A Tale of Two Theories

Reconciling random matrix theory and shrinkage estimation as methods for covariance matrix estimation

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Estimation error in asset returns covariance matrices has plagued the portfolio optimization process ever since Markowitz first proposed the mean-variance approach. To combat this problem, two competing theories have developed to eliminate (or more appropriately, reduce) this estimation error: random matrix theory and shrinkage estimators. While attempting to solve the same problem, the approaches are substantially different. Random matrix theory states that a truly random matrix has a characteristic limit distribution of its eigenvalues. This distribution can be used as a null hypothesis to remove, by scaling to a lower bound, all such eigenvalues associated with this idiosyncratic noise in the sample covariance matrix. In contrast, shrinkage estimation leverages the central limit theorem to shrink covariances toward a biased covariance matrix that lacks estimation error (and hence better represents the unobserved true covariance matrix).

This paper explores these two competing approaches using an R package developed by the author to estimate a covariance matrix of asset returns. This analysis attempts to identify the constraints under which each method best performs and whether there is space to reconcile the two approaches into a single unified framework. In order to compare the performance of the theories, empirical and generated data is used to compute standard portfolio performance metrics. The package itself contains methods for calculating the theoretical Marcenko-Pastur eigenvalue distributions and functions for fitting empirical eigenvalue distributions to the closest Marcenko-Pastur curve. Implementations are also provided for shrinkage using a variety of shrinkage targets, including a single factor model, a constant correlation model, and a multi-factor model.

References

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