Introduction

Objectives of Sensitivity Analysis (examples):

- Help identify key sources of variability (to assist policy making, risk management strategy)
- Help identify key sources of uncertainty (to prioritize additional data collection to reduce uncertainty)
- Variance of an output
- What causes worst/best outcomes
- What are critical control points, critical limits

Local vs. Global Sensitivity Analysis

Model Dependent vs. Model Independent Sensitivity Analysis

Applicability of methods often depends upon characteristics of a model (e.g., nonlinear, thresholds, categorical inputs, etc.)

Moving from Uncertainty Analysis to Sensitivity Analysis

Uncertainty Analysis UA (Janssen, RIVM, The Netherlands):
The study of the uncertain aspects of a model and of their influence on the (uncertainty of the) model output

Sensitivity Analysis SA (Saltelli, EU JRC, Ispra):
The study of how the uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input
**Ideal SA Method**

- **Cope with scale and shape of the input factors:** Range of the factor variation and shape / parameters of the pdf.
- **Include multi-dimensional averaging:** Global versus local methods
- **Model independent (model free):** Cope with non-linear / non-additive, non-monotonic models
- **Grouping of factors:** Treat grouped factors as if they were single factors

**Cost efficient**

Pay attention to computational costs

**SA types**

- Local or global
- Qualitative or quantitative

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**Sobol’ Sensitivity Measures**

**First-order Sensitivity Measure \( (S_i) \)**

Measures the fractional contribution of \( x_i \) to the variance of \( f(x) \) without accounting for interactions of \( x_i \) with the other factors.

\[
S_i \equiv \frac{V_{x_i} (E_{x_{\{i\}}}(Y|X_i))}{V_Y}
\]

**Total-order Sensitivity Measure \( (TS_i) \)**

The sum of all the sensitivity measures involving the factor in question.

E.g. for a model with three input factors, \( TS_1 = S_1 + S_{12} + S_{13} + S_{123} \).

\[
TS_i \equiv \frac{E_{x_{\{i\}}}(V_{x_i}(Y|X_{\{i\}}))}{V_Y}
\]

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**Sobol’ LP\( _{\tau} \) sampling**

- Each Sensitivity Measure is a quotient of integrals in a multidimensional space, which can be approximated via MC integration.
- For large or computer-intensive models it is important that the integral be approximated with as few model evaluations as possible.
- The LP\( _{\tau} \) sequences have the property of always generating points which are regularly distributed in the factor space.

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**2002 Knowledge Economy Index**

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<th>GERD</th>
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<th>RES</th>
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21 missing values

value above the mean

value below the mean
Acknowledging assumptions in the development of the Index

1. **Selecting Indicators**
   - Inclusion-Exclusion of one indicator-at-a-time

2. **Imputation**
   - Trend model: least squares polynomial regression + t-test for the estimates of the std for regression coefficients

3. **Weighting**
   - Equal weights
   - Conceptual model
   - Country-specific weights

4. **Aggregation**
   - Linear
   - Geometric

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Uncertainty analysis results

Investing in the Knowledge Economy (EU-15):
AT has a 35% probability to be among the top 5 countries and 0% probability to be among the bottom 5 countries

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Sensitivity analysis results (Sobol’ method)

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<td>0.718</td>
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**First order**: Capture individual impact

**Total effect**: Capture interactions/synergies

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Sensitivity analysis as a tool to identify thresholds

Selected countries rank versus two important imputed values:
- PhD FR \(\sim N(6428,476)\)
- TES UK \(\sim N(4.52,0.17)\)

Regardless of the changes in the other factors (imputed values, aggregation, weighting, set of indicators)...  
- **France** will not fall behind the 6th position if the expected number of PhD students is 7200.  
- **UK** will not fall behind the 8th position if the expected value for TES = 4.52% is the correct one.
Further reading

JRC Information Server on Composite Indicators at http://farmweb.jrc.cec.eu.int/ci/