frequencyConnectedness

Computing Diebold & Yilmaz and Barunik & Krehlik measures of connectedness

Tomáš Křehlík

Institute of Economic Studies, Charles University in Prague, Czech Republic
Econometric Department, Institute of Information Theory and Automation of Academy of Sciences of the Czech Republic

Abstract

The package operationalizes estimation of the standard multivariate system connectedness measures defined in Diebold & Yilmaz (2010, 2012, 2014) and their frequency decomposition defined in Barunik & Krehlik (2015).
To estimate, we build on the `vars` and `urca` packages that provide the coefficients estimate of the system. The dependency within the system is then computed based on the forecast error variance decomposition (FEVD) or the generalized FEVD estimates.
We provide parallelized version of the estimation functions that allow for rolling window estimation of the dependency.

Spillovers

Consider a covariance stationary N-variable VAR(p),

$$x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \epsilon_t$$

with error term having the covariance matrix Σ. The system can be represented in MA infinite form as the following,

$$x_t = \sum_{i=0}^{\infty} A_i \epsilon_{t-i}$$

The KPSS H-step-ahead forecast error variance (Pesaran and Shin, 1998) decomposition is then computed as

$$\theta_{i,j}(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' A_h \Sigma e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h \Sigma A_h' e_i)}$$

and the resulting spillover is defined as:

$$S(H) = \frac{100}{N} \sum_{i,j=1, i\neq j}^{N} \frac{\theta_{i,j}(H)}{\sum_{j=1}^{N} \theta_{i,j}(H)}$$

Frequency decomposition

Instead of IRFs, we consider a frequency response function, or transfer function

$$\Psi(e^{-i\omega}) = \sum_h e^{-i\omega h} \Psi_h.$$

we define the generalised causation spectrum over frequencies $\omega \in (-\pi, \pi)$ as

$$(\mathfrak{f}(\omega))_{j,k} \equiv \frac{\sigma_{kk}^{-1} \left| \left(\Psi\left(e^{-i\omega}\right)\Sigma\right)_{j,k} \right|^2}{\left(\Psi(e^{-i\omega})\Sigma\Psi'(e^{+i\omega})\right)_{j,j}}$$

Let us also define the spectral power at a given frequency

$$\Gamma_j(\omega) = \frac{\left(\Psi(e^{-i\omega})\Sigma\Psi'(e^{+i\omega})\right)_{j,j}}{\frac{1}{2\pi}\int_{-\pi}^{\pi} \left(\Psi(e^{-i\lambda})\Sigma\Psi'(e^{+i\lambda})\right)_{j,j} d\lambda}$$

Now suppose that we have system x_t that is wide-sense stationary. We establish the correspondence between the original FEVD measure and the decomposed measure as

$$(\theta_\infty)_{j,k} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \Gamma_j(\omega)\,(\mathfrak{f}(\omega))_{j,k}\, d\omega.$$

The FEVD measure can be arbitrarily decomposed on various frequency bands of interest.

$$(\theta_d)_{j,k} = \frac{1}{2\pi}\int_{d} \Gamma_j(\omega)\,(\mathfrak{f}(\omega))_{j,k}\, d\omega$$

Based on the decomposed FEVD we define two decomposed connectedness measures

Within connectedness

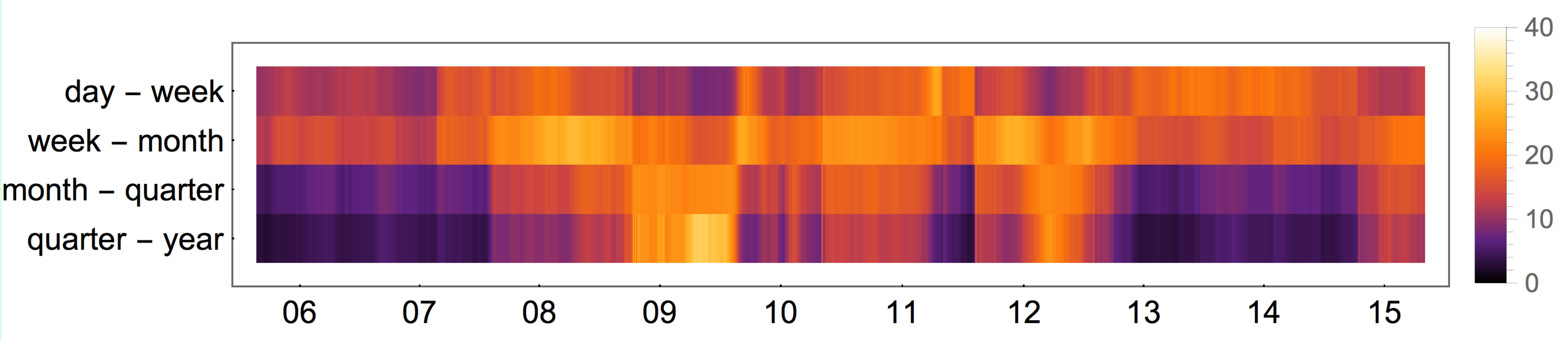
$$\mathcal{C}_d^{\mathcal{W}} = 1 - \frac{\operatorname{Tr}\left\{\widetilde{\theta}_d\right\}}{\sum \left(\widetilde{\theta}_d\right)_{j,k}}$$

Frequency connectedness

$$\mathcal{C}_d^{\mathcal{F}} = \frac{\sum \left(\widetilde{\theta}_d\right)_{j,k}}{\sum \left(\widetilde{\theta}\right)_{j,k}} \cdot \mathcal{C}_d^{\mathcal{W}}$$

Empirical example

We select the most liquid stocks from seven sectors in the American stock market. The selected stocks are: Bank of America Corp, Coca Cola, Microsoft, Pfizer, Walt Disney Co, Exxon Mobile Co, AT&T Inc. The data cover the period 2006 -- 2015, hece we cover the turbulent times of crisis as well as the pre- and post-crisis periods.
To capture the heterogeneity of the market transactions we investigate the periodicities up to one week, week to one month, month to one quarter, and longer.
In many cases, the structure of connectedness is dynamically evolving through the time, hence not strictly stationary. To capture this dynamism, we adopt the Starica & Granger (2015) framework of thinking about the time-series and estimate the connectedness within rolling window. We opt for window of length 250 days.




```
library(frequencyConnectedness)
cl <- makeCluster(16)
data <- read.csv(file = "../Data/spilloverdatastocksRV2016.txt", header = F)
window <- 250
H <- 600
p <- 2
typ <- "both"
bounds <- c(pi + 0.0001, pi/c(5, 20, 60), 0)# Intersector application
rolling_absolute12 <- spilloverRollingBK12(data = data_intersector,
                                           p = p, type = typ,
                                           window = window,
                                           n.ahead = H,
                                           partition = bounds,
                                           table = F, absolute = T,
                                           cluster = cl, no.corr = F)
```

We can see that in our case, the dynamics are pronounced and the connectedness of the system is evolving heavily over time. It starts with low long-cycle connections that however grow during the post-crisis period and then cease during the years 2013 and 2014 mounting again in 2015.
The short-cycle connections seem to be stable over the whole inspected period of time.
The estimation procedure is fully parallel and scales almost linearly with the number of processors.

Further work

Currently, the package supports standard computation of the dependence measures. We aim to further provide bootstrap estimation of the confidence intervals, where we however have to resort to C implementation because of computational exigencies. We will also work to provide facilities to compute directional connectedness as defined in Diebold & Yilmaz (2012) and convenient plotting functions.

Literature

- Diebold, F. X. and K. Yilmaz (2012). Better to give than to receive: Predictive directional measurement of volatility spillovers. International Journal of Forecasting 28(1), 57–66.
- Starica, C. and C. Granger (2005). Nonstationarities in stock returns. Review of economics and statistics 87(3),503–522.
- Barunik, J. and T. Krehlik (2015). Measuring the frequency dynamics of financial connectedness and systemic risk. arXiv preprint arXiv:1507.01729

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