Simulating Games on Networks with \texttt{R}

Application to Coordination in Dynamic Social Network Under Heterogeneity

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Introduction: Networks in social sciences

- “No man is an island!”
- Outcomes of social and economic processes are determined not only by actors’ attributes but also by the structure of relations between them (Granovetter, 1985).
- Actors (nodes/vertexes): individuals, organizations, states...
- Relations (links/edges): cooperation, friendship, communication, joint activities...
- Node attributes: gender, race, age, music tastes...
- Dyadic attributes: geographical distance, taste similarity...
In this talk

- Using **R** to aid theory development
- (No data, ... at least from the real world)
- Framework for simulating certain kinds of models of social network dynamics
- Example of specific theoretical model
Outline

1. The model
2. Simulation
3. Results
   - Types of stable networks
4. Implementation
5. Summing-up
Network games

- Game-theoretical models of network and behavior dynamics
- Actors’ action spaces include both relational and behavioral alternatives
- Network utility function of actor $i$

$$U_i : G, \mathcal{X} \mapsto \mathbb{R}$$

$U_i(g, X)$

where $g \in G$ is the graph, and $X \in \mathcal{X}$ is the matrix of individual (node-level) attributes

- Existing models: Connections, Co-author (Jackson & Wolinski 1996), coordination (Goyal & Vega-Redondo 2005, Jackson & Watts 2002), R&D collaboration (Goyal, 2007) and more
- “Solving” by looking for various forms of equilibria/stability
Coordination in dynamic social network

- Fixed population of \( n \) actors composed of two groups (types) \( A \) and \( B \)
- Every actor chooses one of the two behavioral options \( x \) or \( y \) (behavior)
- Actors form an undirected network \( g = [g_{ij}]_{n \times n} \)

Utility of actor \( i \):

\[ +w \text{ for every relation with actor behaving the same as } i \]
\[ +b \text{ if } i \text{ is of type } A \text{ choosing } x \text{ or of } B \text{ choosing } y \]
\[ -\alpha \mu_i - \beta \mu_i^2 \text{ Cost of maintaining ties where } \mu \text{ is the total number of relations of } i \]
Network benefits

\[
\begin{array}{c|cc|c}
  \text{A} & x & y \\
  \hline 
  A & b + w, b + w & b, 0 \\
  x & w, w & 0, b \\
  y & 0, b & w, w \\
  \hline 
  \text{Within type A} & 0 < b \leq w & \\
\end{array}
\]

\[
\begin{array}{c|cc|}
  \text{B} & x & y \\
  \hline 
  B & b + w, b + w & b, b \\
  x & w, w & 0, b \\
  y & 0, 0 & w, b + w \\
  \hline 
  \text{Between types} & \\
\end{array}
\]

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Solution concept

**Pairwise stability** (Jackson & Wolinsky, 1996)

**Definition (Pairwise stability)**

The network \( g \) is **stable** if and only if the following three conditions are jointly satisfied:

1. There is no pair of actors in \( g \) who would benefit from creating a tie.
2. There is no actor in \( g \) who is interested in deleting a tie.
3. No actor would benefit from changing his behavior.
Simulation setup

- Generate a set of initial conditions: model parameters, initial network, type and behavior distribution
- Actors update their network or behavior in a random order
- Tie formation requires consent from the other player, deletion not (bilateral formation, unilateral deletion)
- The process is run until no change is possible. The final state is pairwise-stable
Results

- Small scale: qualitative analysis (visualizations with pictures and movies)
- Large scale: statistical analysis of generated data
Types of stable networks

Connected center-periphery structures
Behavior-segregated components
Sparse “lines”
Types of stable networks

Fairly integrated populations of “native” players
Used packages

- **network** for storing networks with vertex attributes
- **simecol** as the simulation workhorse
- **sna** and **rSoNIA** for network analysis and visualization
- plus a lot of tweaking (saving results, reading condition data)
Conclusions

- **R** is a convenient simulation platform, although perhaps not the most efficient
- Already some social network analysis functionality (network, dynamicnetwork, igraph, sna, ergm and more)

On the agenda:

- Finishing development of a package for simulating any network utility function
  - Modular architecture: type of dynamics, modeling dyadic interactions, tie cost functions, reputation, beliefs
  - Flexible result saving
  - Visualization