CDO Hedging and Risk Management with R

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Outline

1. Motivation
   - Credit risk instruments in Financial institutions’ books.

2. Theoretical framework
   - The insurance contract for exchanging credit risk.
   - Pricing a portfolio of CDO Tranches.

3. Risk quantification of synthetic CDOs
   - Synthetic CDO risk factors.

4. Optimal composition of an hedging tranche
   - Objective function choice.

5. Concluding Remarks
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Credit risk
A borrower can default its loan obligations

Financial institutions hold many illiquid assets in their books. Credit risk is an intrinsic feature of these assets.

Credit risk is the risk of loss arising from a borrower who might default its loan obligations.

Default events are the manifestation of credit risk. They are assumed to happen randomly and at unforeseeable times.

Default intensity, correlation and contagion effects affect the relevance of credit risk.
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Insurance against a default event. Single name instrument

Credit Default Swap

Risk averse

Protection Buyer

Periodic Payments (spread)

Risk seeker

Protection Seller

Reference entity

Risk averse

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Insurance contract for exchanging credit risk.

Pricing a portfolio of CDO Tranches.
Insurance against multiple defaults.

Collateralised Debt Obligation

Synthetic CDO

- CDS #1 (1m$)
- CDS #2 (1m$)
- CDS #3 (1m$)
- CDS #125 (1m$)

Special Purpose vehicle

- Senior tranche (12-100%)
- Mezzanine tranche (6-12%)
- Junior mezz (3-6%)
- Equity tranche (0-3%)
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Hedging and Risk Management of CDOs portfolio with R
A pool of Credit Default Swaps on different names

We consider a portfolio of CDO tranches on an underlying pool of CDS. We make the following assumptions:

- the value of each tranche in the portfolio is computed with a Monte Carlo simulation;
- the total portfolio value is obtained by summing the value of each tranche;
- the spreads and correlation remain constant over the optimization period.

The previous assumptions might be loosened by inserting a dynamics in the spreads and some contagion effects of the obligors default.
Computing the tranche’s spread

The loss distribution of each portfolio tranche can be computed employing the following scheme:

Monte Carlo Simulation with Gaussian Copula

1: procedure CDO LOSS DISTRIBUTION ( )

2: for (α = 1 to Nsims) do

3: draw $\varepsilon^\alpha \sim N(0, I)$ ▶ Uncorrelated deviates

4: compute $\phi^\alpha = A \cdot \varepsilon^\alpha$ ▶ $A$: Cholesky factor of disturbance cov matrix

5: for (obligor $i = 1$ to $n$) do

6: compute $\tau_i = F_i^{-1}(N(\phi))$ ▶ default times for each obligor

7: if ($\tau_i \leq T_j$) then

8: \{Loss$^\alpha_{Pool}(T_j) + = (1 - R_i) \cdot Notional_i$\}

EndFor loop $i$ over obligors

9: compute $Loss^\alpha_{Cum}(T_j) = \sum_{k=0}^{i} Loss^\alpha_{Pool}(T_k)$

EndFor loop $\alpha$ over replications

10: return (Loss$_{Cum}$)
Computing the Total portfolio value

Considering the protection buyer standpoint we have:

\[ V^\gamma(t) = -s^\gamma \cdot V_{\text{Fee}}(t) + V_{\text{Cont}}(t) \]  \hspace{1cm} (1)

the spread \( s^\gamma \) is computed at contract inception while \( V_{\text{Fee}}(t) \) and \( V_{\text{Cont}}(t) \) depend on the tranche losses.

Each of the tranche position can be long (protection buyer) or short (protection seller). For the total portfolio we have:

\[ \Pi(\{s_i\}, \rho, J) = \sum_{\gamma=1}^{n_{\text{tranches}}} \phi^\gamma \cdot V^\gamma(\{s_i\}, \rho, J) \]  \hspace{1cm} (2)
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Market risk factors

The CDO portfolio is exposed to all the risks driving changes in the market value of each tranche:

1. movements in the interest and exchange rates;
2. movements in the credit spread of obligors;
3. movements in the correlations among the obligors;

The last two factors are far more important than the first one.
Credit risk factors refer to the event of a default of an obligor in the underlying pool. For a CDO tranche the credit risk depends on:

1. tranche attachment point or degree of subordination;
2. tranche thickness;
3. degree of contagion effects in the defaults among obligors;
Credit spread sensitivities.

In the code snippet we show the computation of the PV for three different tranches as function of bumps applied to the spread of the underlying CDS.

```r
for (j in ssp) {
  ib <- j + (-ssp[1] + 1)
  spreade <- spreadbas + (j-1)*step
  lambda <- (spreade) / fden
  cequit <- TranPricing(nobl, delta, lambda, rho, Notio[1], c_0,
                        attp, dequi, rfree, Nsim,fleq)
  Vequ_base[ib] <- Vtranche(cequit, spbase, fleq)
  cmezz <- TranPricing(nobl, delta, lambda, rho, Notio[1], c_0,
                        amez, dmez, rfree, Nsim,fleq)
  Vmez_base[ib] <- Vtranche(cmezz, spbmez, fleq)
  ..... }
```
Tranche Present Value sensitivity with the spread

Experiment with $\rho = 0.2$

- 0 – 3% Tranche
- 3 – 6% Tranche
- 12 – 22% Tranche

Spread bump (bp)

Tranche PV

40 60 80 100 120 140

−0.5 0.0 0.5 1.0

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An important credit spread sensitivity figure is the $\Delta$ value of the tranche which is defined as

$$\Delta_i^\gamma = \frac{\Delta V^\gamma(\{s_i\})}{\Delta s_i}$$

where: $\Delta V^\gamma(\{s_i\})$ is the change in tranche $\gamma$ Present Value for a $\Delta s_i$ bump in the obligor $i$ spread.
Credit spread sensitivities.

Equity tranche Marginal Credit Spread for different correlation

- rho=0.
- rho=.2
- rho=.4
- rho=.6
- rho=.8

Delta value

Obligor Credit Spread (bps)

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Credit spread sensitivities.

Mezz tranche Marginal Credit Spread for different correlation

- $\rho = 0$
- $\rho = 0.2$
- $\rho = 0.4$
- $\rho = 0.6$
- $\rho = 0.8$

Delta value vs. Spread bump (bp)
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Objective function choice.

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The goal of the optimization exercise is to figure out the composition of a new CDO tranche with the wish to immunize the portfolio P/L against adverse market movements:

\[
\mathbb{E}(\hat{J}) = \min_{\{\hat{J}\}} \left( \sum_{\forall \beta} \Pi(\{s_i\}, \beta, \hat{J}) \right)
\]

where: \( J \) is the pool-obligor connectivity matrix, and \( s_i \) is the spread on obligor \( i \). Our goal is to minimize the spread sensitivity over a range of spread bumps.
Hedging the portfolio spread sensitivity.

When the number of obligors exceeds 6 or 7 the optimization cannot be tackled with standard derivative based methods. Here we have tried to employ the most popular stochastic heuristic method based on:

- Differential Evolution (*DEoptim*);
- Genetic Algorithms (*ga*);
- Simulation Annealing (*GenSA*);
Hedging the portfolio spread sensitivity.

```r
while (bet < 3) {
  attp <- trprop[it,1]; detp <- trprop[it,2]
  tposit <- trprop[it,3]
  spcur <- sp[,it] *(1+bet)
  lambda <- spcur/(1-delta) * 1e-4
  nobl <- length(lambda)
  if (nobl > 0) {
    tvalue <- TranPricing(nobl, delta, lambda, rho,
                          Notio[1], c_0, attp, detp, rfree, Nsim, tposit)
    spbatr <- spbase[it]
    PVpor <- PVpor + Vtranche(tvalue, spbase, flag)
  }
  spcur <- spx * (1 + bet)
  spcur <- x1 *spcur
  lambda <- spcur /(1-delta) * 1e-4
  nobl <- length(lambda)
  if (nobl > 0) {
    tvalue <- TranPricing(nobl, delta, lambda, rho,
                          Notio[1], c_0, attp, detp, rfree, Nsim, tposit)
    spbatr <- spbase[it]
    PVpor <- PVpor + Vtranche(tvalue, spbase, flag)
  }
  bet <- bet + .25
}
```

Objective function choice.

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Objective function choice.

Calling the Genetic Algorithm with binary variables.

```r
spvar <- spread
dimension <- length(spvar)
fn.call <- 0
tol <- 1e-3
fitness <- function(x1,spx,flag,sp,spbase,trprop, rho,ntranc) - myobj(x1,spx,flag,spm,spbase,
sink(file="CDOexa2GA.out",type=c("output"),split=T)

# now we run GA optimization
GAb <- ga(type = "binary", fitness = fitness, spx,flag, spm, spbase, trprop, rho, nTranc,
        nBits=length(x1), popSize = 100, pmutation = 0.2, maxiter = 50, run = 20,seed=712343)
summary(GAb)
print(GAb@solution)
```
Hedging the portfolio spread sensitivity.

For these optimizations we have only preliminary results:
- Heuristic optimization algorithms require a careful tuning;
- different seeds should be tested;
- some speed-up technique such as parallelization should be implemented.
Concluding Remarks

- We have written some *R* functions for evaluating portfolio P/L composed of CDO tranches;
- we have considered risk management issues in CDO portfolios;
- we have written some *R* functions for computing P/L sensitivities to correlation and spread variations;
- we have made a first attempt in employing evolutionary algorithm for computing the tranche composition minimizing the spread sensitivity of a CDO portfolio.
For Further Reading

A. De Servigny and N. Jobst.
The Handbook of Structured Finance.

G. Löffer and P.N. Posch.
Credit risk modeling using Excel and VBA.

C.C. Mounfield.
Synthetic CDOs Modelling, Valuation and Risk Management.
*Cambridge University Press, 2009.*
Our deeper understanding of Credit derivatives
Thank you for your attention.
Tak for din opmærksomhed.

Any questions?