BAYESIAN COVARIANCE SELECTION IN HIERARCHICAL LINEAR MIXED MODELS

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We present an MCMC algorithm to parsimoniously estimate the random-effects covariance matrix in hierarchical linear mixed models. The definite structure of zero and non-zero elements in the variance-covariance matrix is chosen in a data-driven manner in the course of the modeling procedure. Thereby model selection with regard to fixed versus random effects is automatically included. We specify a straightforward MCMC scheme for joint selection of elements of the random-effects covariance matrix and parameter estimation.

We write the model in the non-centered parameterization, see e.g. [3], which is based on the Cholesky decomposition of the random-effects covariance matrix: $Q = C \cdot C'$, with a lower triangular C. The structure of this model representation allows to identify zeros in the Cholesky factors by common variable selection methods, [2]. This approach is related to ideas of [4], who introduced covariance selection for multivariate normal data.

We contribute to on-going research about random-effects models in various respects. The non-centered parameterization with the above Cholesky decomposition allows us to choose a conditionally conjugate normal prior for C and automatically leads to non-negative definite covariance matrices. A straightforward Gibbs sampling scheme may easily be derived but contrary to the common inverted Wishart prior our new prior is less influential on posterior inference. Existing approaches to estimate a parsimonious variance-covariance matrix are by [4], who select *off-diagonal* elements and by [1], who determine *whole rows and columns* as zero or non-zero. Our choice of the Cholesky decomposition makes it possible to determine zeros and non-zeros for *each* element of the variance-covariance matrix. The ability to specify the finer structure of the random-effects covariance matrix turned out to be of hight importance in real applications with higher dimensional parameters.

New R-code is developed for this method and a real-data example from marketing is given as an illustration.

References

[1] Z. Chen and D. Dunson. Random effects selection in linear mixed models. *Biometrics*, 59:762–769, 2003.

[2] E. I. George and R. E. McCulloch. Approaches for Bayesian variable selection., *Statistica Sinica*, 7:339–373, 1997.

[3] X.-L. Meng and D. van Dyk. Fast EM-type implementations for mixed effects models. Journal of Royal Statistical Society B, 60:559–578, 1998.

[4] M. Smith and R. Kohn. Parsimonious covariance matrix estimation for longitudinal data. *Journal of the American Statistical Association*, 97:1141–1153, 2002.