VAR Model: Theoretical Basics

- **VAR(p) model for k-variate process** $Y_t$:
  \[ Y_t = \nu + A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + \varepsilon_t. \]

**Standard Modeling Steps:**
1. Identification of relevant variables and data initial analysis
2. Lag order selection
3. Parameter estimation for selected model
4. Forecasting
5. Residual analysis

Special Characteristics of AuFVAR

1. Incorporation of time trend and season as exogenous variables:
   \[ Y_t = \nu + n_{1t} \nu_1 + \ldots + n_{(s-1)t} \nu_{s-1} + \gamma t_t + A(L) Y_t + \varepsilon_t. \]

- **Season Dummies**
- **Time Trend**

2. Definition of a VAR model with $n$ time series in each model
3. Automated model structure selection (settings und lag order) by means of the information criteria and backtesting
4. Repeated forecasts for different VAR models with similar time series structure
Data Initial Analysis in AuFVAR

**Phase 1**

- **Initial Analysis of Time Series**
  - **Stationarity Test**
  - **Granger Causality**
  - **Log Transformation**

**Empirical Example**

1. **Stationarity Test: ADF Test**
   - Function: `adf.test` (modified)
2. **Granger Causality Test**
   - Function: `grangertest` (standard)
3. **Log Level Transformation**
   - Natural Logarithm of Time Series

Test for Structural Breaks

- Testing for the structural breaks:
  - Function `breakpoints`, library `strucchange`
  - Determining the number of breakpoints
  - Computation of the optimal breakpoints
  - Adjustment of the model definition

Model Selection

**Phase 2**

- **Automated Model Selection**
  - Lag Order
  - Model Settings

**User Defined Settings**

1. **Information Criteria, e.g.**
   \[
   AIC(p) = \ln \text{det}(\hat{\Sigma}(p)) + \frac{2}{T}pK^2, \]
2. **Backtesting (out-of-sample):**
   \[
   RMSE = \sqrt{\frac{1}{h} \sum_{i=1}^{h} (y_{T+h} - y_{T})^2}.\]

Estimation and Forecasting

**Phase 3**

- **Test for Structural Breaks**
  - **Determining the Number of Break Points**
  - **Computation of the Optimal Break Points**

**Phase 4**

- **Coefficient Estimation and Forecasting**
  - **VAR Model (OLS)**
  - **VAR with Trend and Season (OLS)**
  - **Recursive Forecast**

1. **Estimation of the appropriate model after testing for structural breaks**
   - Function: `estVARXls` for the models with intercept and lagged variables (standard)
   - Function: `estVARXlsM` for the models with additional time trend and/or season (modified)
2. **Forecasting the time series, computation of confidence bounds and corresponding plots**
Residual Analysis

- Portmanteau Test for Autocorrelation:
  - The standard test statistic, if time series length $T \geq 100$
  - The modified test statistic otherwise

- Test for Nonnormality based on Skewness and Kurtosis (Lütkepohl, 1993)

Example: Time Series and Forecasting

Data
- Variables to be forecasted:
  - Common Advertisement Earnings (with/without media)

- Further possible variables for the VAR model:
  1. ZEW Index (Centre for European Economic Research)
  2. Incoming Orders (Germany)
  3. CDAX (Composite DAX)