

robKalman — a package on Robust Kalman Filtering

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Classical setup: Linear state space models (SSMs)

- State equation:

$$X_t = F_t X_{t-1} + v_t$$

- Observation equation:

$$Y_t = Z_t X_t + \varepsilon_t$$

- Ideal model assumption:

$$X_0 \sim \mathcal{N}_p(a_0, \Sigma_0), \quad v_t \sim \mathcal{N}_p(0, Q_t), \quad \varepsilon_t \sim \mathcal{N}_q(0, V_t),$$

all independent

- (preliminary ?) simplification: Hyper parameters F_t, Z_t, V_t, Q_t constant in t



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Problem and classical solution

- Problem: Reconststruction of X_t by means of $Y_s, s \leq t$
- Criterion: MSE
- \rightsquigarrow general solution: $\mathbb{E} X_t | (Y_s)_{s \leq t}$
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Kalman filter

- 0 Initialization ($t = 0$):

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- 1 Prediction ($t \geq 1$):

$$X_{t|t-1} = FX_{t-1|t-1}, \quad \text{Cov}(X_{t|t-1}) = \Sigma_{t|t-1} = F\Sigma_{t-1|t-1}F' + Q$$

- 2 Correction ($t \geq 1$):

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + K_t(Y_t - ZX_{t|t-1}) \\ K_t &= \Sigma_{t|t-1}Z'(Z\Sigma_{t|t-1}Z' + V)^{-1}, \quad (\text{Kalman gain}) \\ \text{Cov}(X_{t|t}) &= \Sigma_{t|t} = \Sigma_{t|t-1} - K_tZ\Sigma_{t|t-1} \end{aligned}$$



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Types of outliers and robustification

- IOs (system intrinsic): state equation is distorted — not considered here
- AO/SOs (exogeneous): observations are distorted:
 - either error ε_t is affected (AO)
 - or observations Y_t are modified (SO)
- a robustifications as to AO/SOs is to
 - retain recursivity (three-step approach)
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Considered approaches

Approximate conditional mean (ACM): [Martin(79)]

- $\dim Y_t = 1$
- particular model: $Y_t \sim \text{AR}(p)$
 - $\rightsquigarrow X_t = (Y_t, \dots, Y_{t-p+1})$,
 - hyper parameters $Z = (1, 0, \dots, 0)$, $V^{\text{id}} = 0$, F , Q unknown
- estimation of F , Q by means of GM-Estimators
- modified Corr.step: for suitable location influence curve ψ

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + \Sigma_{t|t-1} Z' \psi(Y_t - ZX_{t|t-1}) \\ \Sigma_{t|t} &= \Sigma_{t|t-1} - \Sigma_{t|t-1} Z' \psi'(Y_t - ZX_{t|t-1}) Z \Sigma_{t|t-1} \end{aligned}$$



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rLS filter: [P.R.(01)]

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Goal: package robKalman

Contents

- Kalman filter: filter, Kalman gain, covariances
- ACM-filter: filter, GM-estimator
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- further recursive filters?
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- Programming language
 - completely in S
 - perhaps some code in C (much) later
- Use existing infrastructure
 - from where to “borrow” :
 - univariate setting: KalmanLike (package stats);
time series classes: ts, its, irts, zoo, zoo.reg, tframe
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 - internal functions: no S4-objects
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Concept and strategy III

- Use of S4
 - Hierarchic Classes:
 - state space models (SSMs) (Hyper-Parameter, distributional assumptions, outlier types)
 - filter results (specific subclass of (multivariate) time series)
 - control structures for filters (tuning parameters)
 - Methods:
 - filters (for different types of SSMs)
 - accessor/replacement functions
 - simulate for SSMs
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 - tests?
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Implementation so far: interfaces

- preliminary, “S4-free” interfaces
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 - ↔ close contact to
 - RCore,
 - Paul Gilbert,
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 - casting/conversion functions for various time series classes
 - User interface robfilter (?)
 - goal: four arguments: data, SSM, control-structure, filter type
 - should cope with various definitions of SSMs, data in various time series classes,
 - possibly simpler interfaces for ACM (Splus-compatibility) ↔ ACMfilt-like
 - Release schedule
 - wait for results of discussion as to class definition
 - guess: end of 2006

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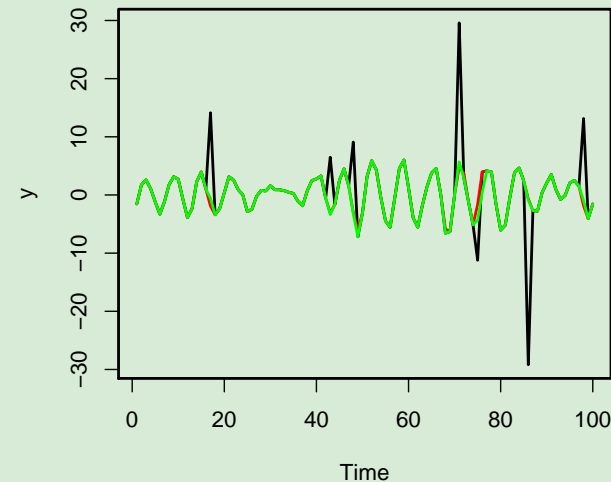
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Demonstration: `ACMfilt`

```
## generation of data from AO model:
set.seed(361)
Eps ← as.ts(rnorm(100))
ar2 ← arima.sim(list(ar = c(1, -0.9)),
               100, innov = Eps)
Binom ← rbinom(100, 1, 0.1)
Noise ← rnorm(100, sd = 10)
y ← ar2 + as.ts(Binom*Noise)

## determination of GM-estimates
y.arGM ← arGM(y, 3)
## ACM-filter
y.ACMfilt ← ACMfilt(y, y.arGM)

plot(y)
lines(y.ACMfilt$filt, col=2)
lines(ar2, col="green")
```



green: ideal time series,
black: AO contam. time series,
red: result ACM

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Demonstration: rLSFilter

```
## specification of SSM: (p=2, q=1)
a0 ← c(1, 0); S0 ← matrix(0, 2, 2)
F ← matrix(c(.7, 0.5, 0.2, 0), 2, 2)
Q ← matrix(c(2, 0.5, 0.5, 1), 2, 2)
Z ← matrix(c(1, -0.5), 1, 2)
Vi ← 1;
## time horizon:
TT←50
## AO-contamination
mc ← -20; Vc ← 0.1; ract ← 0.1
## for calibration
r1←0.1; eff1←0.9

#Simulation::
X ← simulateState(a, S0, F, Q, TT)
Yid ← simulateObs(X, Z, Vi, mc, Vc, r=0)
Yre ← simulateObs(X, Z, Vi, mc, Vc, ract)
```

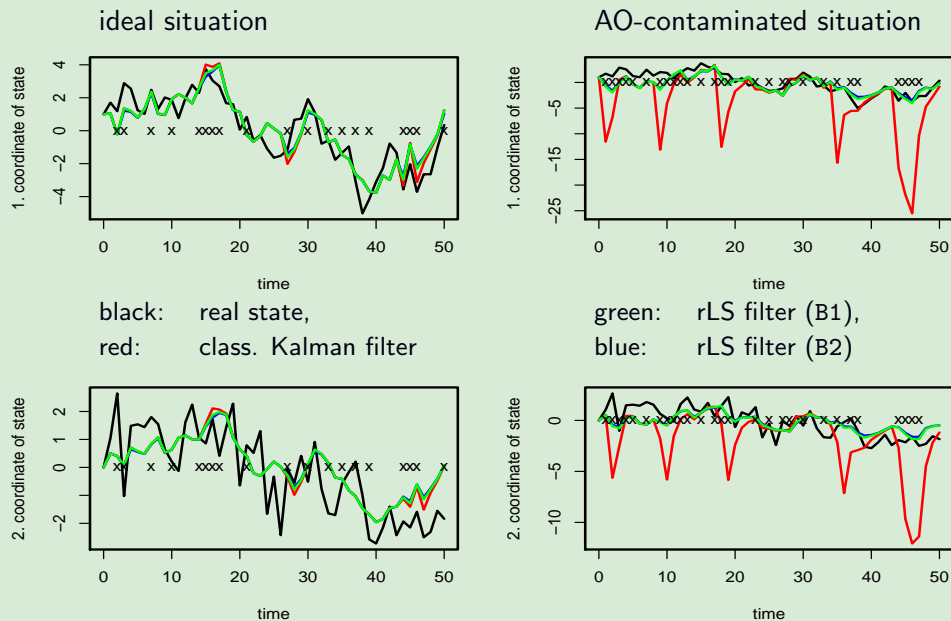
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Demonstration: rLSfilter II

```
### calibration b
#limiting S_{t|t-1}
SS ← limitS(S, F, Q, Z, Vi)
# by efficiency in the ideal model
(B1 ← rLScalibrateB(eff=eff1, S=SS, Z=Z, V=Vi))
# by contamination radius
(B2 ← rLScalibrateB(r=r1, S=SS, Z=Z, V=Vi))






### evaluation of rLS
rerg1.id ← rLSFilter(Yid, a, Ss, F, Q, Z, Vi, B1$b)
rerg1.re ← rLSFilter(Yre, a, Ss, F, Q, Z, Vi, B1$b)
rerg2.id ← rLSFilter(Yid, a, Ss, F, Q, Z, Vi, B2$b)
rerg2.re ← rLSFilter(Yre, a, Ss, F, Q, Z, Vi, B2$b)
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