Investigating ODEs with R and Spreadsheets

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Topics

Motivation

Examples

Discussion

Motivation

Project for support of introductory service course for ODEs
(together with TU Sofia)

R Package deSolve has a set of ODE solvers

Beginning students (neither math nor statistics majors, but engineers) should be able to study ODEs interactively

Students do not know R

Interface should be very simple (and somehow familiar)

Preparation

	A	B	C	D	E
1	N of dependent variables				
2	Name of dependent variables				
3	N of parameters				
4	Name of parameters				
5	Name of indepependent variable				
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					
21					
22					
23					

Preparation

	A	B	C	D	E
1	N of dependent variables			2	
2	Name of dependent variables			x	
3	N of parameters			2	
4	Name of parameters			p	
5	Name of indepependent variable			t	
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					
21					
22					
23					

Setup

	A	B	C	D	E
1	N of dependent variables			2	
2	Name of dependent variables			x	
3	N of parameters			2	
4	Name of parameters			p	
5	Name of indepependent variable			t	
6					
7	params	p[1]	p[2]		
8	labels	p[1]	p[2]		
9	paramvals				
10					
11	function	x[1]	x[2]		
12	labels	x[1]	x[2]		
13	deriv				
14	initvals				
15					
16					
17	t	x[1]	x[2]		
18					
19					
20					
21					
22					
23					

Setup

	A	B	C	D	E
1	N of dependent variables			2	
2	Name of dependent variables			x	
3	N of parameters			2	
4	Name of parameters			p	
5	Name of indepependent variable			t	
6					
7	params	p[1]	p[2]		
8	labels	p[1]	p[2]		
9	paramvals	1	-1		
10					
11	function	x[1]	x[2]		
12	labels	x[1]	x[2]		
13	deriv	p[1]*x[2]	p[2]*x[1]		
14	initvals	1	0		
15					
16					
17	t	x[1]	x[2]		
18					
19					
20					
21					
22					
23					

Setup

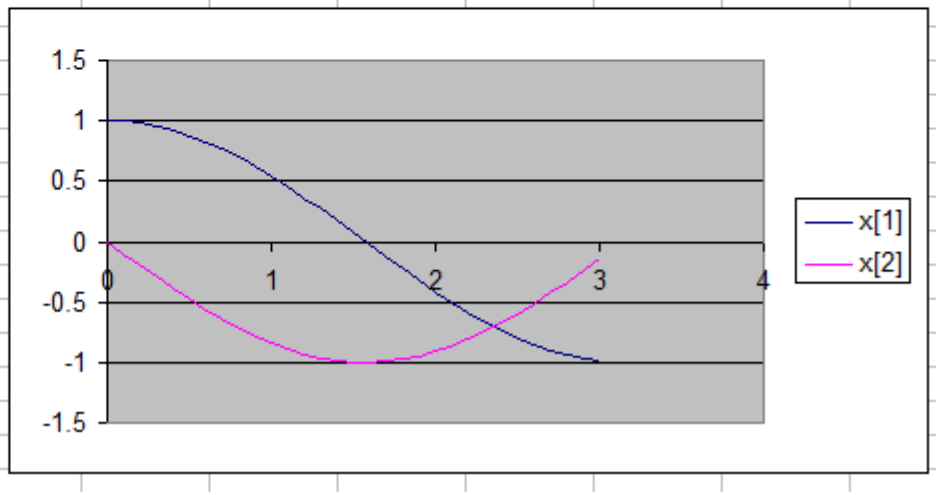
	A	B	C	D	E
1	N of dependent variables			2	
2	Name of dependent variables			x	
3	N of parameters			2	
4	Name of parameters			p	
5	Name of indepependent variable			t	
6					
7	params	p[1]	p[2]		
8	labels	p[1]	p[2]		
9	paramvals	1	-1		
10					
11	function	x[1]	x[2]		
12	labels	x[1]	x[2]		
13	deriv	p[1]*x[2]	p[2]*x[1]		
14	initvals	1	0		
15					
16					
17	t	x[1]	x[2]		
18	0				
19	0.1				
20	0.2				
21	0.3				
22	0.4				
23	0.5				

Solve

	A	B	C	D	E
1	N of dependent variables			2	
2	Name of dependent variables			x	
3	N of parameters			2	
4	Name of parameters			p	
5	Name of indepependent variable			t	
6					
7	params	p[1]	p[2]		
8	labels	p[1]	p[2]		
9	paramvals	1	-1		
10					
11	function	x[1]	x[2]		
12	labels	x[1]	x[2]		
13	deriv	p[1]*x[2]	p[2]*x[1]		
14	initvals	1	0		
15					
16					
17	t	x[1]	x[2]		
18	0	1	0		
19	0.1	0.995004	-0.09983		
20	0.2	0.980067	-0.19867		
21	0.3	0.955337	-0.29552		
22	0.4	0.921061	-0.38942		
23	0.5	0.877582	-0.47943		

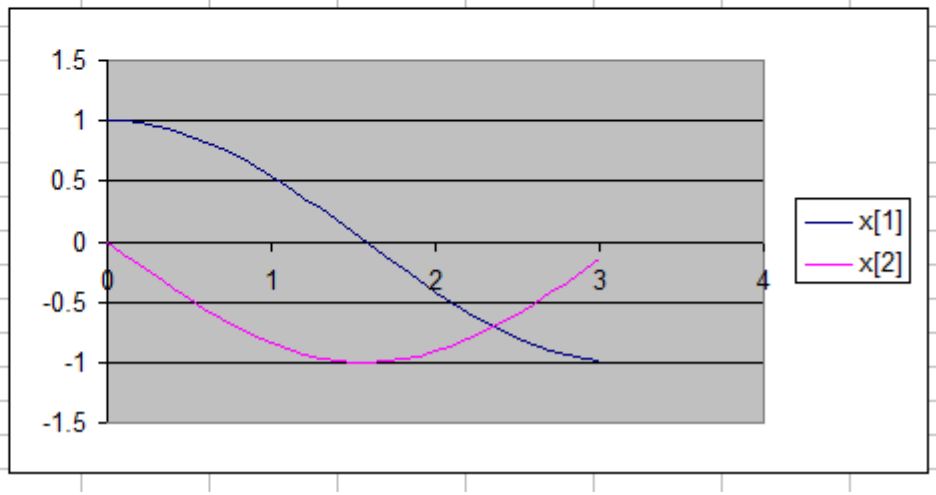
Graph of solution

	A	B	C	D	E	F	G	H	I	J	K
1	N of dependent variables			2							
2	Name of dependent variables			x							
3	N of parameters			2							
4	Name of parameters			p							
5	Name of indepependent variable			t							
6											
7	params	p[1]	p[2]								
8	labels	p[1]	p[2]								
9	paramvals	1	-1								
10											
11	function	x[1]	x[2]								
12	labels	x[1]	x[2]								
13	deriv	p[1]*x[2]	p[2]*x[1]								
14	initvals	1	0								
15											
16											
17	t	x[1]	x[2]								
18	0	1	0								
19	0.1	0.995004	-0.09983								
20	0.2	0.980067	-0.19867								
21	0.3	0.955337	-0.29552								
22	0.4	0.921061	-0.38942								
23	0.5	0.877583	-0.47943								
24	0.6	0.825336	-0.56464								



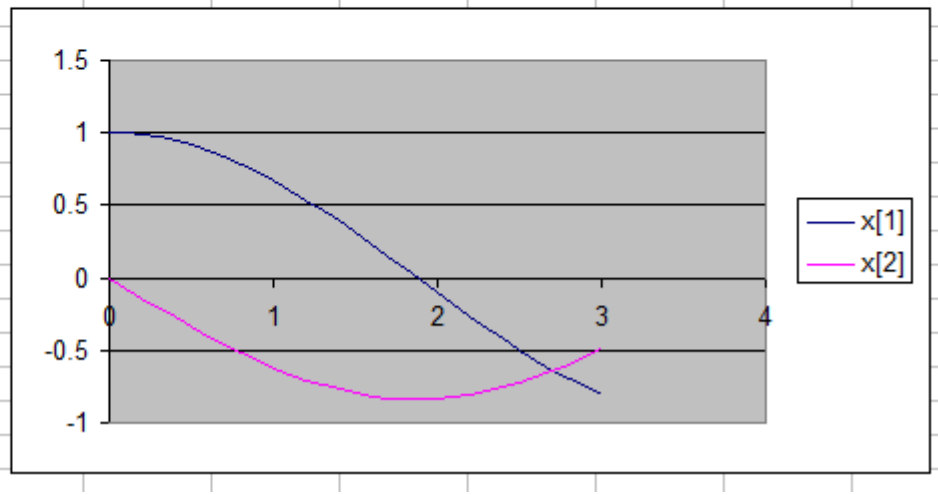
Sliders for parameters

	A	B	C	D
1	N of dependent variables			2
2	Name of dependent variables			x
3	N of parameters			2
4	Name of parameters			p
5	Name of indepependent variable			t
6				
7	params	p[1]	p[2]	
8	labels	p[1]	p[2]	
9	paramvals	1	-1	
10				
11	function	x[1]	x[2]	
12	labels	x[1]	x[2]	
13	deriv	p[1]*x[2]	p[2]*x[1]	
14	initvals	1	0	
15				
16				
17	t	x[1]	x[2]	
18	0	1	0	
19	0.1	0.995004	-0.09983	
20	0.2	0.980067	-0.19867	
21	0.3	0.955337	-0.29552	
22	0.4	0.921061	-0.38942	
23	0.5	0.877583	-0.47943	
24	0.6	0.825336	-0.56464	



Sliders for parameters

	A	B	C	D
1	N of dependent variables			2
2	Name of dependent variables			x
3	N of parameters			2
4	Name of parameters			p
5	Name of indepependent variable			t
6				
7	params	p[1]	p[2]	
8	labels	p[1]	p[2]	
9	paramvals	1	-0.7	
10				
11	function	x[1]	x[2]	
12	labels	x[1]	x[2]	
13	deriv	p[1]*x[2]	p[2]*x[1]	
14	initvals	1	0	
15				
16				
17	t	x[1]	x[2]	
18	0	1	0	
19	0.1	0.996502	-0.06992	
20	0.2	0.986033	-0.13935	
21	0.3	0.968665	-0.2078	
22	0.4	0.944521	-0.2748	
23	0.5	0.913769	-0.33988	
24	0.6	0.876624	-0.40258	



Example

x[1]'=p[1]*x[2]
x[2]'=p[2]*x[1]

For p[1]=1 we have x[1]'=x[2], so x[2] is the derivative of x[1]

We can interpret x[1] as distance and x[2] as speed

Naming of variables and parameters

Labels for variables and parameters can be used in equations (pendulum example):

x[1]	distance
x[2]	speed
p[1]	acc_constant
x[1]'=x[2]	distance'=speed
x[2]'=p[1]*x[1]	speed'=acc_constant*distance

Meaningful names help understanding the problem under consideration

Tools for investigation (value added by spreadsheets)

Automatic updating when parameters or initial values change

Sliders for parameters and initial values
(direct manipulation interface)

Comparison of different integration methods
(Currently Euler-Cauchy, Runge-Kutta 4th order,
LSODR (Livermore solver))