Mixed-effects Maximum Likelihood Difference Scaling

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Difference scaling is a psychophysical procedure used to estimate interval, perceptual scales along a stimulus continuum. On each of a set of trials, an observer is presented with a quadruple, (I_a, I_b, I_c, I_d), drawn from an ordered set of stimuli, $\{I_1 < I_2 < \ldots < I_p\}$. The observer judges which pair shows the greater perceptual difference. Alternatively, a trial can consist of a triple, (I_a, I_b, I_c) and the observer judges whether the difference between (I_a, I_b) or (I_b, I_c) appears greatest. The fitting procedure estimates perceptual scale values, $\{\psi_1, \psi_2, \ldots, \psi_p\}$, that best capture the observer's judgments of the perceptual differences between the stimuli. The perceived difference between a pair of stimuli, (I_a, I_b), is modeled as the difference of perceptual scale values, $L_{ab} = \psi_b - \psi_a$. In comparing, pairs of pairs, we assume that the observer forms the noise-contaminated decision variable, $\Delta = L_{ab} - L_{cd} + \epsilon, \epsilon \sim \mathcal{N}(0, \sigma^2)$ and judges the difference between the pair (I_a, I_b) to be greater than that between (I_c, I_d) precisely when $\Delta > 0$. The parameter σ represents the observer's precision in judgment. We estimate σ and the remaining free parameters, $\psi_2, \ldots, \psi_p$, either by directly maximizing the likelihood, using the `optim` function or, because the decision rule is linear, via a generalized linear model, using `glm`. The package **MLDS** on CRAN implements these two approaches.

We extend these analyses to mixed-effects models, using `glmer` and `lmer` in the **lme4** package. This allows us to incorporate variability of observers and other sources as random effects, The design matrix constructed for `glm`, however, distributes the stimulus levels over different columns, somewhat like a factor variable, making it difficult to treat them as a single entity. We consider two strategies to deal with this situation using `glmer` and one using `lmer`.

1. redefining the decision variable in terms of a parametric function that describes well the perceptual scale and using this decision variable as a regressor in the model formula,
2. redefining the decision variable in terms of an observer's own estimated perceptual scale and using this decision variable as a regressor in the model formula,
3. fitting the estimated perceptual scale values as a linear mixed-effects model, using `lmer`.

We demonstrate the three approaches with example data sets and discuss their pros and cons.

References

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