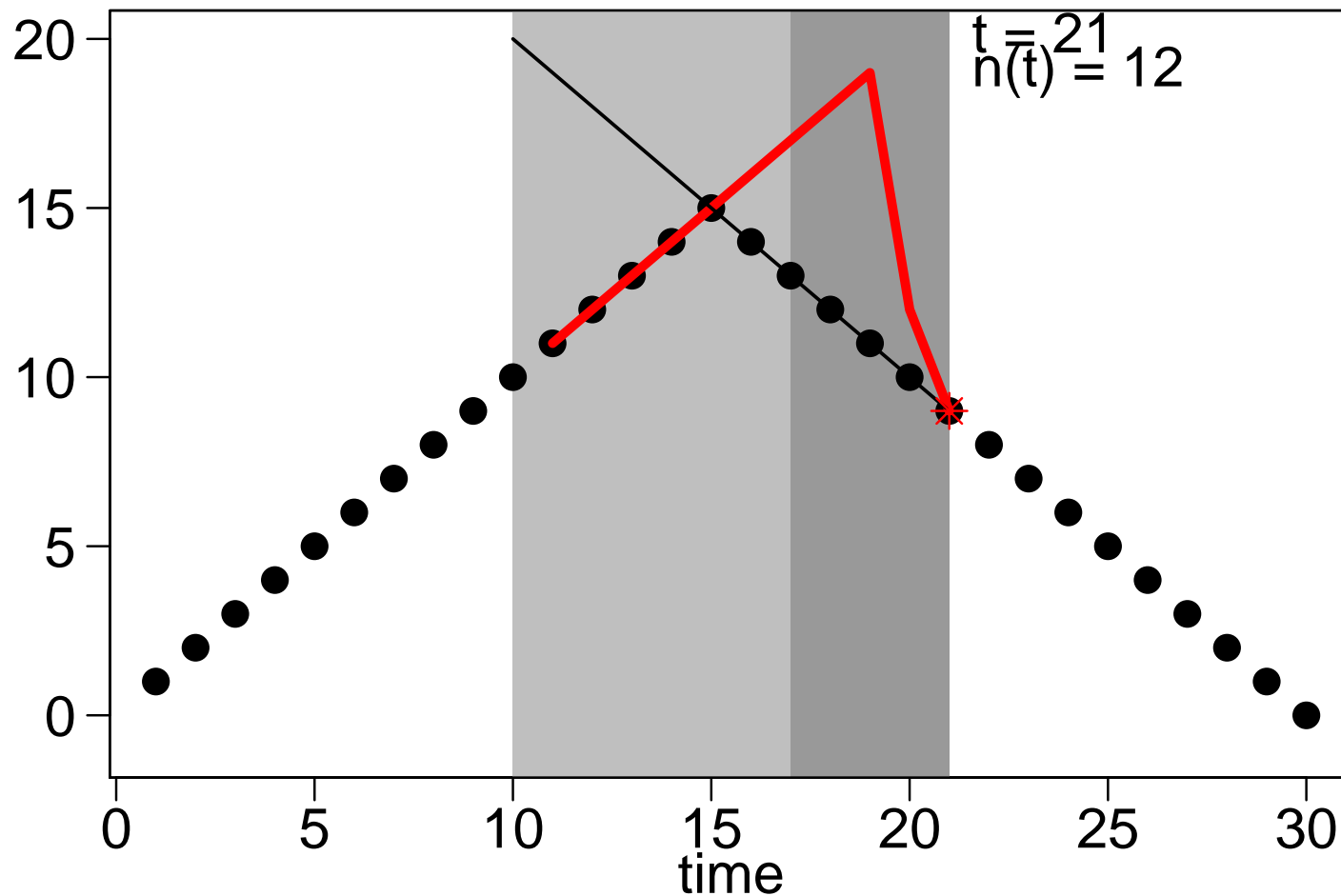
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



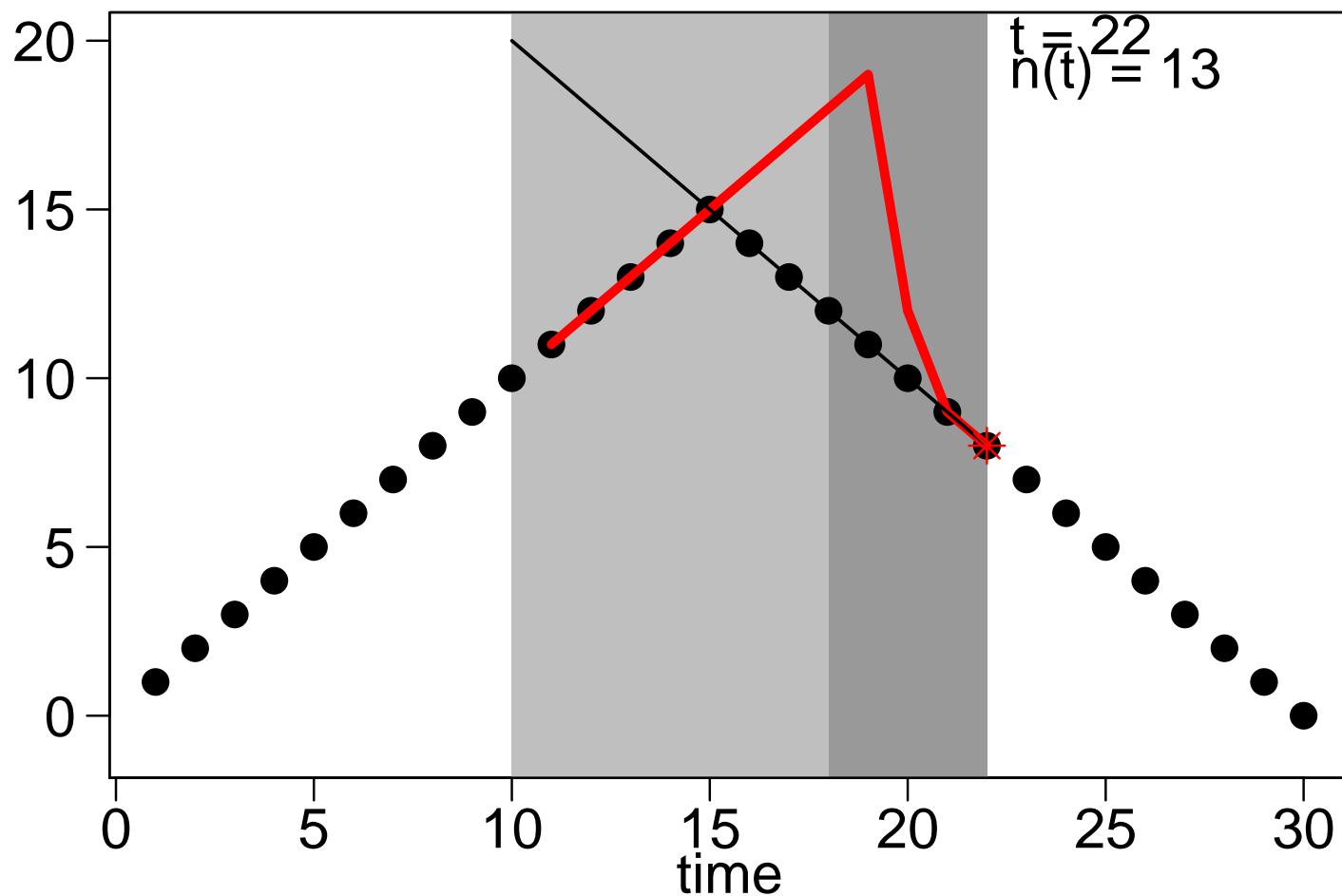
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



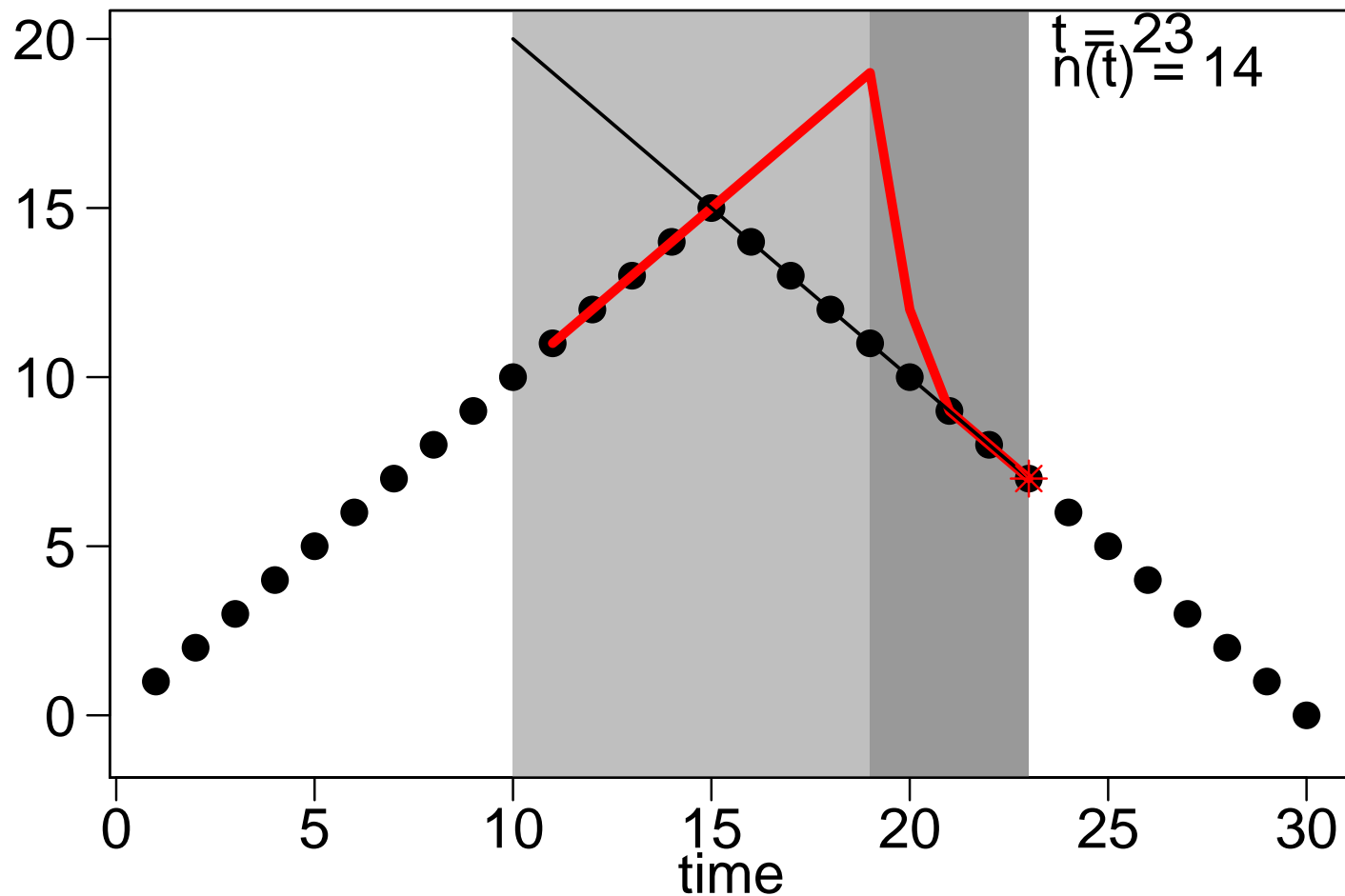
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



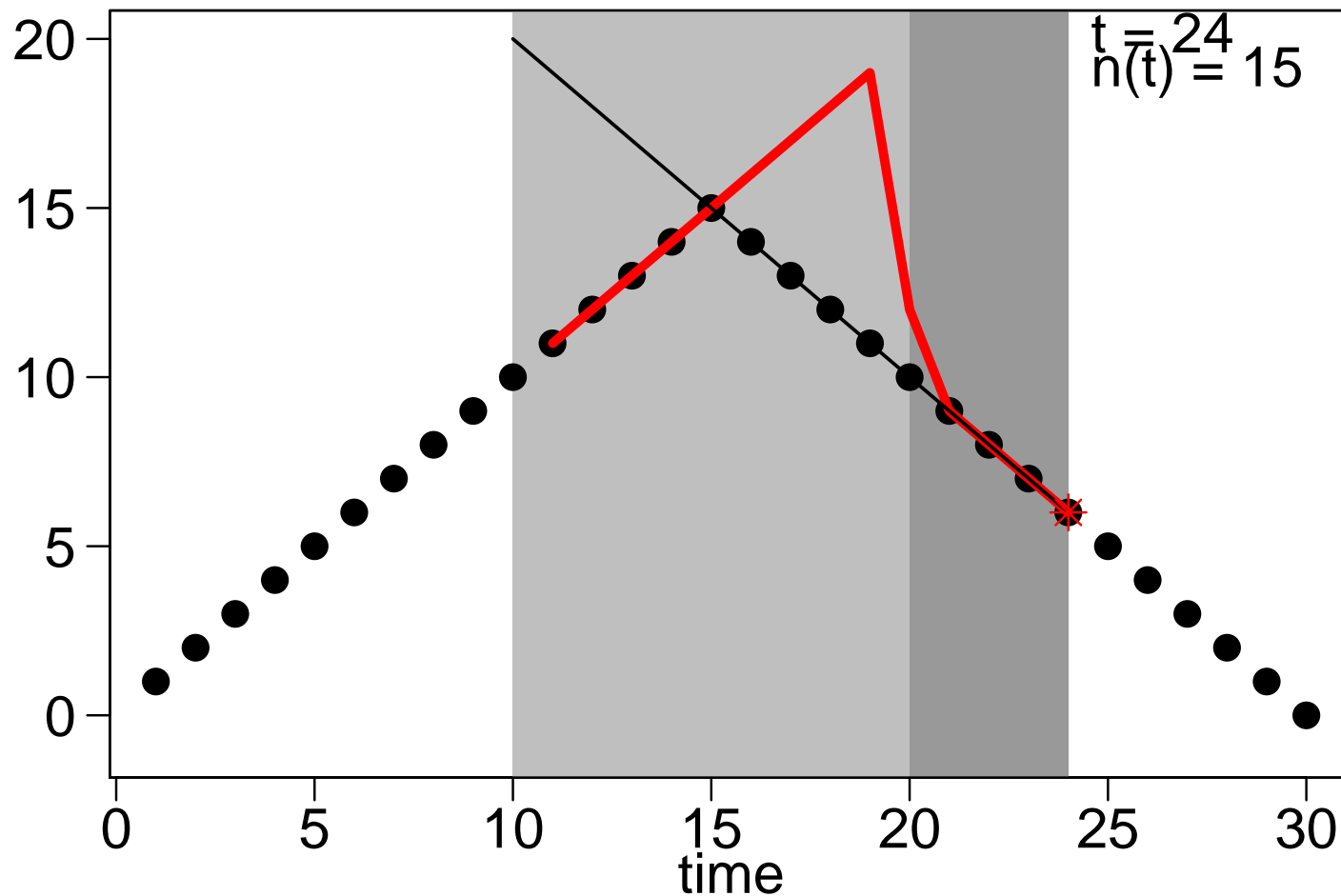
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



## adore.filter – Illustration

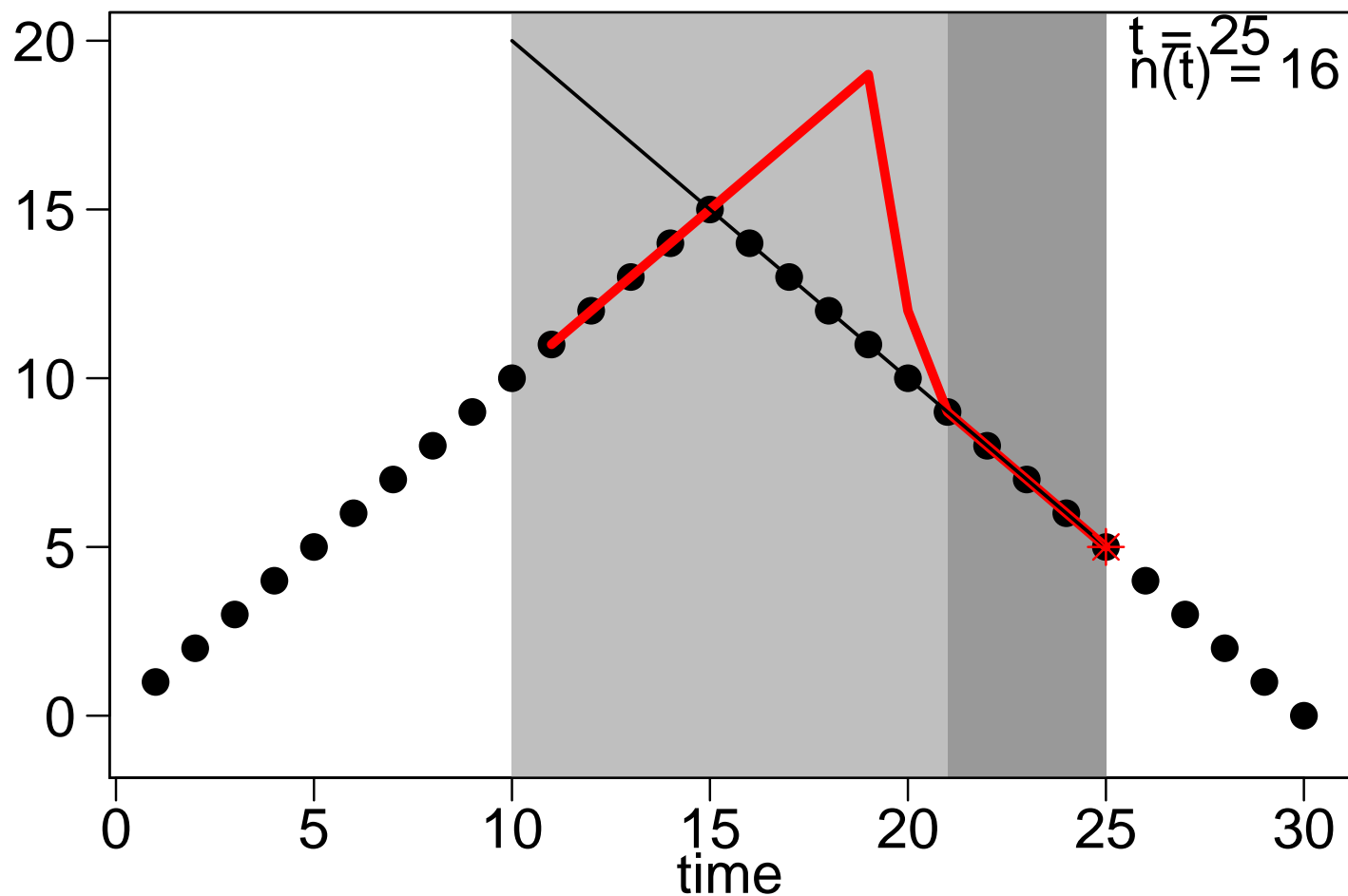
min.width=11    p.test=5    max.width=100





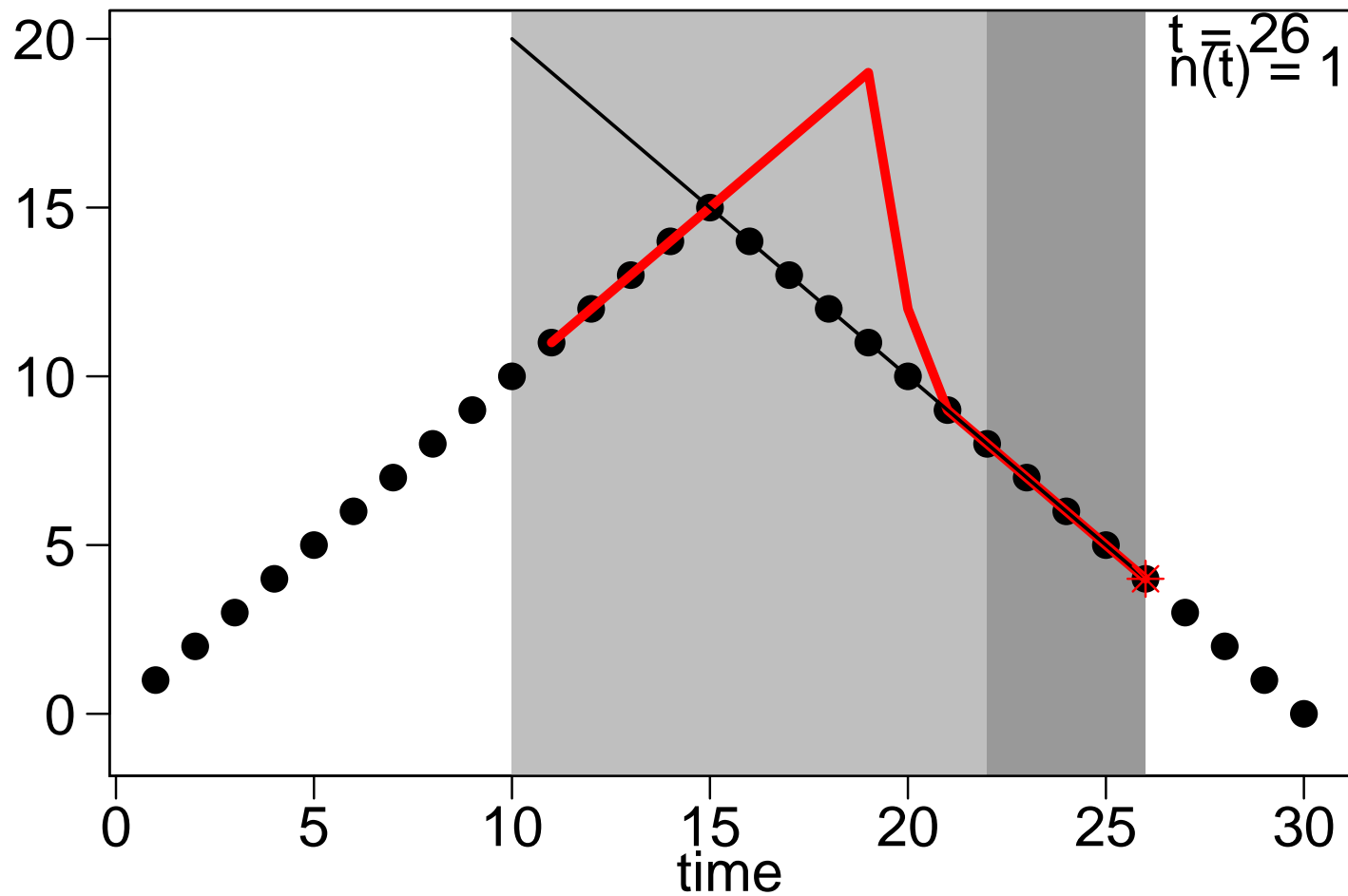
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



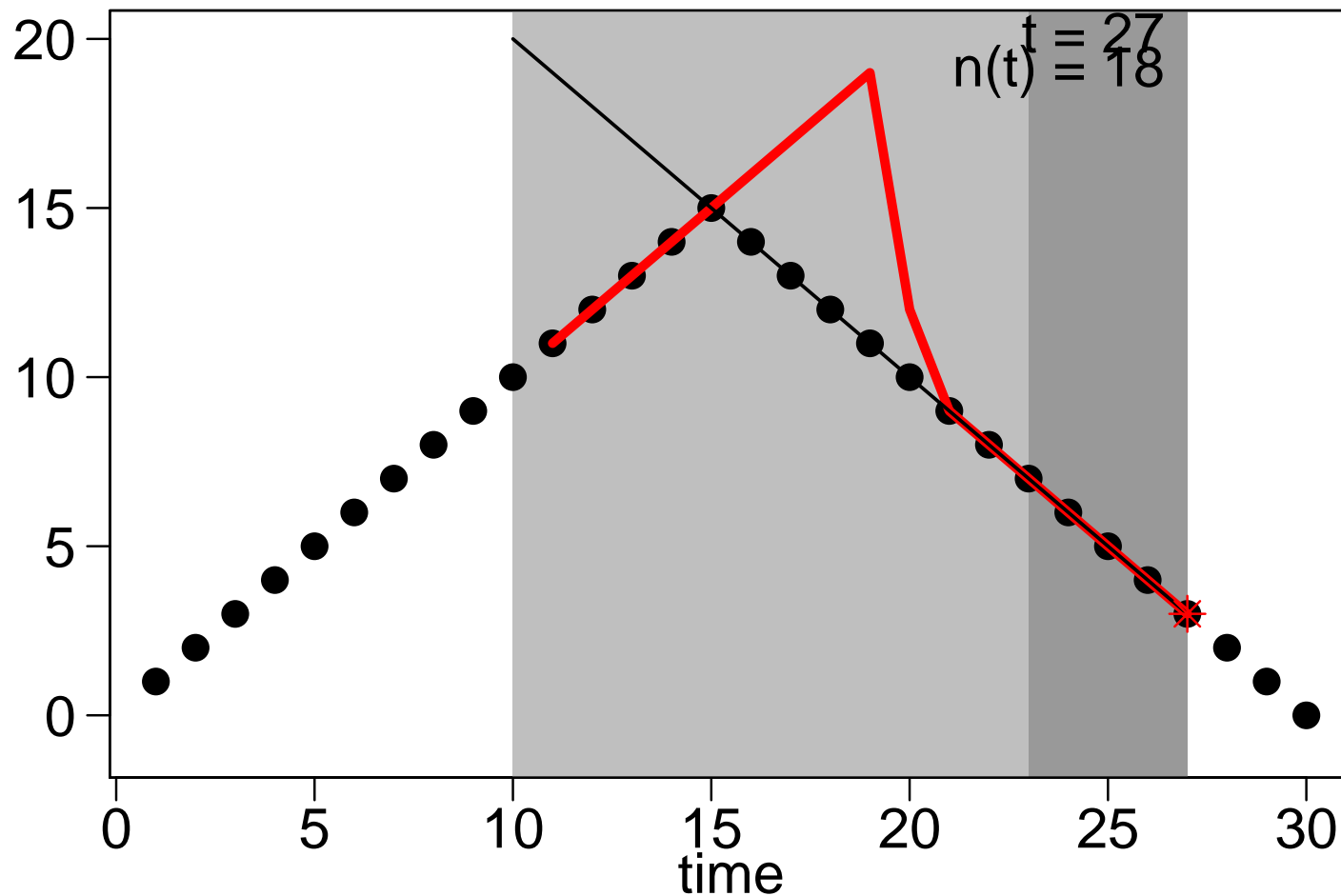
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



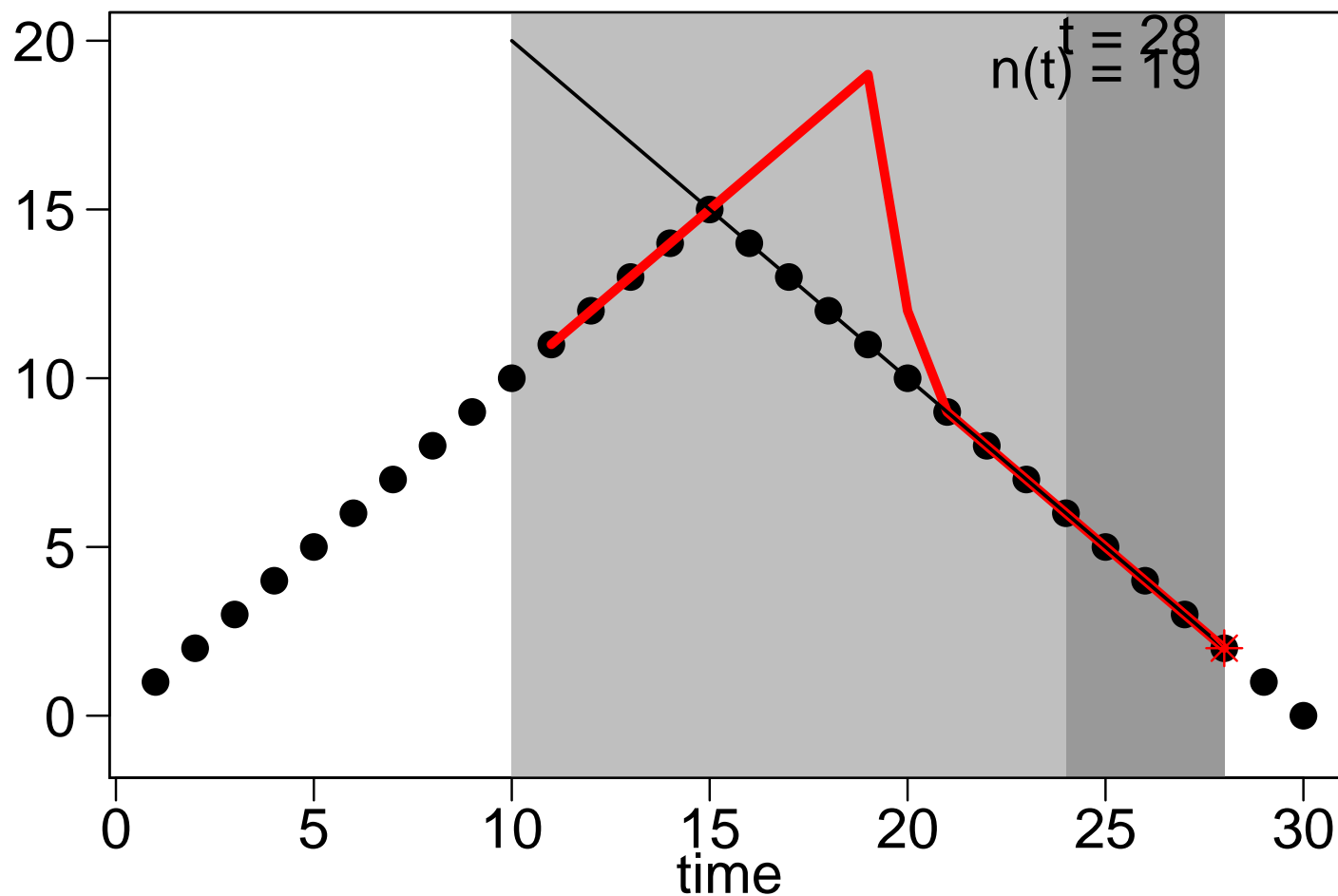
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



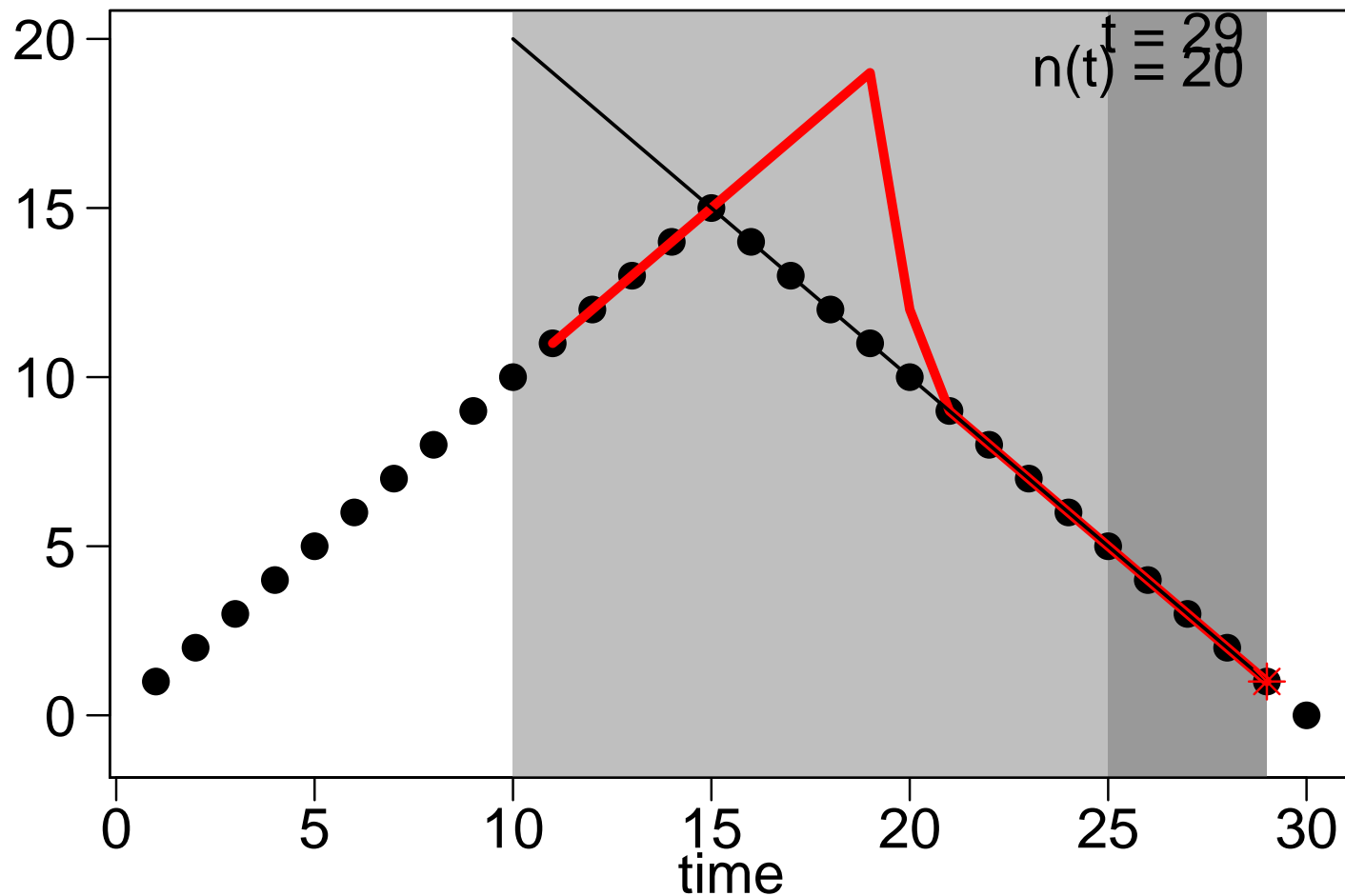
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



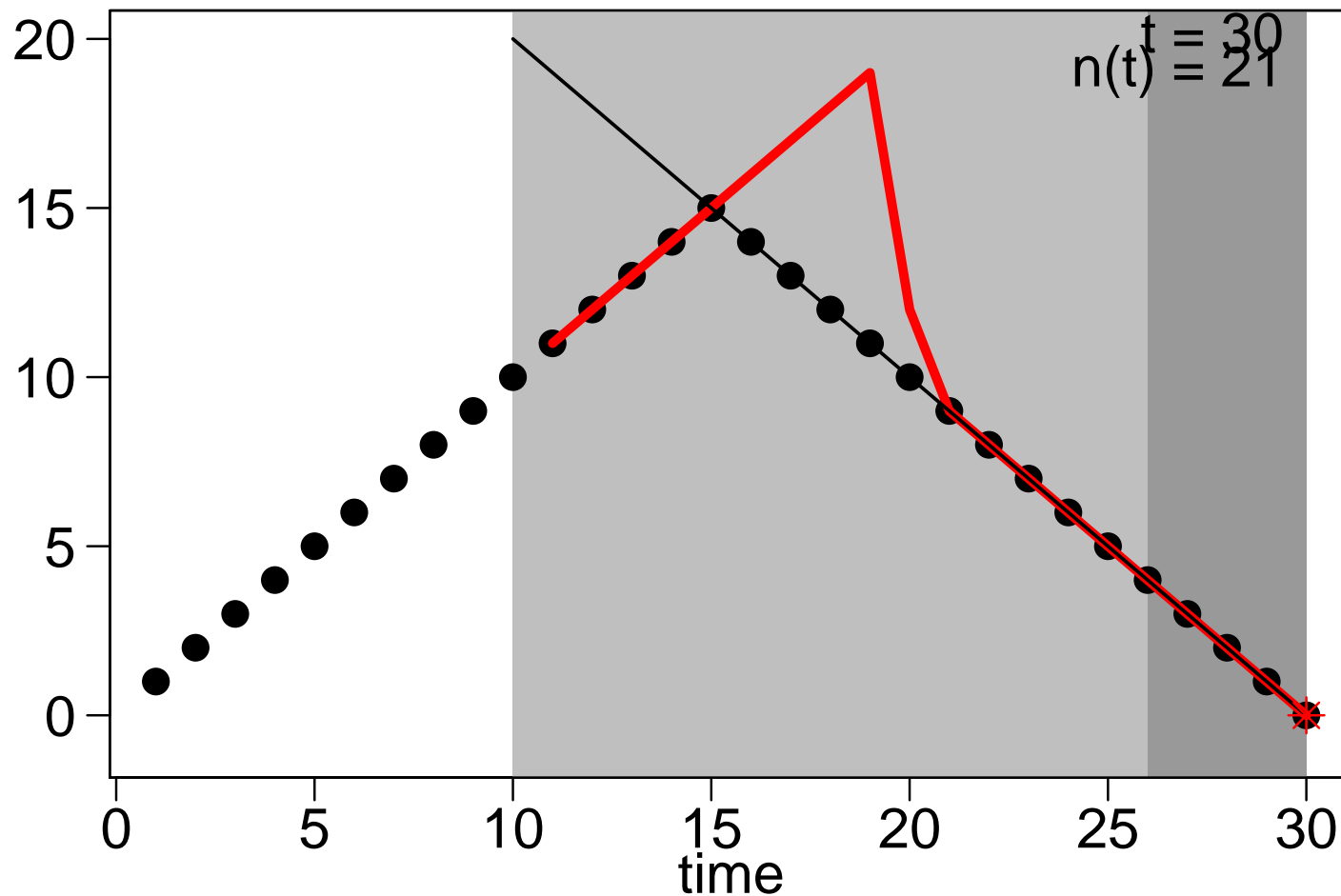
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



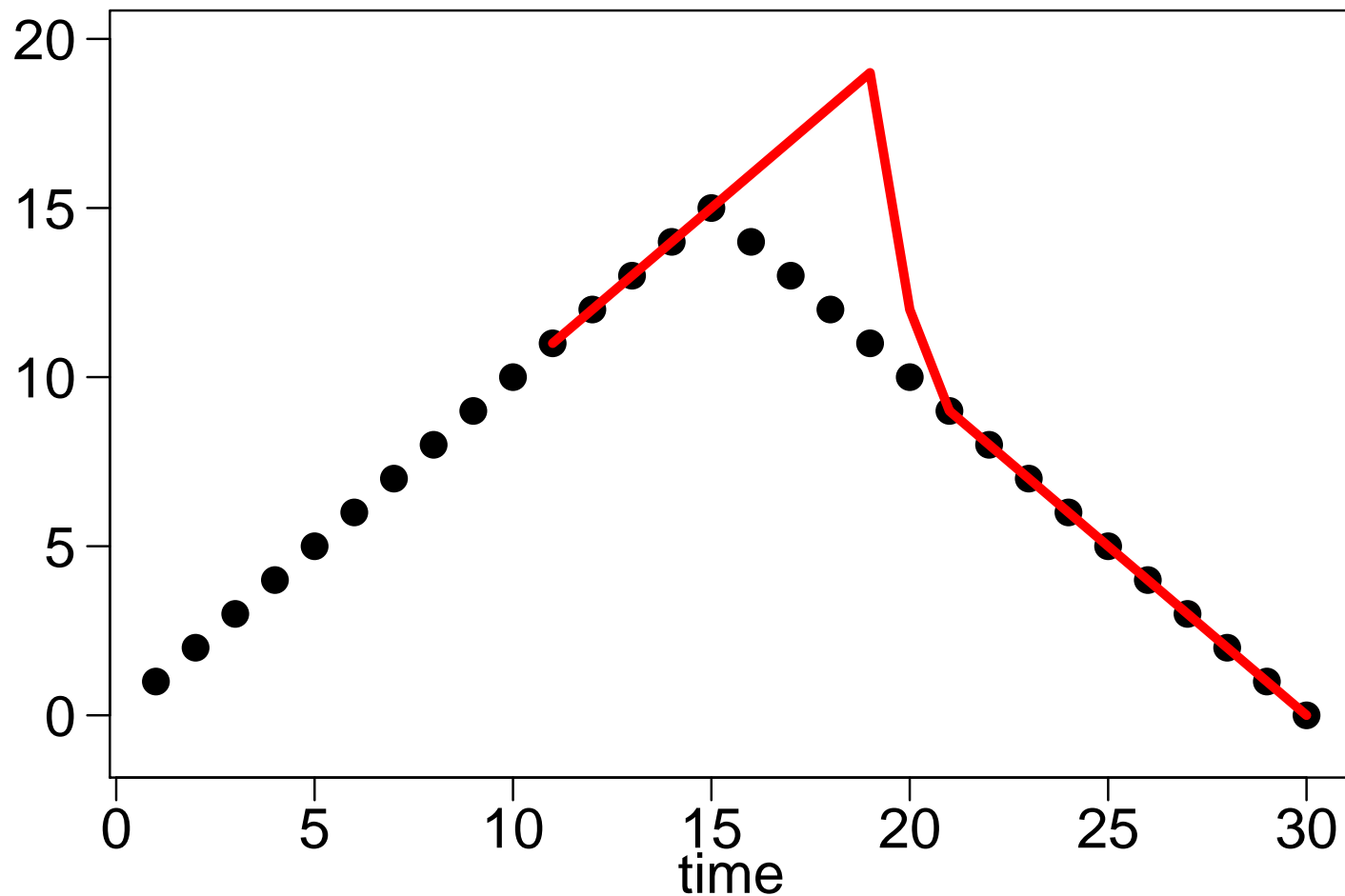
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100



## adore.filter – Illustration

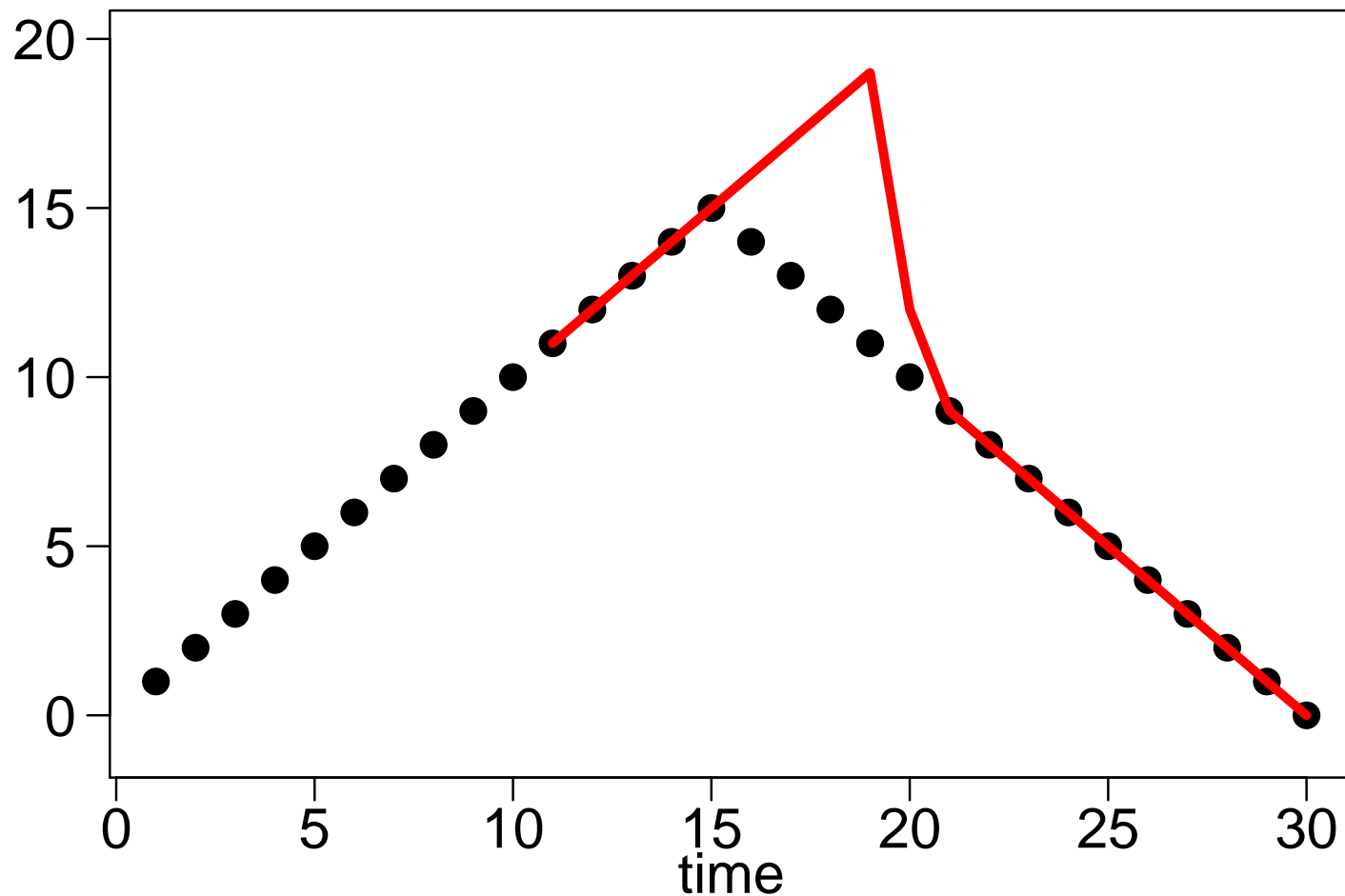
min.width=11    p.test=5    max.width=100



## adore.filter – Illustration

`min.width=11`   `p.test=5`   `max.width=100`

`rtr=0`: No restriction on the estimated signal level

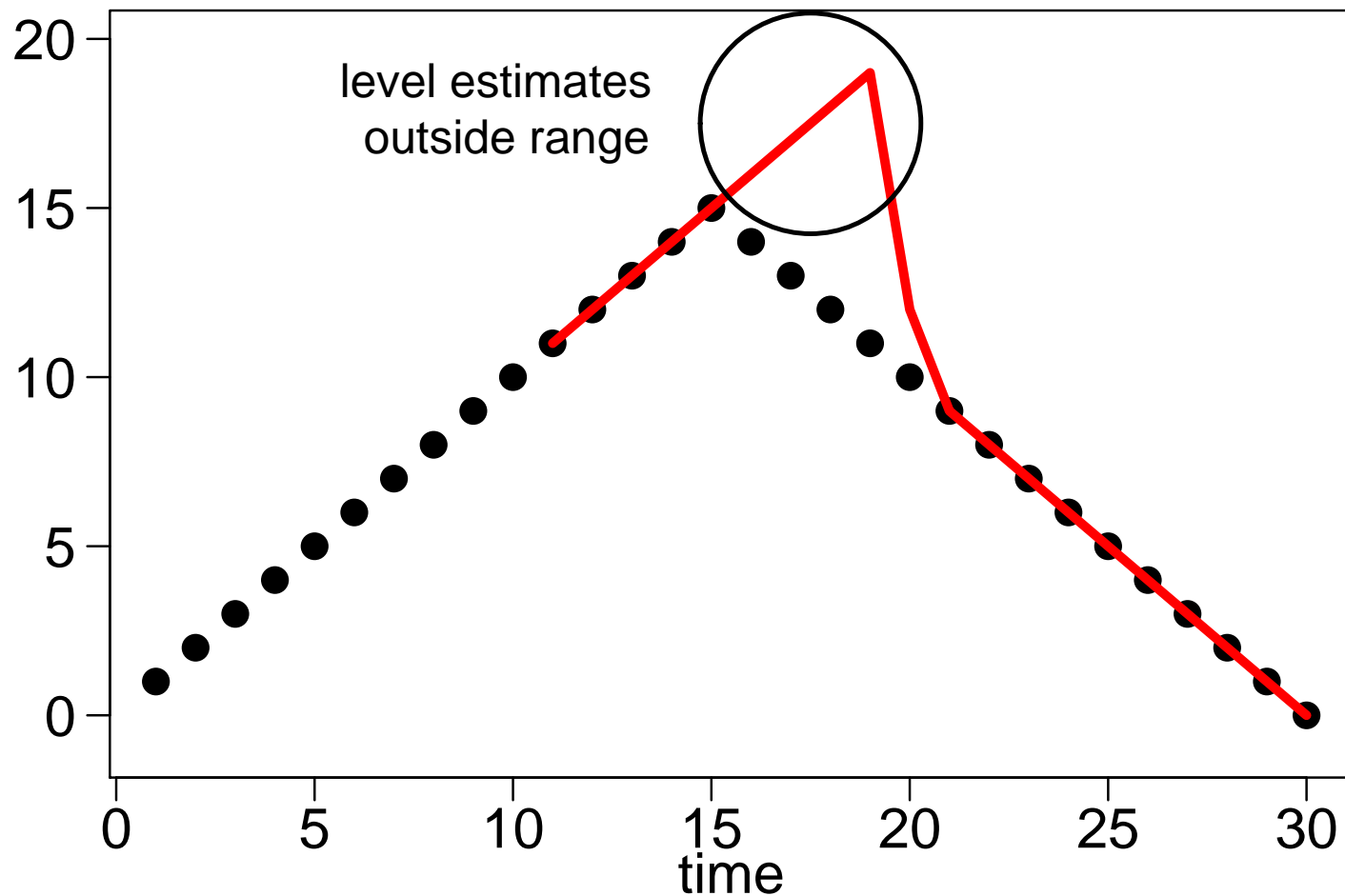




## adore.filter – Illustration

`min.width=11`    `p.test=5`    `max.width=100`

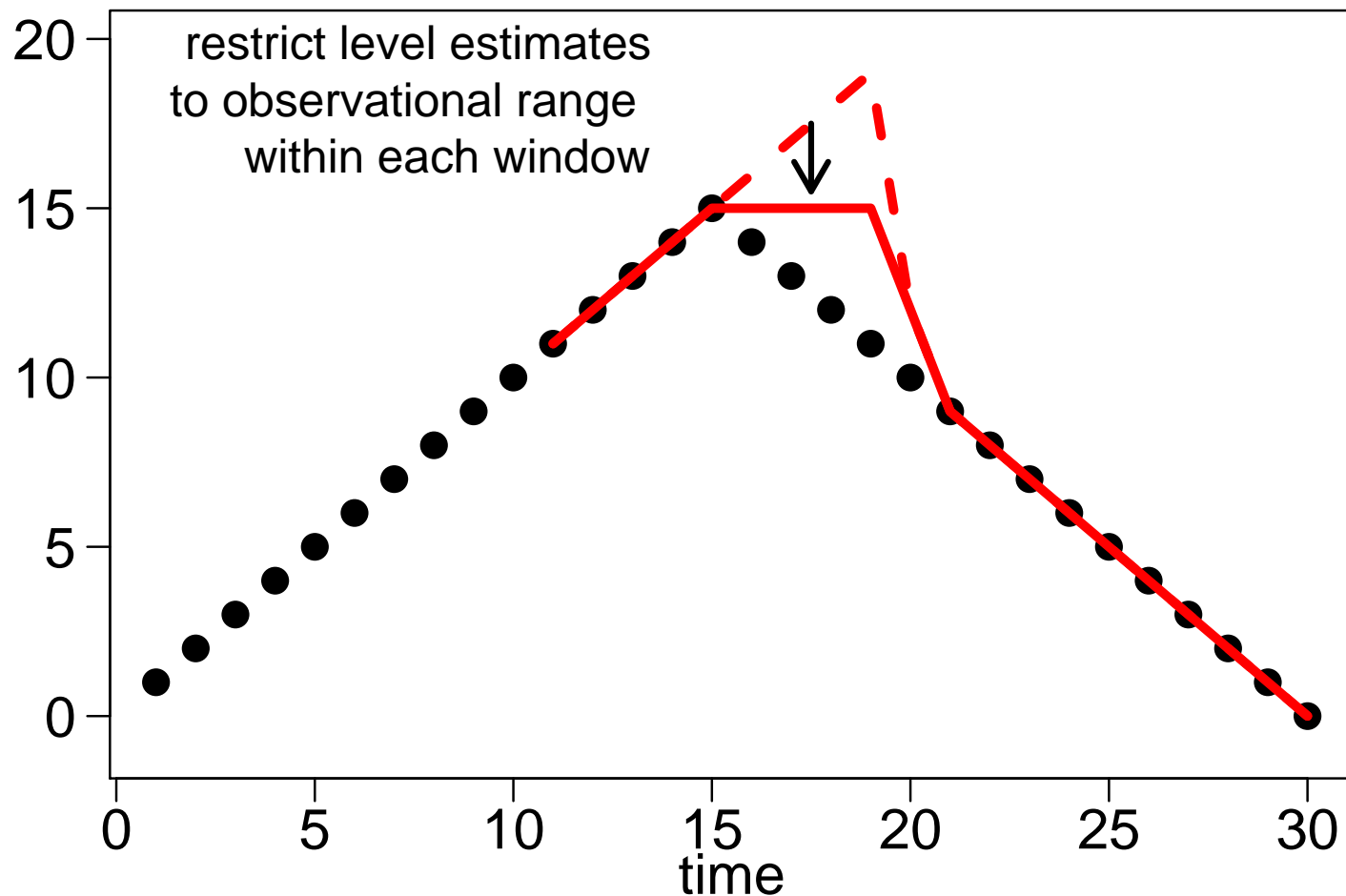
`rtr=0`: No restriction on the estimated signal level



## adore.filter – Illustration

`min.width=11`    `p.test=5`    `max.width=100`

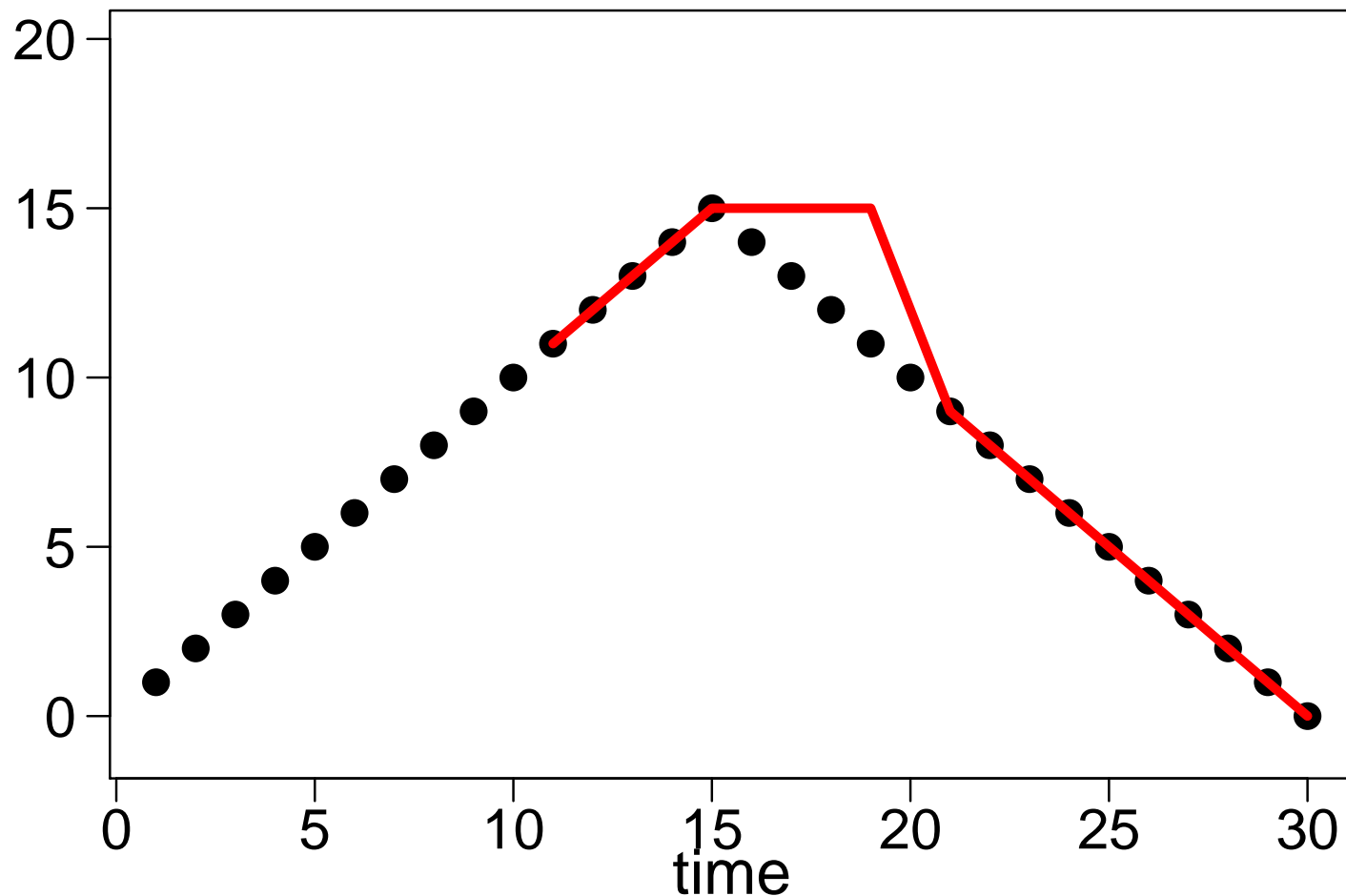
`rtr=0`: No restriction on the estimated signal level



## adore.filter – Illustration

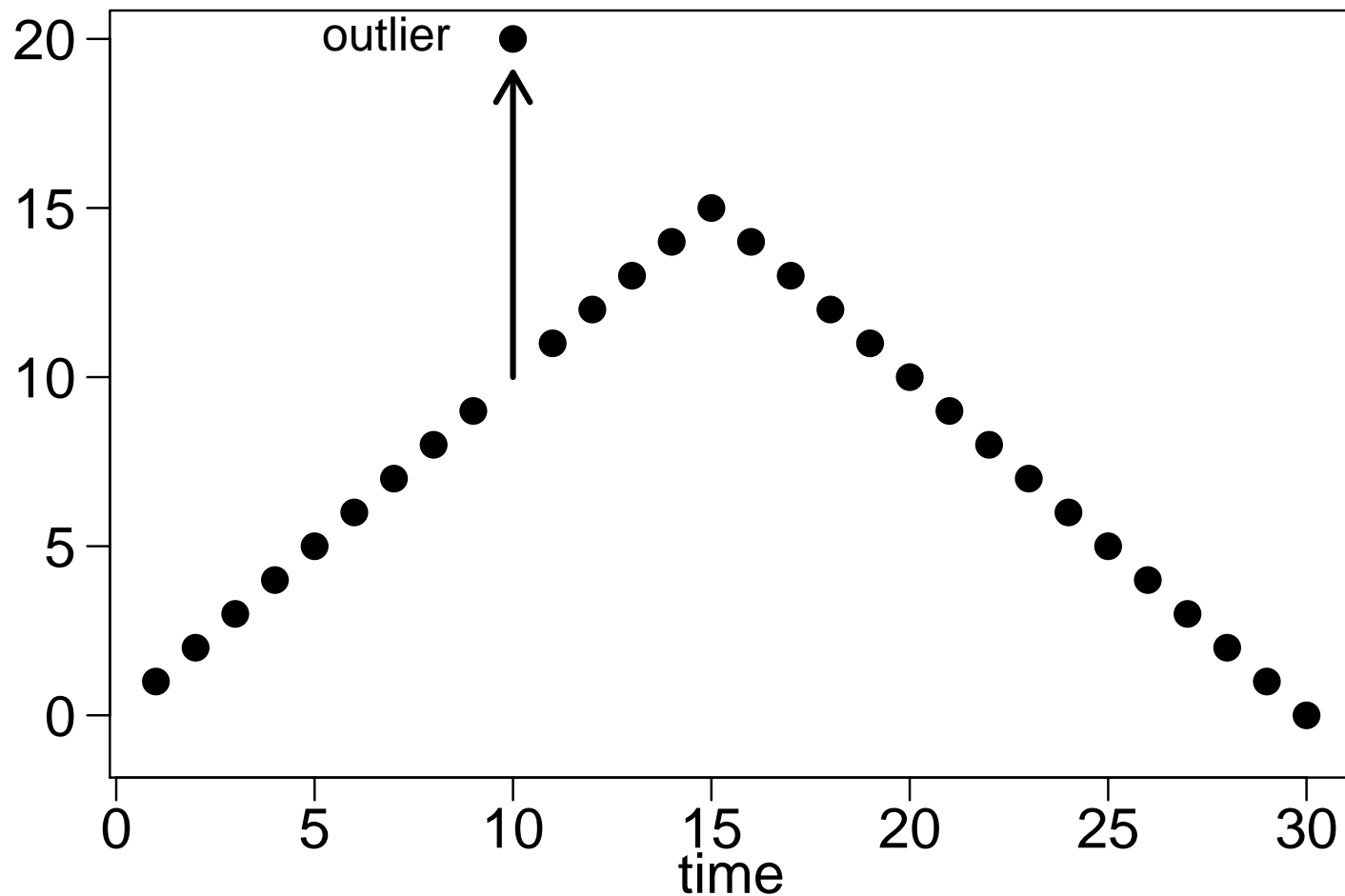
`min.width=11`    `p.test=5`    `max.width=100`

`rtr=1`: 'restrict to range' of observations in the current window



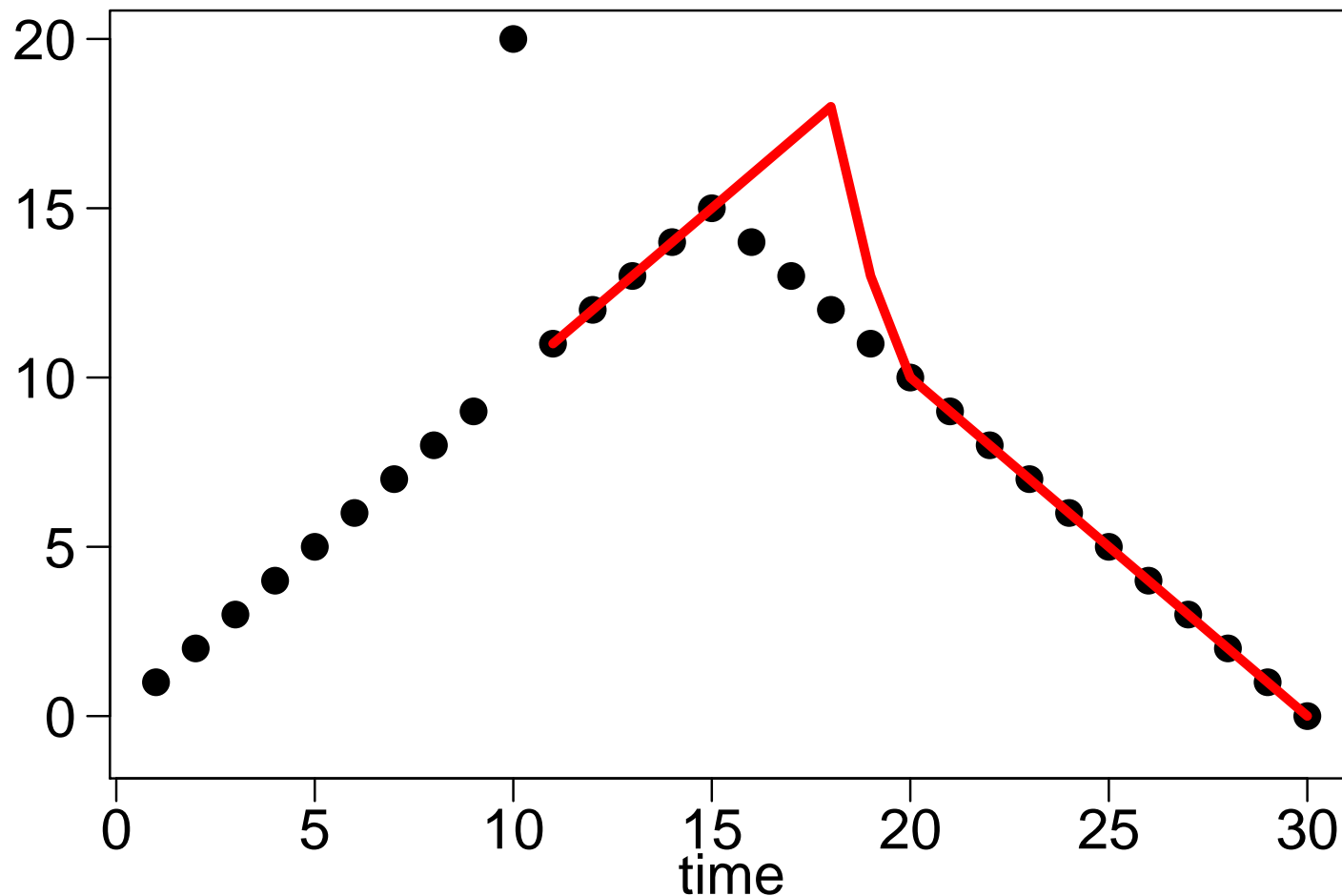
## adore.filter – Illustration

`min.width=11`    `p.test=5`    `max.width=100`



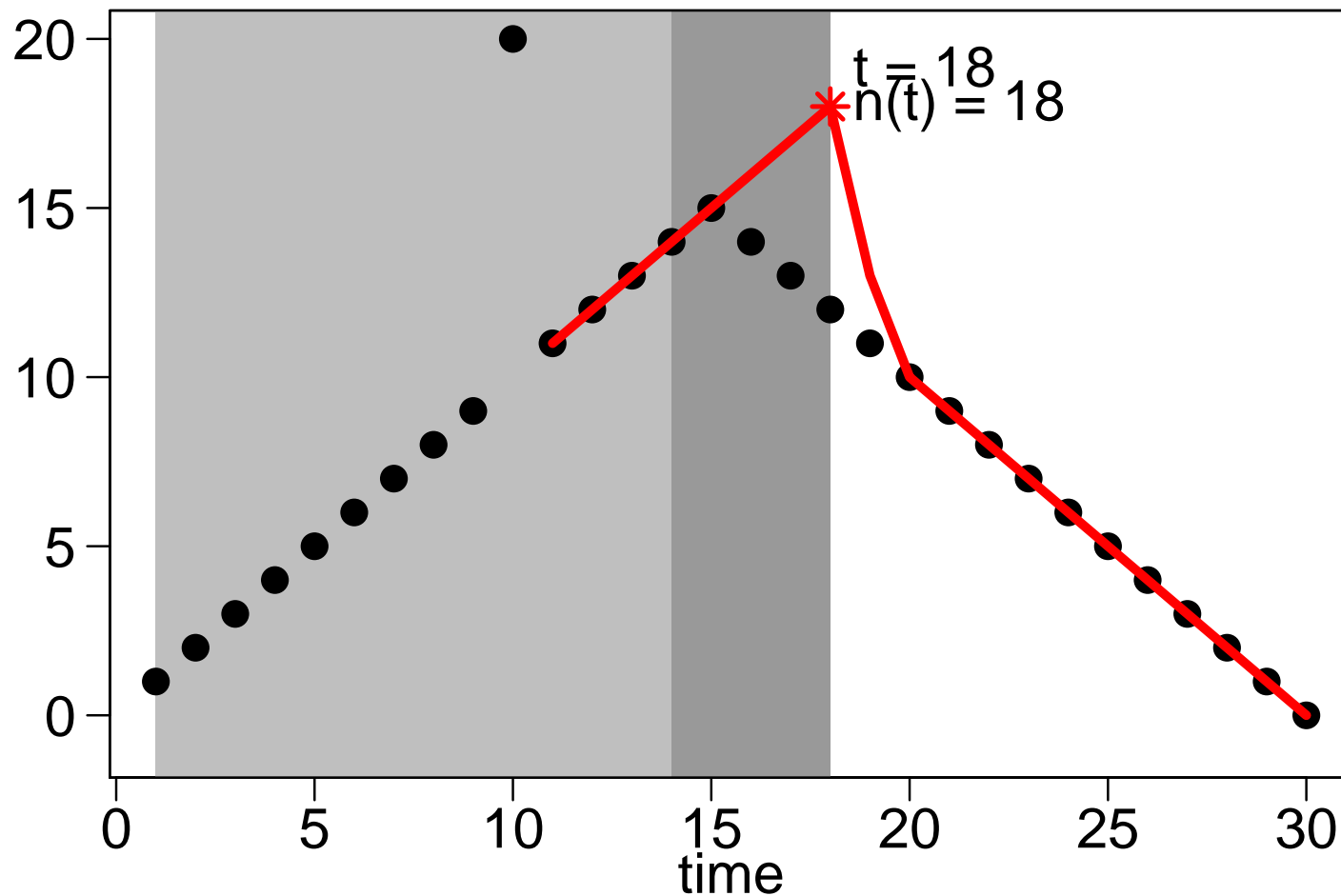
## adore.filter – Illustration

min.width=11    p.test=5    max.width=100  
rtr=1: 'restrict to range' of observations in the window



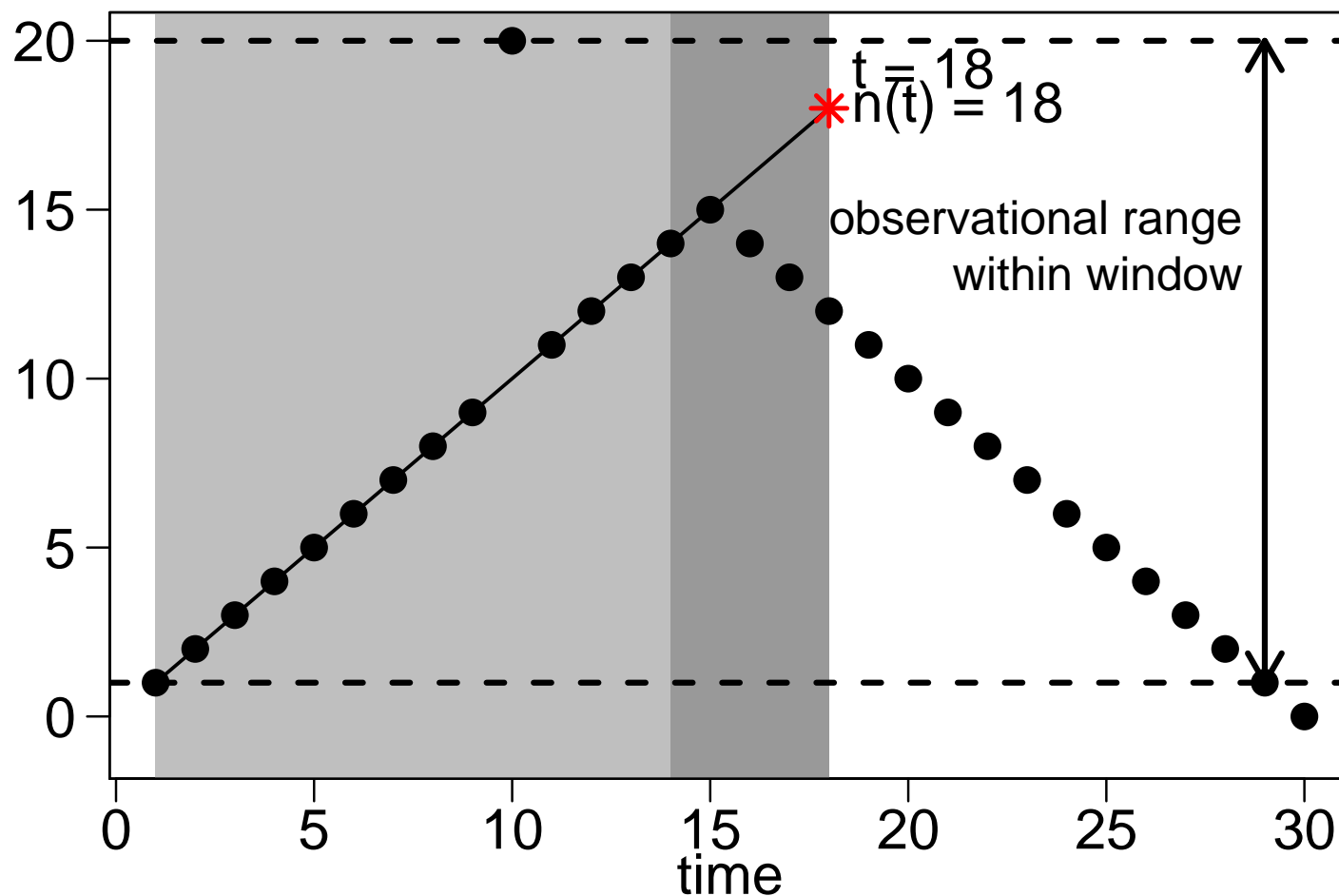
## adore.filter – Illustration

`min.width=11`   `p.test=5`   `max.width=100`  
`rtr=1`: 'restrict to range' of observations in the window



## adore.filter – Illustration

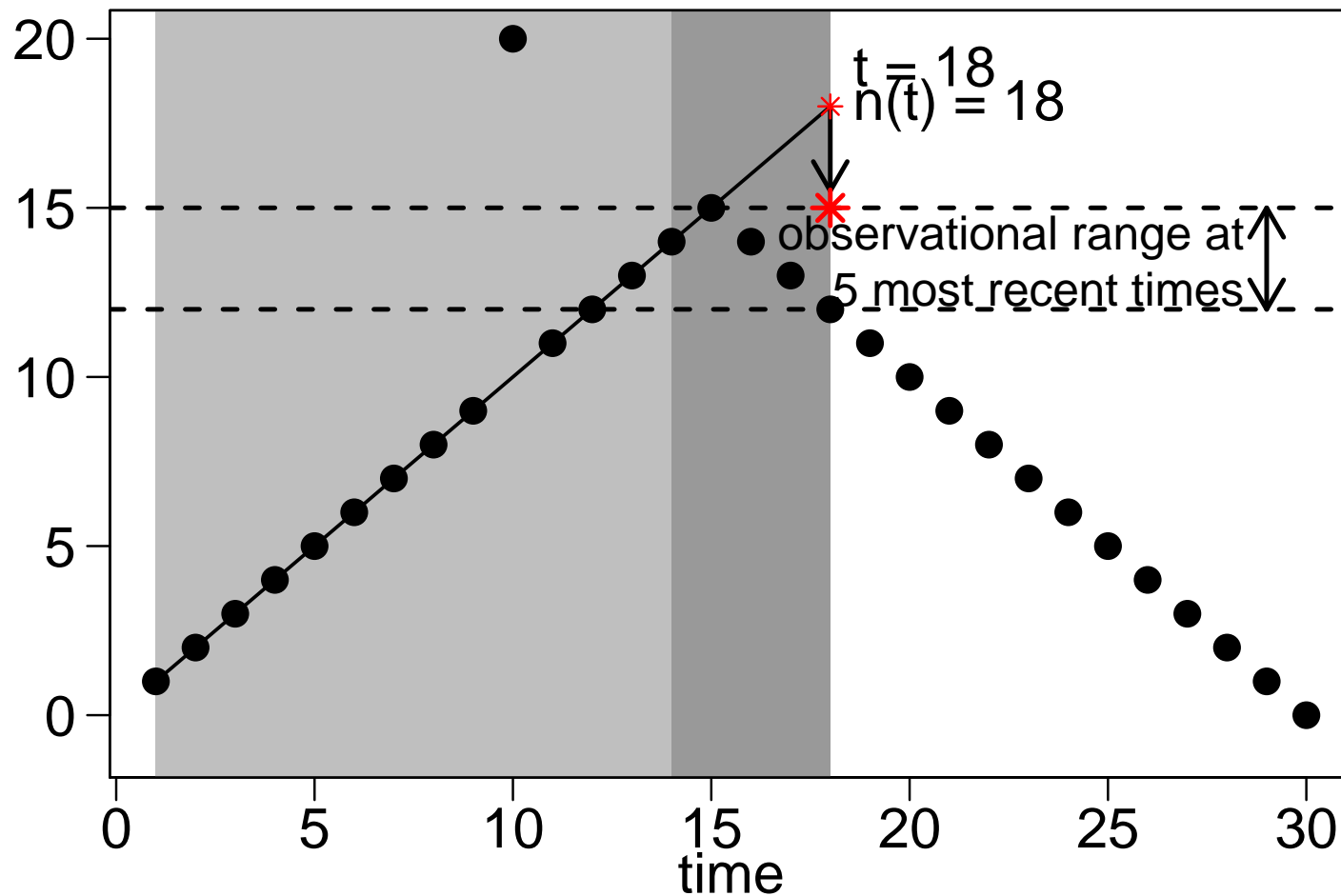
`min.width=11`   `p.test=5`   `max.width=100`  
`rtr=1`: 'restrict to range' of observations in the window



## adore.filter – Illustration

`min.width=11`   `p.test=5`   `max.width=100`

`rtr=2`: 'restrict to range' of `p.test` most recent observations

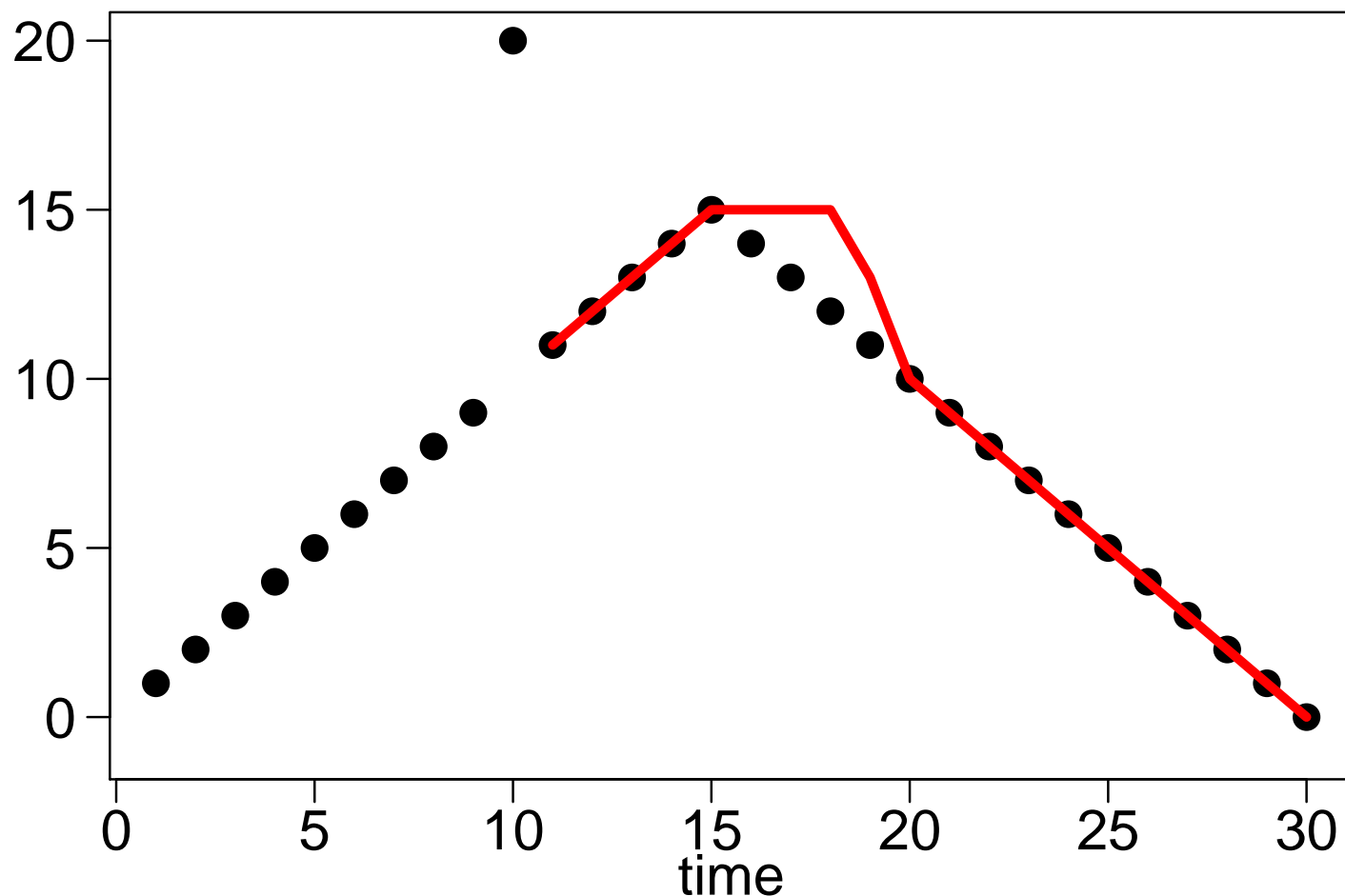




## adore.filter – Illustration

`min.width=11`   `p.test=5`   `max.width=100`

`rtr=2`: 'restrict to range' of `p.test` most recent observations



## madore.filter

(Borowski, Schettlinger, Gather 2008)

Time window of length  $n_t$  /  $k$ -variate sample with  $n_t$  observations

$$(\mathbf{y}_{t+i}) = \begin{pmatrix} y_{1, t-n_t+1} & \cdots & y_{1, t} \\ y_{2, t-n_t+1} & \cdots & y_{2, t} \\ \vdots & & \vdots \\ y_{k, t-n_t+1} & \cdots & y_{k, t} \end{pmatrix}$$

with  $i = t - n_t + 1, \dots, t$

## madore.filter

(Borowski, Schettlinger, Gather 2008)

Time window of length  $n_t$  /  $k$ -variate sample with  $n_t$  observations

$$(\mathbf{y}_{t+i}) = \begin{pmatrix} y_{1, t-n_t+1+\text{lag}} & \cdots & y_{1, t+\text{lag}} \\ y_{2, t-n_t+1+\text{lag}} & \cdots & y_{2, t+\text{lag}} \\ \vdots & & \vdots \\ y_{k, t-n_t+1+\text{lag}} & \cdots & y_{k, t+\text{lag}} \end{pmatrix}$$

with  $i = t - n_t + 1 + \text{lag}, \dots, t + \text{lag}$

and  $\text{lag} = 0$  for online estimation

## madore.filter

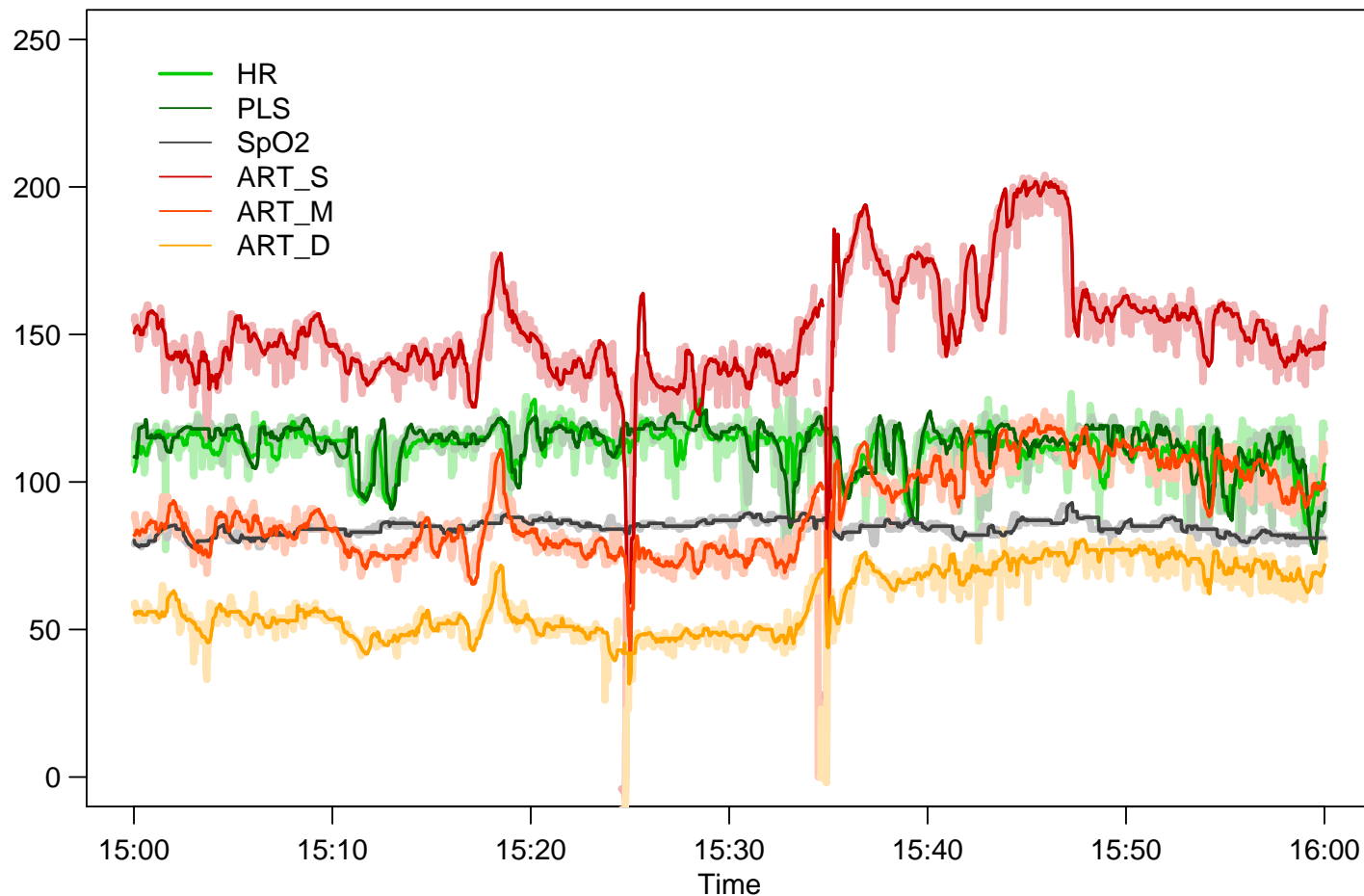
(Borowski, Schettlinger, Gather 2008)

Within a time window of length  $n_t$  / a  $k$ -variate sample ( $\mathbf{y}_{t+i}$ ):

1. Use adaptive univariate **RM** to find  $\hat{\boldsymbol{\mu}}_t$  and  $\hat{\boldsymbol{\beta}}_t$   
and to determine overall window width  $n_t$  ( $\rightarrow$  `adore.filter`)
2. Estimate local covariance matrix of multivariate residuals  $\mathbf{r}_{t+i}$
4. Trim observations  $\mathbf{y}_{t+i}$  where the Mahalanobis distance of the corresponding residuals  $\mathbf{r}_{t+i}$  is larger than  $d_{n_t}$  ( $\stackrel{e.g.}{=} \chi_{k;\alpha}^2$ )
5. Use multivariate Least Squares on the trimmed observations  
to find the level estimate  $\hat{\boldsymbol{\mu}}_t$   
( $\rightarrow$  `dw.filter`)

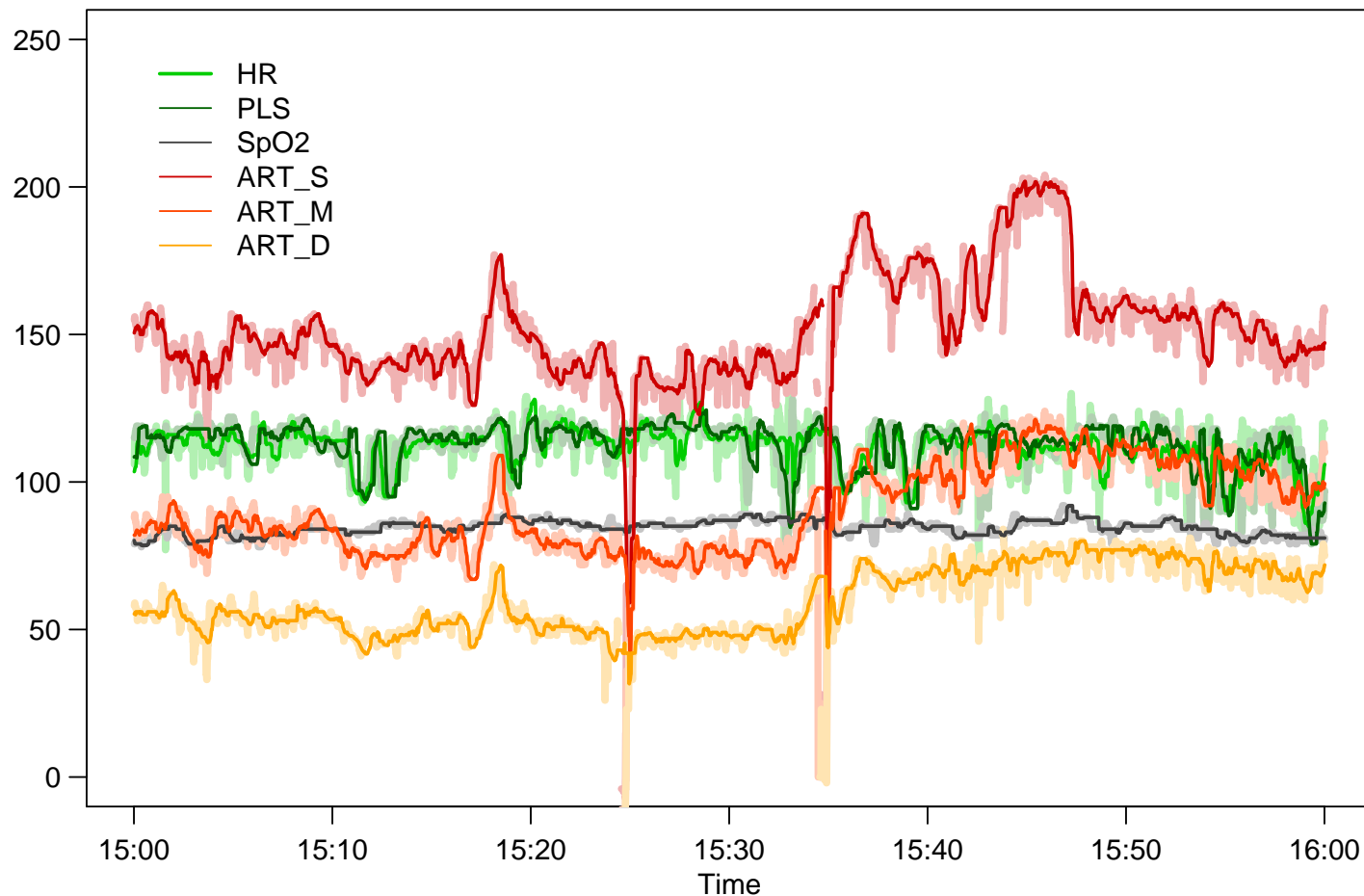
## (m)adore.filter – Application

```
adore.filter(..., min.width=61, p.test=10, rtr=0)  
component-wise application
```



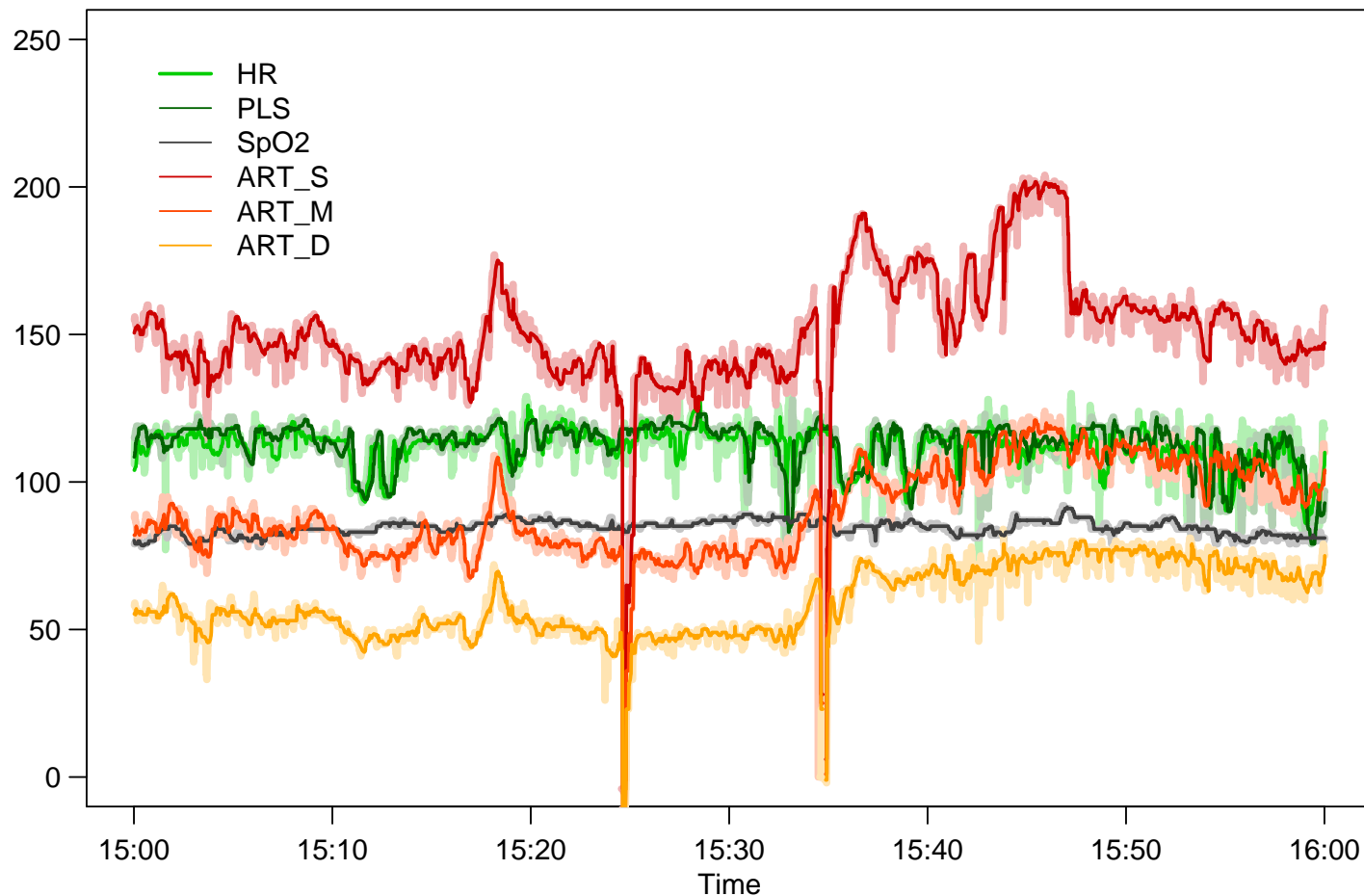
## (m)adore.filter – Application

```
adore.filter(..., min.width=61, p.test=10, rtr=1)  
component-wise application
```



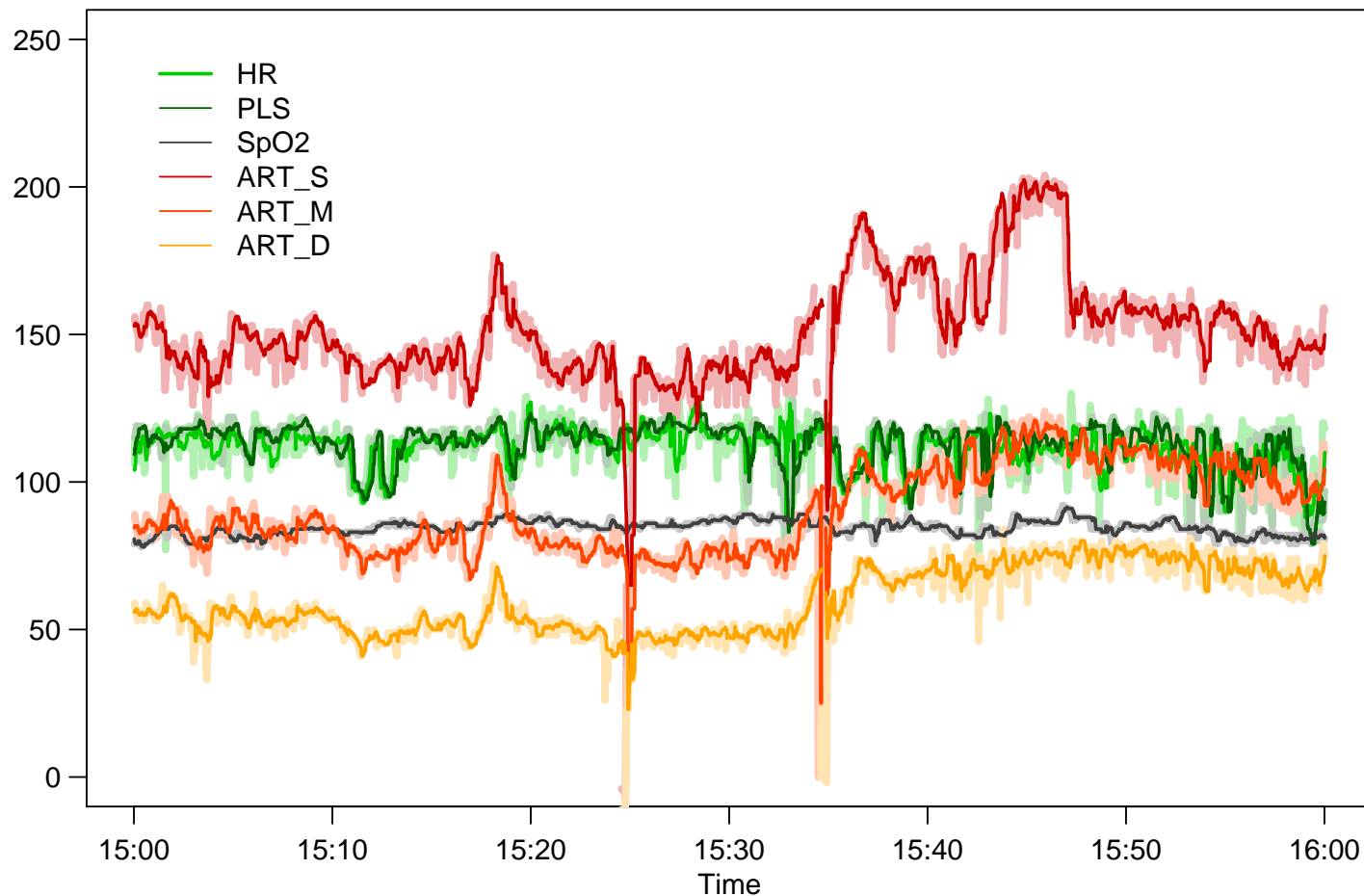
## (m)adore.filter – Application

```
adore.filter(..., min.width=61, p.test=10, rtr=2)  
component-wise application
```



## (m)adore.filter – Application

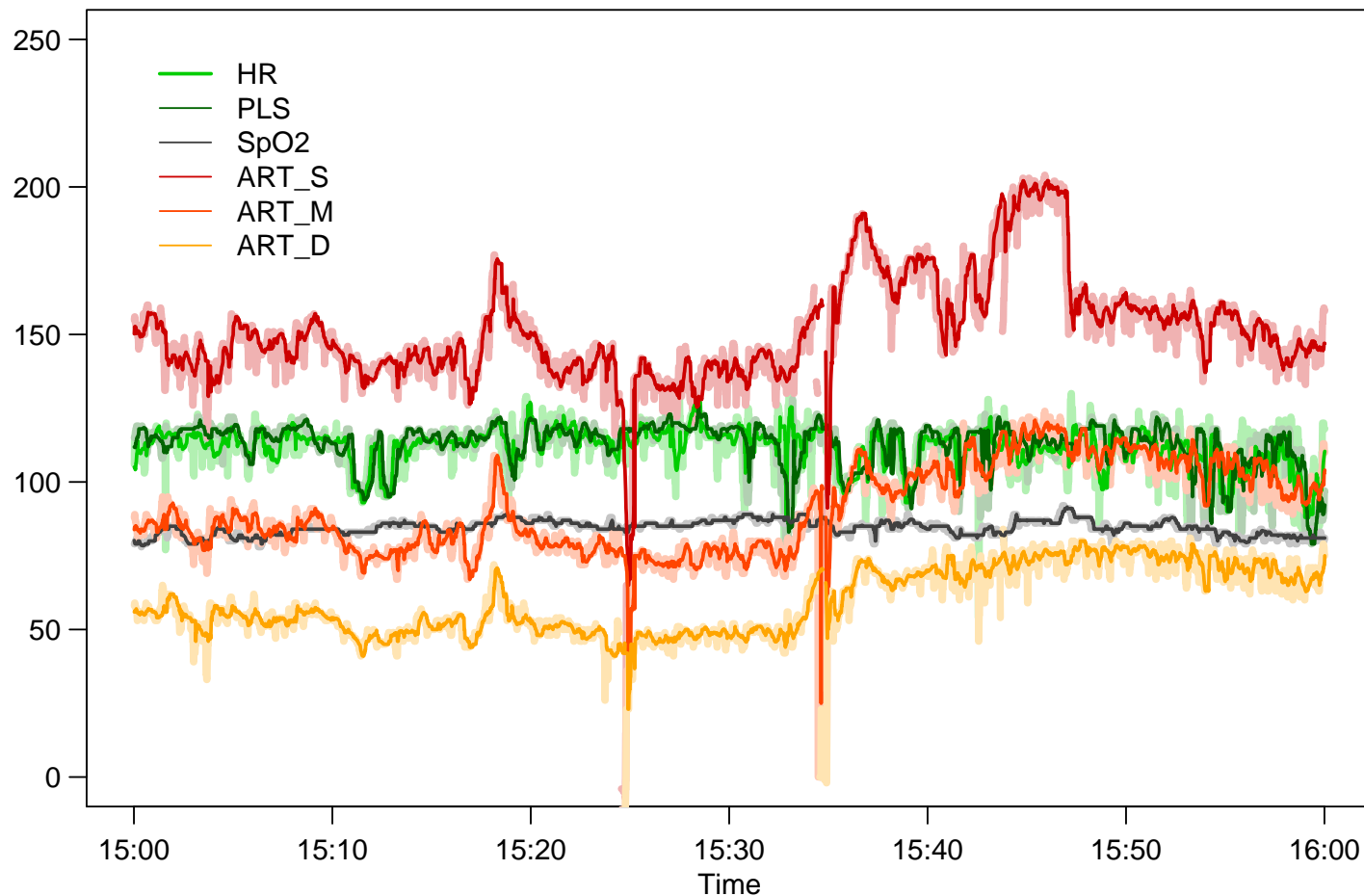
```
madore.filter(..., min.width=61, p.test=10, rtr=2)
```





## (m)adore.filter – Application

`madore.filter(..., min.width=61, p.test=10, rtr=2)`  
applied to blocks of highly correlated variables



## Filter Output

- An object of class `...filter` / a list of the elements:
  - `level` filtered signal level
  - `slope` corresponding slope within each time window

Each of these elements is a `data.frame`  
with column names specified by the applied `method(s)`.

## Filter Output

- An object of class `...filter` / a list of the elements:

`level` filtered signal level

`slope` corresponding slope within each time window

Each of these elements is a `data.frame`

with column names specified by the applied `method(s)`.

- Input parameters
- scale output within each time window possible for `hybrid.filter`, `dw.filter`, `robust.filter`, and `adore.filter`
- Filter specific output

# Summary

	with delay	online	missing values	adaptive widths	multivariate
<code>robreg.filter</code>	✓	✓	✓	✗	✗
<code>hybrid.filter</code>	✓	✗	✓	✗	✗
<code>dw.filter</code>	✓	✓	✓	✗	✗
<code>robust.filter</code>	✓	✓	✗	✓	✗
<code>wrm.filter</code>	✓	✓	✓	✗	✗
<code>adore.filter</code>	✗	✓	✓	✓	✗
<code>madore.filter</code>	✓	✓	✓	✓	✓

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