Stairstep-like dendrogram cut: a permutation test approach

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All computations and graphics were done using the R system (packages: cluster, clusterGeneration, ggplot2)

Slides has been composed using IAT_EX(*beamer* class) and the Sweave tool Stairstep-like dendrogram cut: a permutation test approach

(a not necessarily regular cut for a dendrogram)

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The rep1HighNoise dataset

Yeung KY, Medvedovic M, Bumgarner KY: Clustering gene-expression data with repeated measurements.

Genome Biology, 2003, 4:R34

It is a synthetic data set with error distributions derived from real array data.







Stairstep-like dendrogram cu





Stairstep-like dendrogram cu



Horizontal cut k = 2 (red clusters) k = 3



Stairstep-like dendrogram cu





Stairstep-like dendrogram cu







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Stairstep-like dendrogram cu



Horizontal cut k = 2 (red clusters) k = 3 (green clusters) k = 4 (blue clusters)



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Stairstep-like dendrogram cu





An alternative cut k = 3 (rainbow clusters)







k = 3 (rainbow clusters)







k = 4 (rainbow clusters)







k = 4 (rainbow clusters)







5 clusters







5 clusters

An alternative cut k = 5 (rainbow clusters)







 $\alpha = 0.01$ 5 clusters

An alternative cut k = 5 (rainbow clusters)



The reference framework



The reference framework



The reference framework





A (? not so ?) simple procedure

3 Some results





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A (? simple ?) idea

2 A (? not so ?) simple procedure

3 Some results

The Wishlist



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n the number of objects to classify;



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Stairstep-like dendrogram cu

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- n the number of objects to classify;
- C^k_L and C^k_R the two classes merged at level k (k=1,...,n-1)













- n the number of objects to classify;
- C^k_L and C^k_R the two classes merged at level k (k=1,...,n-1)
- h (C^k_L ∪ C^k_R) the height necessary to merge C^k_L and C^k_R









- n the number of objects to classify;
- C^k_L and C^k_R the two classes merged at level k (k=1,...,n-1)
- $h\left(C_{L}^{k} \cup C_{R}^{k}\right)$ the height necessary to merge C_{L}^{k} and C_{R}^{k}





 C_R^3

Let:

- n the number of objects to classify;
- C^k_L and C^k_R the two classes merged at level k (k=1,...,n-1)
- h (C^k_L ∪ C^k_R) the height necessary to merge C^k_L and C^k_R



 C_l^3



- n the number of objects to classify;
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- h (C_j^k) the height at which C_j^k has been obtained (j ∈ { L, R })





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 C_R^1



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Input: A dataset and its related dendrogram **Output**: A partition of the dataset



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Input: A dataset and its related dendrogram **Output**: A partition of the dataset

initialization:

```
aggregationLevelsToVisit \leftarrow h(C_L^1 \cup C_R^1)
permClusters \leftarrow []
i \leftarrow 1
```



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initialization:

```
aggregationLevelsToVisit \leftarrow h(C_L^1 \cup C_R^1)
permClusters \leftarrow []
i \leftarrow 1
```

repeat

```
if C_L^i \equiv C_R^i then| add C_L^i \cup C_R^i to permClusterselse| add h(C_L^i) and h(C_R^i) to aggregationLevelsToVisitsort aggregationLevelsToVisit in descending orderend
```



Input: A dataset and its related dendrogram **Output**: A partition of the dataset

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```
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```

Input: A dataset and its related dendrogram **Output**: A partition of the dataset

initialization:

```
aggregationLevelsToVisit \leftarrow h(C_L^1 \cup C_R^1)
permClusters \leftarrow []
i \leftarrow 1
```

repeat

```
if C_L^i \equiv C_R^i then| add C_L^i \cup C_R^i to permClusterselse| add h(C_L^i) and h(C_R^i) to aggregationLevelsToVisit| sort aggregationLevelsToVisit in descending orderendremove the first element from aggregationLevelsToVisiti \leftarrow i+1until aggregationLevelsToVisit is empty
```























clusters to compare
$$H_0: C_L^2 \equiv C_R^2 \mapsto \text{reject}$$











clusters to compare
$$H_0: C_L^3 \equiv C_R^3 \mapsto \text{reject}$$











clusters to compare
$$H_0: C_L^4 \equiv C_R^4 \mapsto \text{accept}$$



 $\frac{\text{Iteration}}{i \leftarrow 4}$

aggregationLevelsToVisit $h(C_R^3), h(C_R^2), h(C_L^2), h(C_L^3)$

permClusters $C_L^4 \cup C_R^4 \Leftrightarrow C_R^3$

clusters to compare $H_0: C_L^4 \equiv C_R^4 \mapsto ext{accept}$





 $\frac{\text{Iteration}}{i \leftarrow 9}$

aggregationLevelsToVisit $h(C_R^3), h(C_R^2), h(C_L^2), h(C_L^3)$

permClusters

 $C_{L}^{3},\,C_{R}^{3},\,C_{L}^{2},\,C_{L}^{4},\,C_{R}^{4}$



A (? simple ?) idea

A (? not so ?) simple procedure

3 Some results

The Wishlist



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For each *k*, the difference between $\max_{j \in \{L,R\}} h(C_j^k)$ and $\min_{j \in \{L,R\}} h(C_j^k)$ can be considered as the *minimum cost* necessary to merge the two classes.





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The difference between $h\left(C_{L}^{k} \cup C_{R}^{k}\right)$ and $\max_{j \in \{L,R\}} h\left(C_{j}^{k}\right)$ can be, instead, considered as the *cost* actually incurred for merging C_{L}^{k} and C_{R}^{k} .



The ratio between these two costs:

Let:

- n the number of objects to classify;
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$$\frac{\max_{j \in \{L,R\}} h\left(C_{j}^{k}\right) - \min_{j \in \{L,R\}} h\left(C_{j}^{k}\right)}{h\left(C_{L}^{k} \cup C_{R}^{k}\right) - \max_{j \in \{L,R\}} h\left(C_{j}^{k}\right)}$$

is thus a measure that characterizes the aggregation process resulting in the new class $C_L^k \cup C_R^k$



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Stairstep-like dendrogram cut



The algorithm retraces down-ward the tree, starting from the root of the dendrogram where all objects are classified in a unique cluster.



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The algorithm retraces down-ward the tree, starting from the root of the dendrogram where all objects are classified in a unique cluster.

 \forall *k* a *permutation test* is designed to test the *Null Hypothesis* that the two classes C_L^k and C_R^k really belong to the same cluster, i.e. :

$$H_0: C_L^k \equiv C_R^k$$





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Under H_0 , mixing up (*permuting*) the statistical units of C_L^k and C_R^k should not alter the aggregation process resulting in their merging in.





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$$H_0: C_L^k \equiv C_R^k$$

Under H_0 , mixing up (*permuting*) the statistical units of C_L^k and C_R^k should not alter the aggregation process resulting in their merging in.

Let ${}_{m}C_{L}^{k}$ and ${}_{m}C_{R}^{k}$ be the two new classes obtained by permuting the elements in C_{L}^{k} and C_{R}^{k}





Let ${}_{m}C_{L}^{k}$ and ${}_{m}C_{R}^{k}$ be the two new classes obtained by permuting the elements in C_{L}^{k} and C_{R}^{k} For each of them a new dendrogram is generated.





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For each of them a new dendrogram is generated.

The heights at which each of the two classes are buit up again, clearly correspond to the heights of the root nodes of the corresponding dendrograms.





 $cost \left(C_L^k \cup C_R^k \right)$ should be close enough.
The (? not so ?) simple procedure: detail



The permutation procedure is repeated *M* times and each time a new couple ${}_{m}C_{L}^{k}$, ${}_{m}C_{R}^{k}$ is obtained. The pvalue Montecarlo is thus computed as:

$$p = \frac{\# \left\{ cost\left({_mC_L^k \cup {_mC_R^k}} \right) \le cost\left({C_L^k \cup C_R^k} \right) \right\} + 1}{M+1}$$

A (? simple ?) idea

2 A (? not so ?) simple procedure

3 Some results

The Wishlist



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The yeast galactose dataset

Ideker T, Thorsson V, Ranish JA, Christmas R, Buhler J, Eng JK, Bumgarner RE, Goodlett DR, Aebersold R, Hood L Integrated genomic and proteomic analyses of a systemically perturbed metabolic network.

Science 2001, 292:929-934.

$$n = 205$$

p = 80

It is a subset of 205 genes that

reflect four functional categories

in the Gene Ontology listings.





Settings

distanceMethod = euclidean aggregationMethod = Ward $\alpha = 0.05$ M = 999



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The diabetes dataset

Banfield JD, Raftery AE Model-based Gaussian and Non-Gaussian Clustering. Biometrics, 1993, 49, 803-821. n = 145p=3It contains 145 subjects divided into three groups (normal, chemical diabetes, overt diabetes) on the basis of their oral glucose tolerance descripted by three variables





Settings

distanceMethod = euclidean aggregationMethod = Ward $\alpha = 0.05$ M = 999



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genRandomCluster numClust = 2:7 numNonNoisy = 5 sepVal = 0.01



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Stairstep-like dendrogram cu

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genRandomCluster

numClust = 2:7 numNonNoisy = 5 sepVal = 0.01

Settings

distanceMethod = euclidean aggregationMethod = Ward





genRandomCluster numClust = 2:7 numNonNoisy = 5 sepVal = 0.01

Settings

distanceMethod = euclidean aggregationMethod = Ward M = 999 $\alpha = 0.1$



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genRandomCluster numClust = 2:7 numNonNoisy = 5 sepVal = 0.01

Settings

distanceMethod = euclidean aggregationMethod = Ward M = 999 $\alpha = 0.05$





genRandomCluster numClust = 2:7 numNonNoisy = 5 sepVal = 0.01

Settings

distanceMethod = euclidean aggregationMethod = Ward M = 999 $\alpha = 0.01$



Some results... for 5 variables (100 replications)



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Stairstep-like dendrogram cut

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genRandomCluster numClust = 2:7 numNonNoisy = 10 sepVal = 0.01

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Stairstep-like dendrogram cu

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genRandomCluster

numClust = 2:7 numNonNoisy = 10 sepVal = 0.01

Settings

distanceMethod = euclidean aggregationMethod = Ward





genRandomCluster numClust = 2:7 numNonNoisy = 10 sepVal = 0.01

Settings

< 6 b

distanceMethod = euclidean aggregationMethod = Ward M = 999 $\alpha = 0.1$





genRandomCluster numClust = 2:7 numNonNoisy = 10 sepVal = 0.01

Settings

< 6 b

distanceMethod = euclidean aggregationMethod = Ward M = 999 $\alpha = 0.05$



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genRandomCluster numClust = 2:7 numNonNoisy = 10 sepVal = 0.01

Settings

< 6 b

distanceMethod = euclidean aggregationMethod = Ward M = 999 $\alpha = 0.01$



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Some results... for 10 variables (100 replications)



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Stairstep-like dendrogram cut

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genRandomCluster numClust = 2:7 numNonNoisy = 15 sepVal = 0.01





genRandomCluster

numClust = 2:7 numNonNoisy = 15 sepVal = 0.01

Settings

distanceMethod = euclidean aggregationMethod = Ward





genRandomCluster numClust = 2:7 numNonNoisy = 15 sepVal = 0.01

Settings

< 61 b

distanceMethod = euclidean aggregationMethod = Ward M = 999 $\alpha = 0.1$





genRandomCluster numClust = 2:7 numNonNoisy = 15 sepVal = 0.01

Settings

< 61 b

distanceMethod = euclidean aggregationMethod = Ward M = 999 $\alpha = 0.05$





genRandomCluster numClust = 2:7 numNonNoisy = 15 sepVal = 0.01

Settings

< 61 b

distanceMethod = euclidean aggregationMethod = Ward M = 999 $\alpha = 0.01$



Stairstep-like dendrogram cu

Some results... for 15 variables (100 replications)



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A (? simple ?) idea

A (? not so ?) simple procedure

3 Some results





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The wishlist

Statistical issues

R issues



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Stairstep-like dendrogram cut

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Image: A math a math

The wishlist

Statistical issues

- Quality measures of the obtained partition
- Use of different types of clusters
 - different cardinality of clusters
 - different type of cluster generation
- Study on the stability of the number of Montecarlo replications
- Computational complexity

R issues



The wishlist

Statistical issues

- Quality measures of the obtained partition
- Use of different types of clusters
 - different cardinality of clusters
 - different type of cluster generation
- Study on the stability of the number of Montecarlo replications
- Computational complexity

R issues

- profiling and optimizing the R code
- use of compiled code
- use of S3–S4 methods
- deploying a package