Overview

- Directed Acyclic Graph (DAG) and its skeleton
- The PC-algorithm for finding the skeleton is consistent
- R-package: pcalg

Directed Acyclic Graphs (DAGs)

- Nodes: Random Variables
- Edges: Some Dependence
- Recursive factorization:
  \[ f(GM,C,S) = f(GM) f(C|GM) f(S|GM) \]

Directed Global Markov Property

- DAG implies conditional independence relations
- \( C \perp S | GM \iff C,S \text{ are separated by } GM \)
- In
  \[ \left( G_{An(C\cup S\cup GM)} \right)^m \]

- Ancestral set
- Moralize
- Drop directions
Skeleton of a DAG

- Ignore directions of arrows
- Edge between two nodes $A$ and $B$ 
  $A, B$ are dependent given every subset of 
  remaining nodes

PC-algorithm for finding the skeleton

1. Form complete graph $G$
2. $l = -1$
3. repeat
   1. $l = l + 1$
   2. repeat
      1. select ordered pair of adjacent nodes $A, B$ in $G$
      2. select neighborhood $N$ of $A$ with size $l$ (if possible)
      3. delete edge $A, B$ in $G$ if $A, B$ are cond. indep. 
         given $N$
   until all ordered pairs have been tested
4. until all neighborhoods are of size smaller than $l$

Main Result

- Test Cond. Indep. Relations $A \perp B \mid S$ consistently
- PC-Algorithm is consistent 
  (more detailed results when using assumptions)

R-package: pcalg

- Estimate the skeleton given a data matrix
- Visualize the estimated skeleton