*Using **R** to Assess Mathematical Sense-Making in Introductory Physics Courses*

by Brian Danielak, Eric Kuo, Michael M. Hull, and Andrew Elby

2010 UseR! Conference - July 21, 2010


I work behind here

Students in conventional courses often treat formulas as **algorithms.**

Sherin, B. L. (2001). How Students Understand Physics Equations. Cognition and Instruction, 19(4), p. 479

IF I KNOW THE INITIAL VELOCITY...

THE UNIFORM ACCELERATION...

AND THE TIME...

$$v = v_0 + at$$

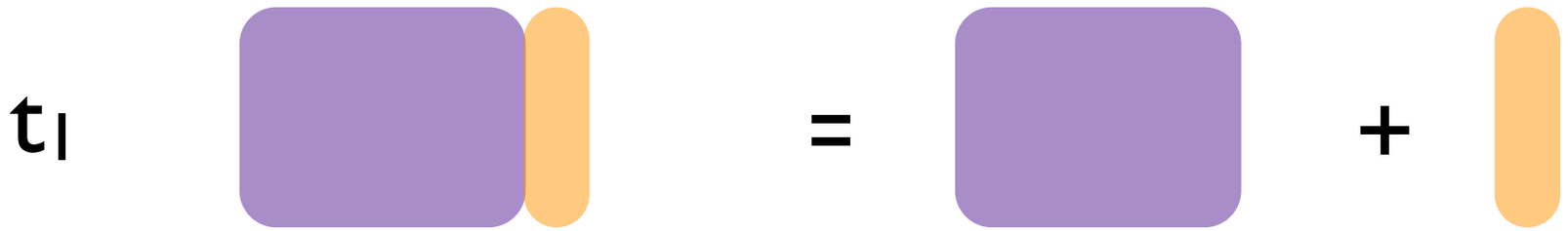
THEN I CAN **CALCULATE** THE FINAL VELOCITY!

Sense-making is about seeking
coherence

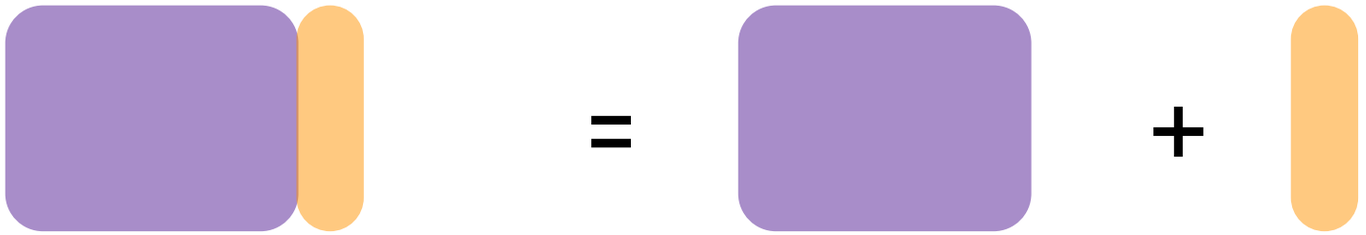
$$v = v_0 + at$$

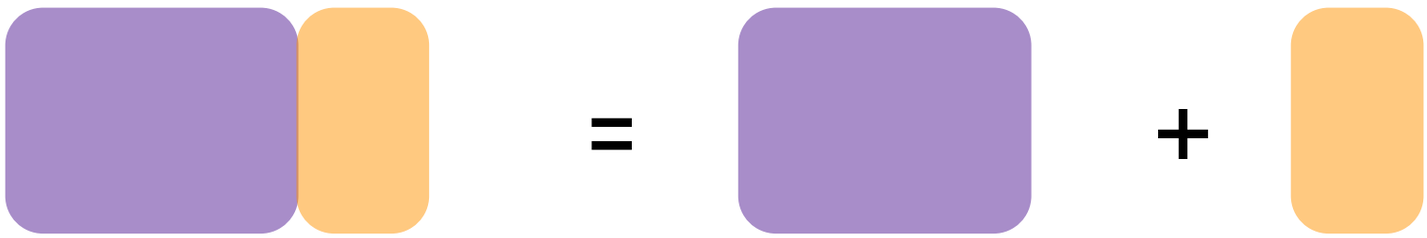
$$v = v_0 + at$$

$$v = v_0 + at$$

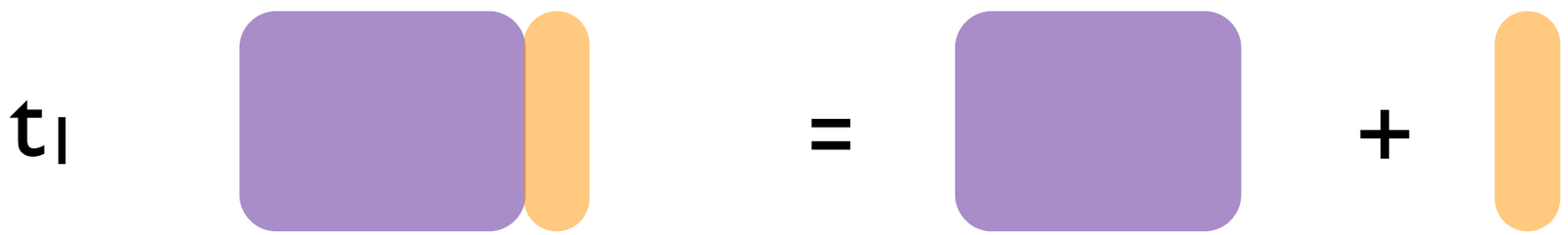


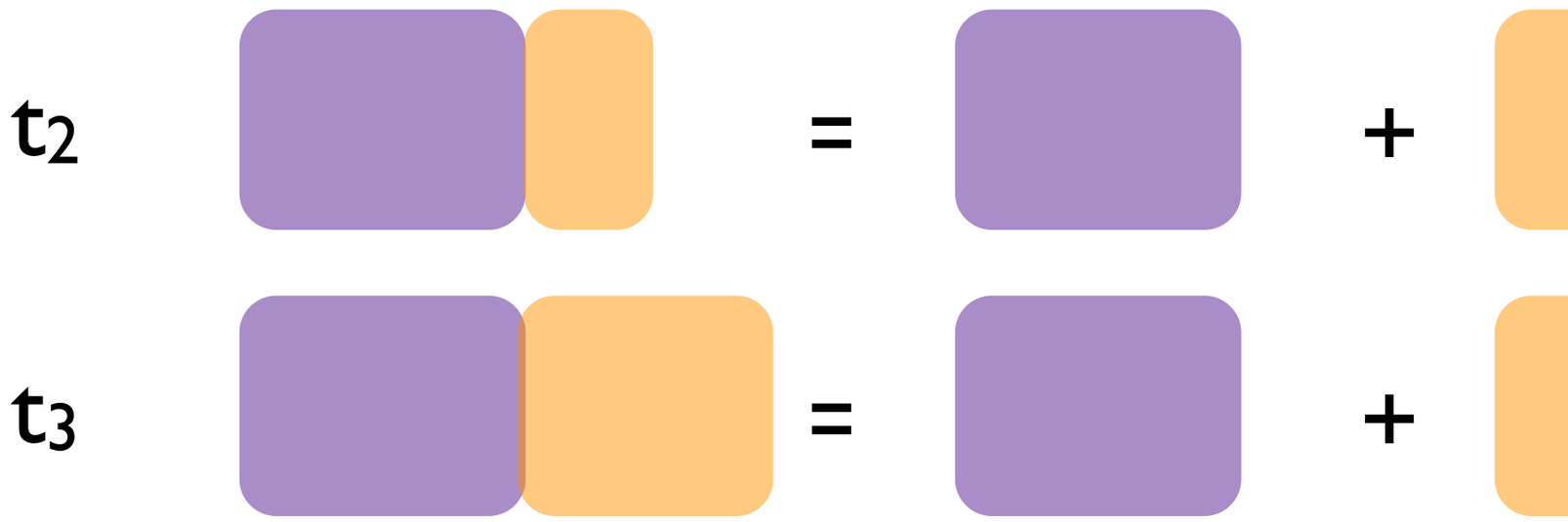
$$v = v_0 + at$$

t_1 

t_2 

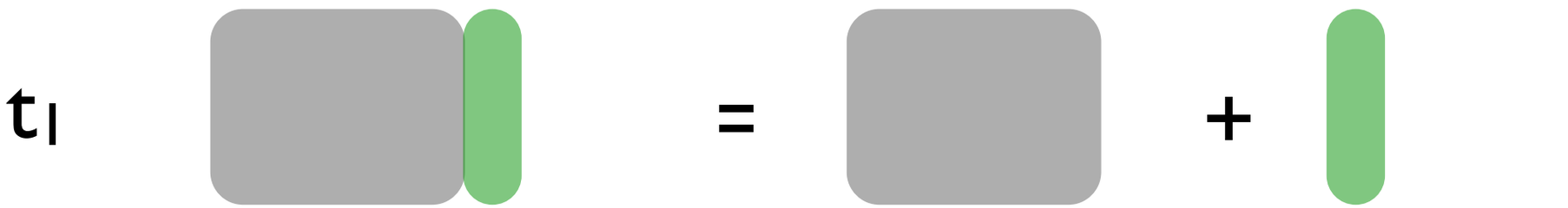
$$v = v_0 + at$$



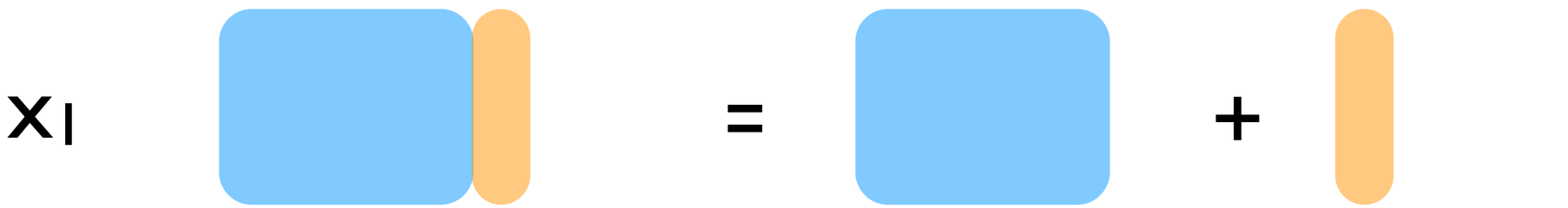


The same mathematical **structure** models lots of other situations

$$\text{money} = P + rt$$



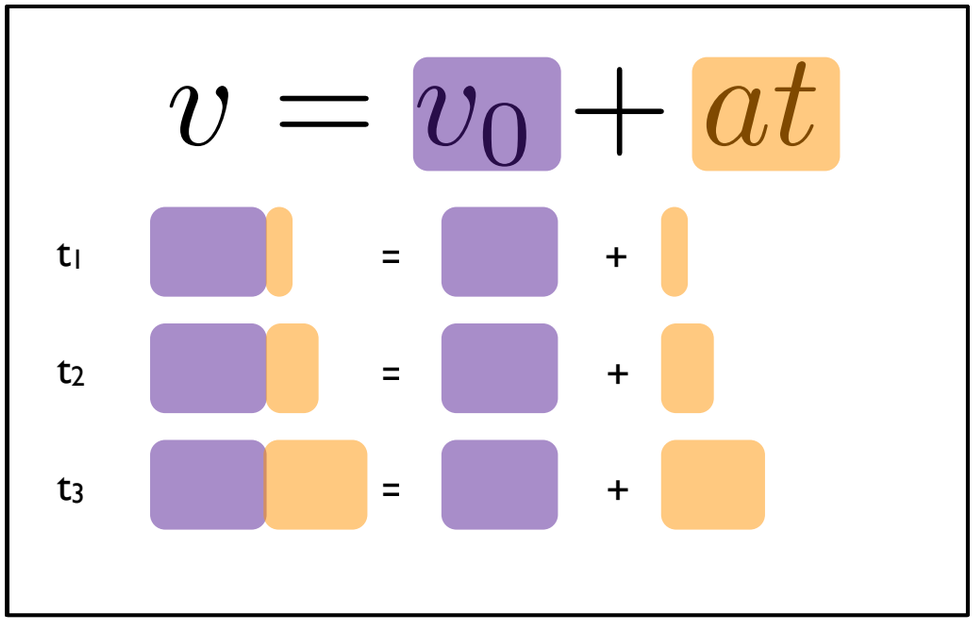
$$\hat{Y} = \beta_0 + \beta_1 X1$$



So here's the big

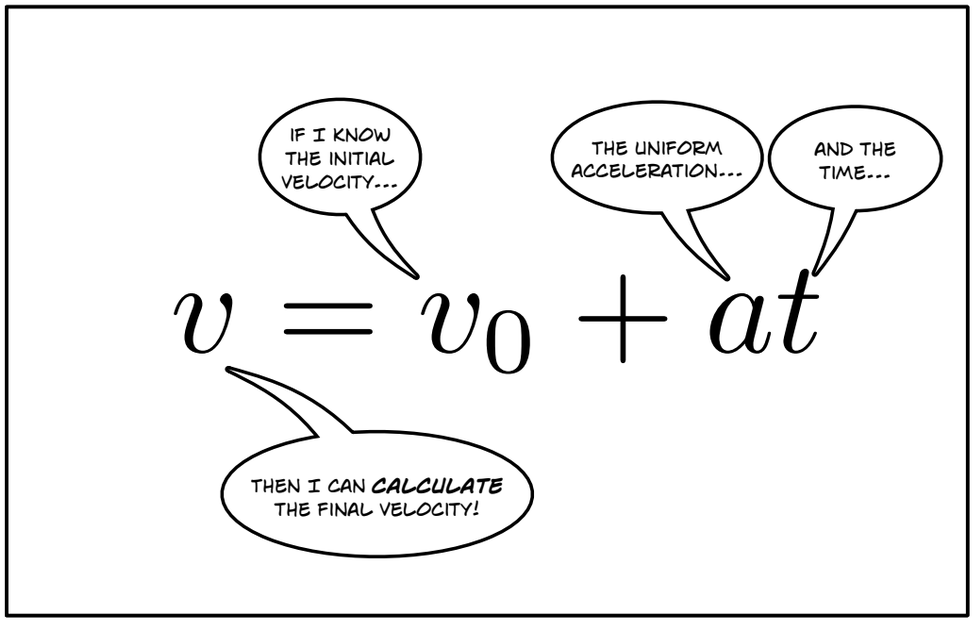
Question

How can we test for thinking like **this**



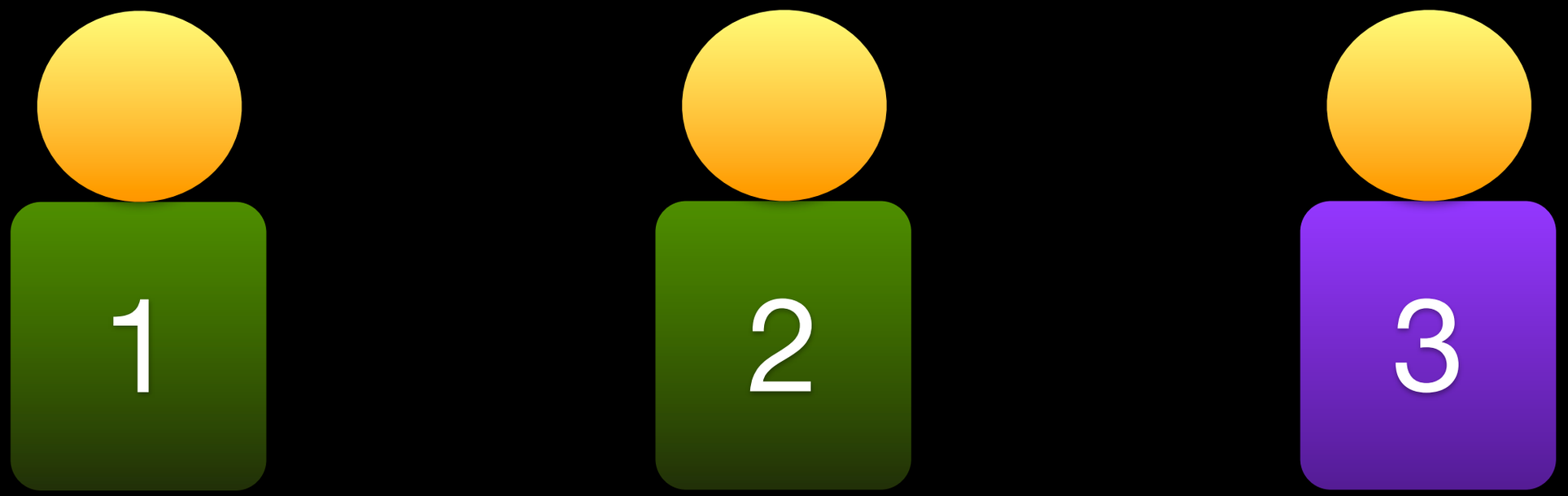
15

When we usually test for thinking like *this*?



10

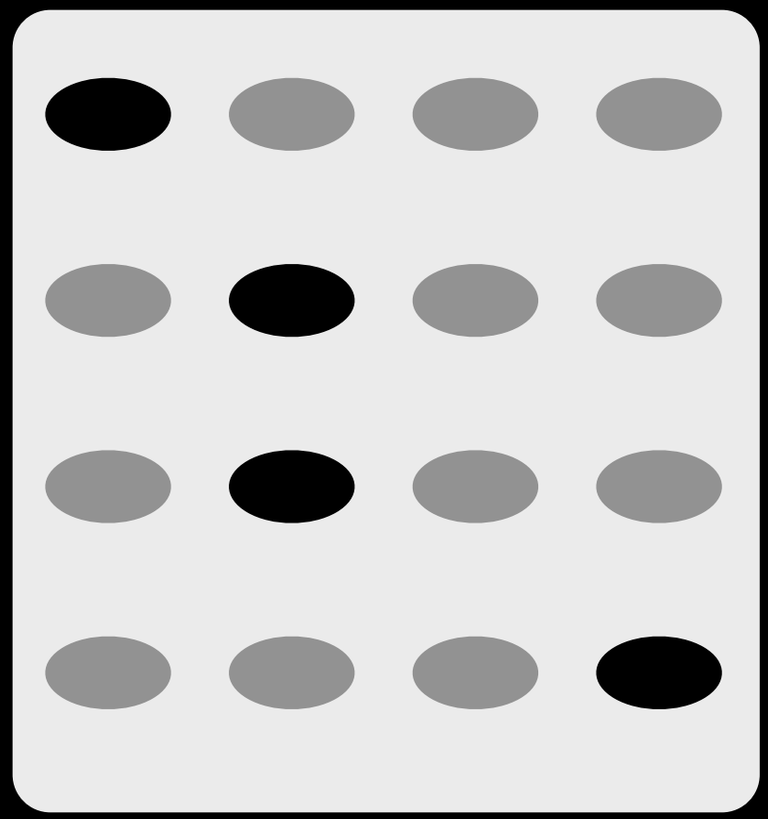
A Tale of 3 Instructors



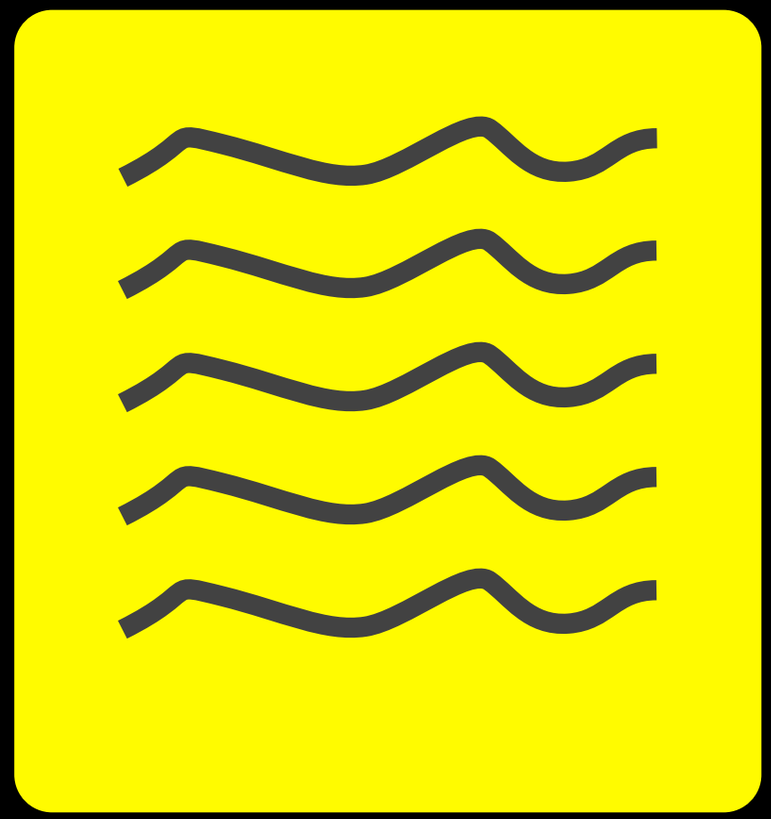
Sense-making

Traditional Approach

And one Final Exam

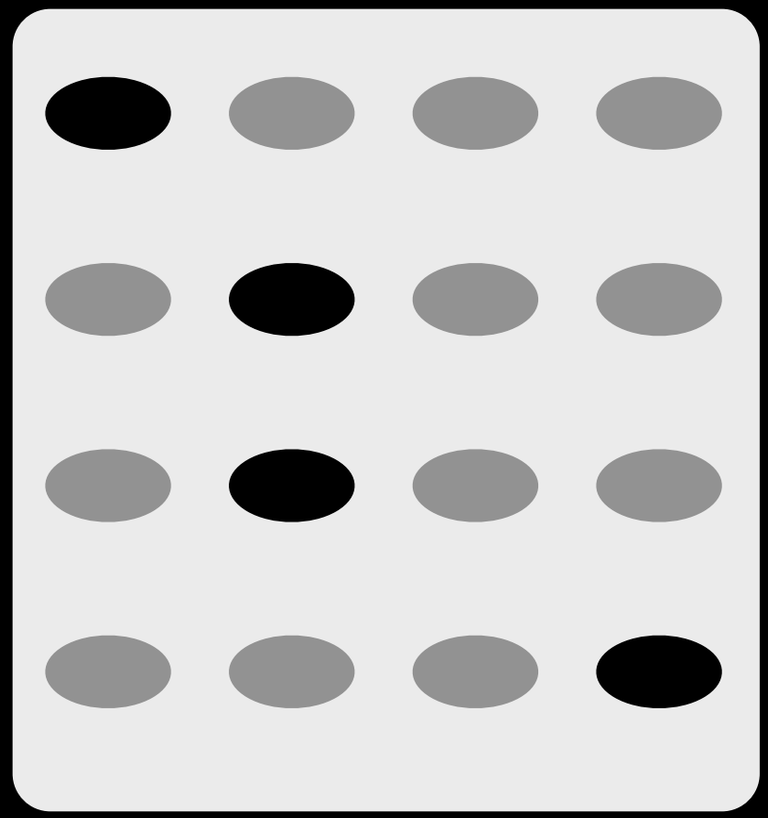


Multiple Choice



Free Response

And one Final Exam



Multiple Choice

Free Response

426 Students

3 Classes

1
2
3
4
5
6
7
8

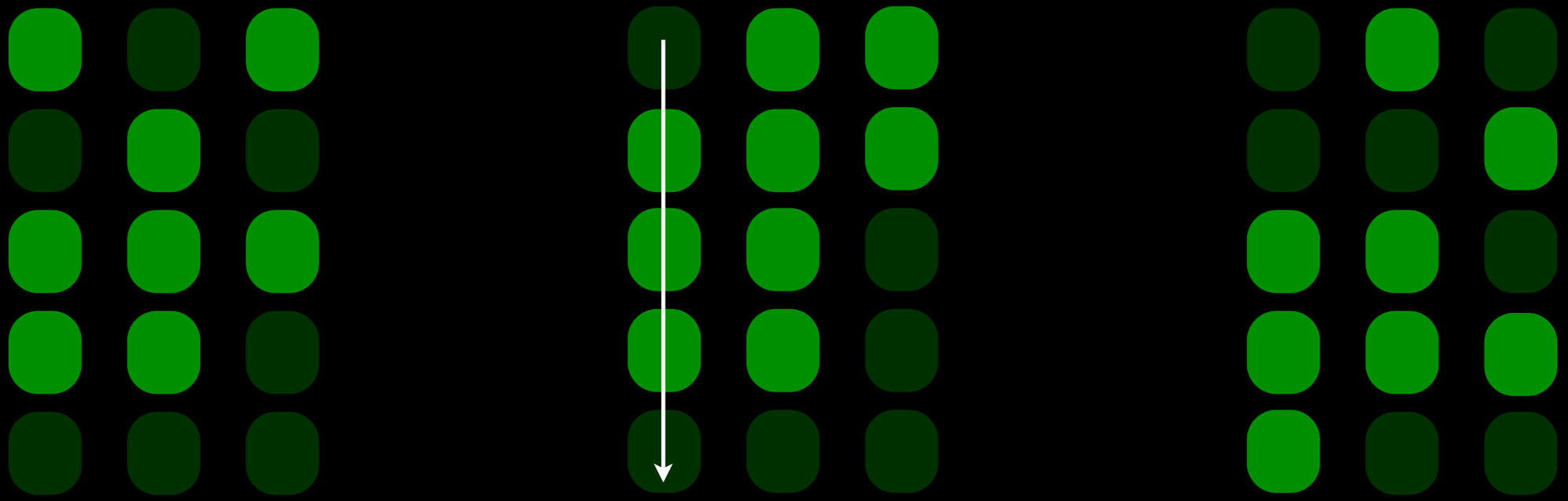
Sense-making items

chosen *a priori*

Class 1

Class 2

Class 3

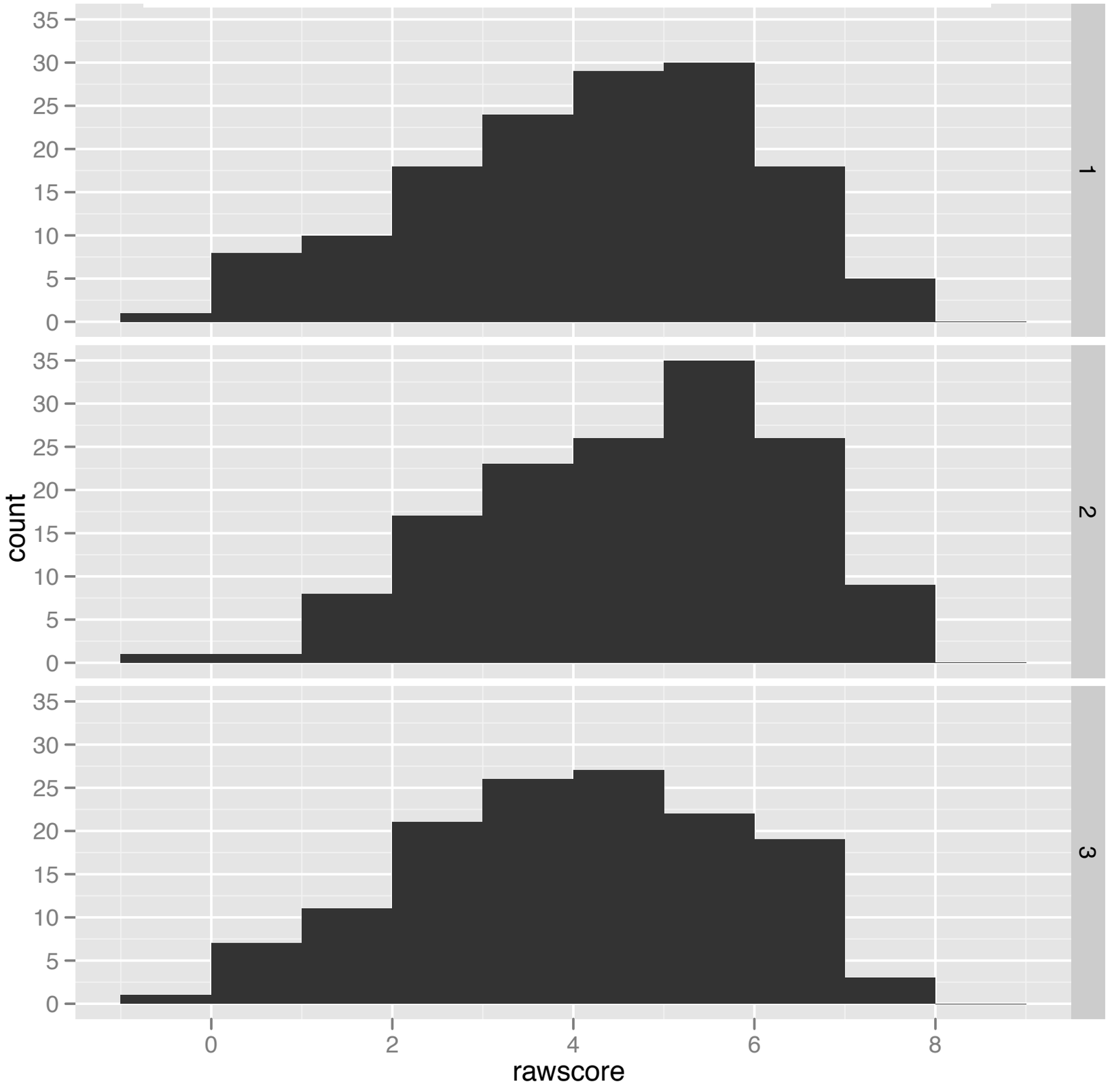


3 4 2

e-score

Looking **graphically** at the data

Across all 8 MC items

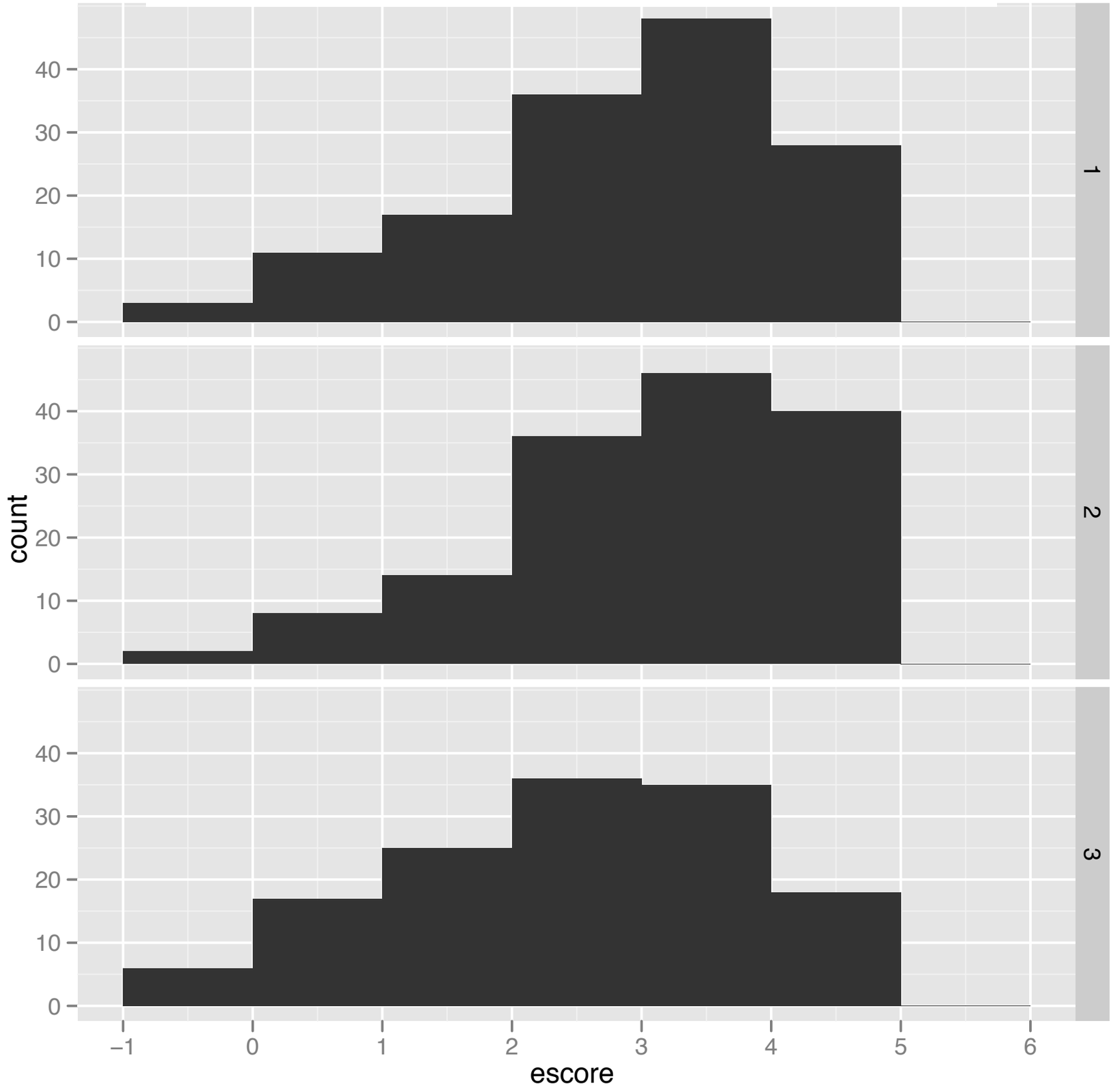


Sense-making (new)

Sense-making (vet)

Traditional (vet)

Just the 5 Sense-making Items



Sense-making (new)

Sense-making (vet)

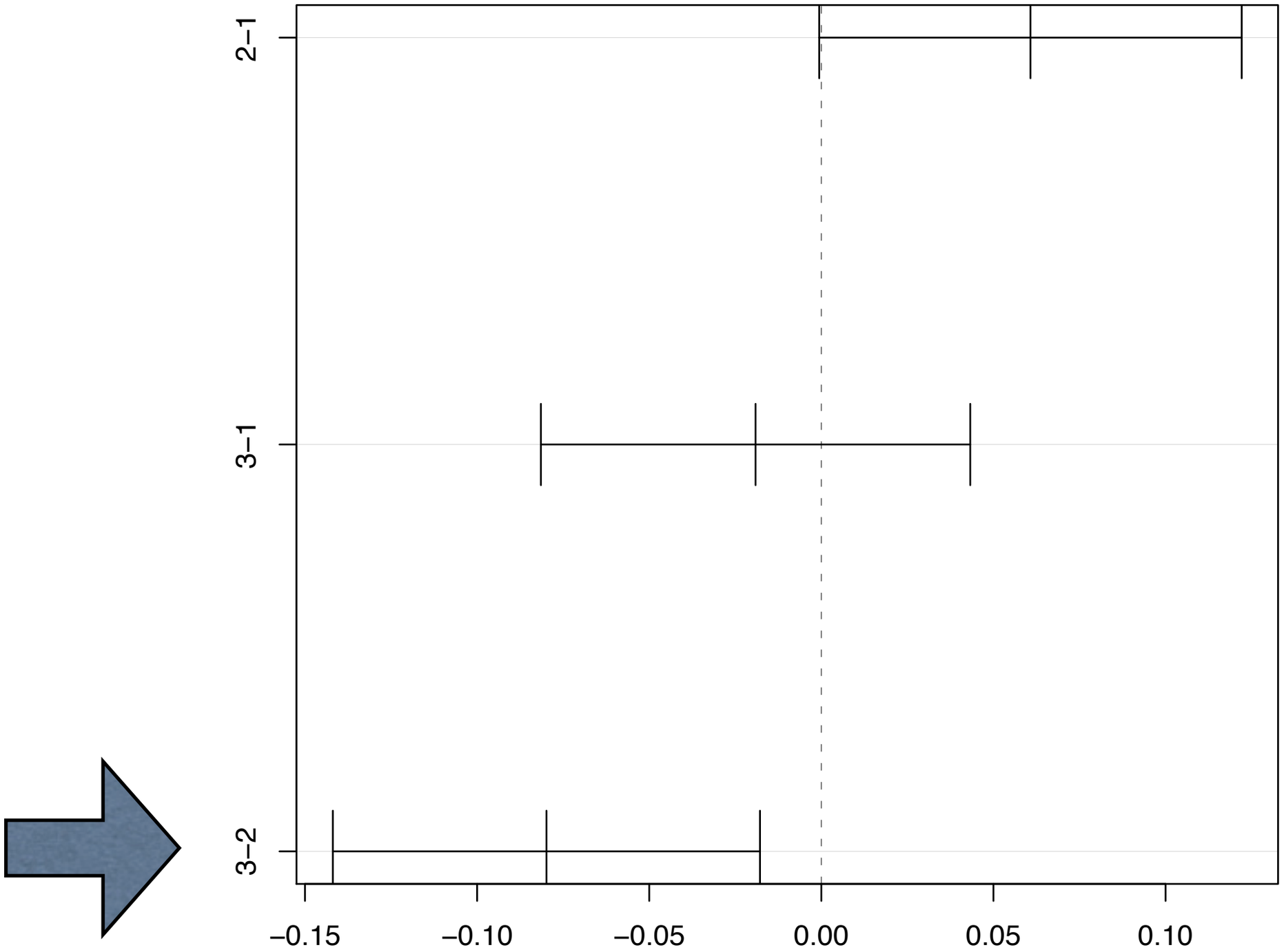
Traditional (vet)

We ran a one-way ANOVA, then MCPs

Result 1

Across all 8 Multiple Choice items,

Sense-making > traditional approach


95% family–wise confidence level
2–1
3–1
3–2
–0.15
–0.10
–0.05
0.00
0.05
0.10
Differences in mean levels of instructor
Rawratio

Across all 8 MC items

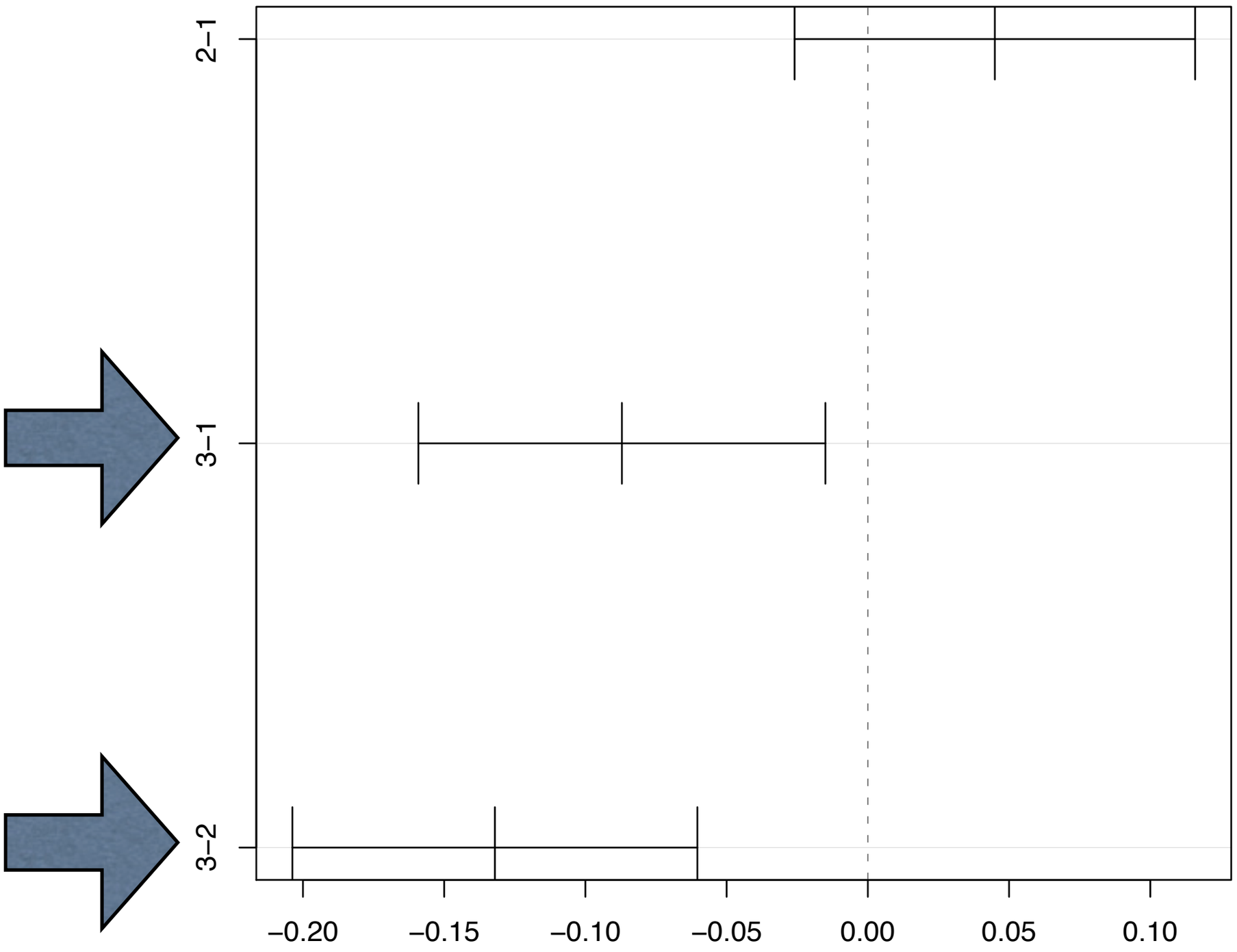
95% Family-wise Confidence Level

			
TukeyHSD	0.061	-0.019	-0.080
p-adj	0.053	0.751	0.007
Cohen's d	0.277	-0.0839	**-0.365**

Result 2

Across the 5 specific sense-making items,

Sense-making >> traditional approach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2−1
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−0.10
−0.05
0.00
0.05
0.10
Differences in mean levels of instructor
Eratio

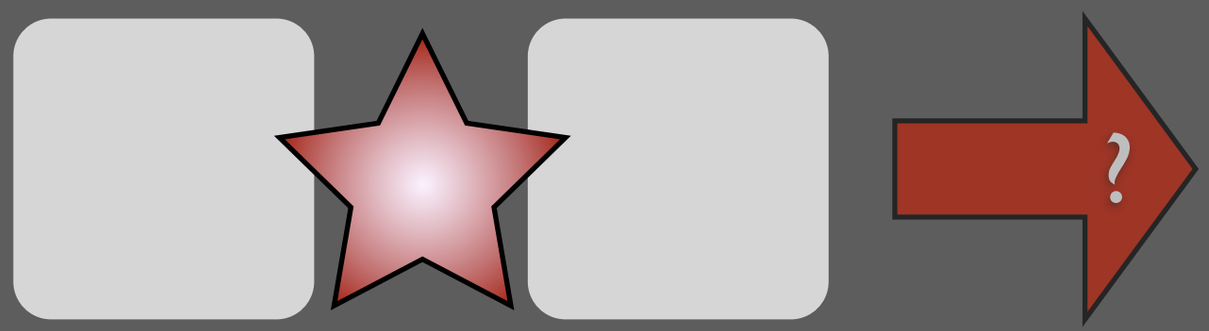
On Sense-making MCs

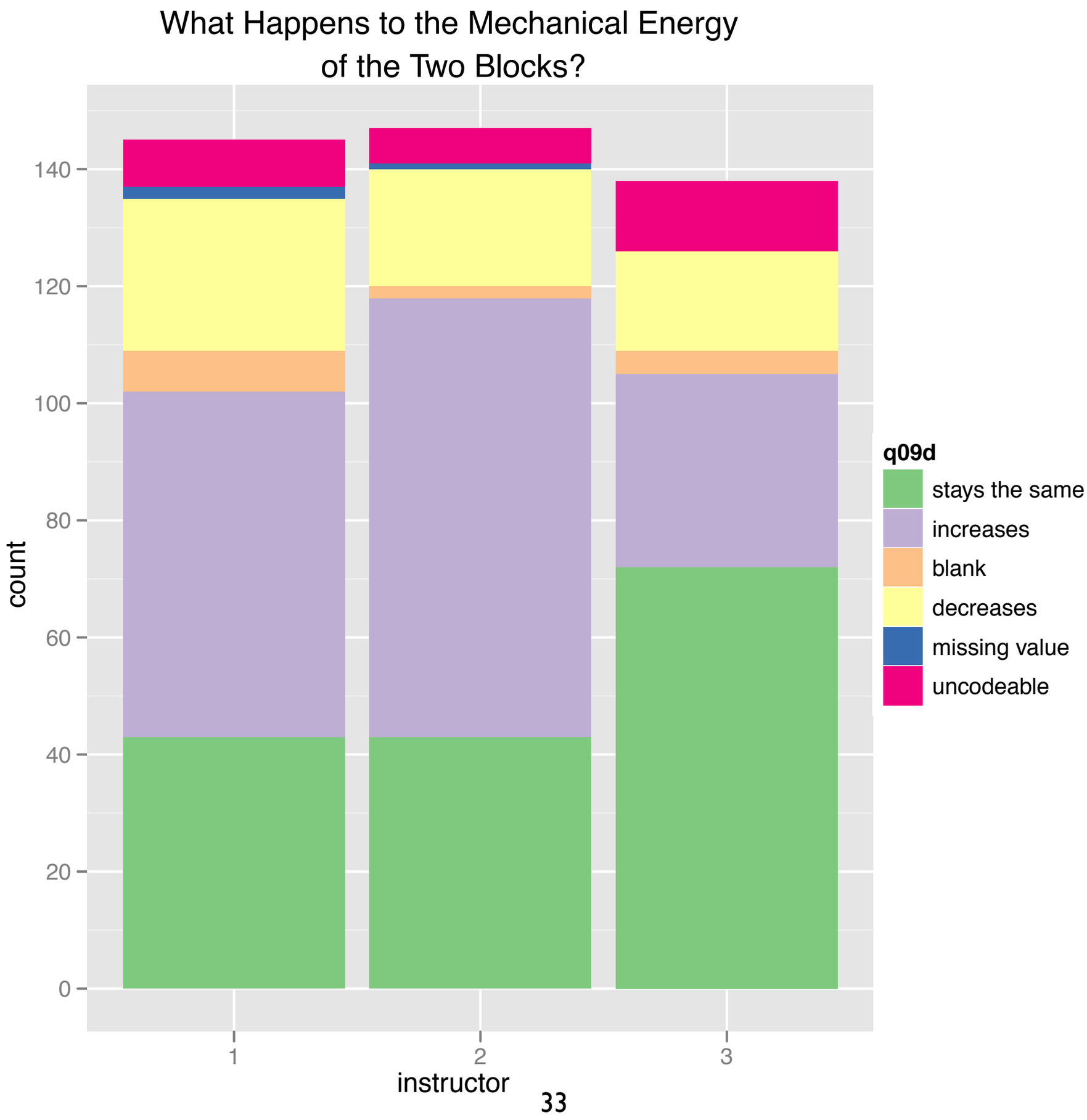
95% Family-wise Confidence Level

			
TukeyHSD	0.0450	-0.0871	-0.132
p-adj	0.296	0.0130	< 0.001
Cohen's d	0.182	**-0.331**	**-0.510**

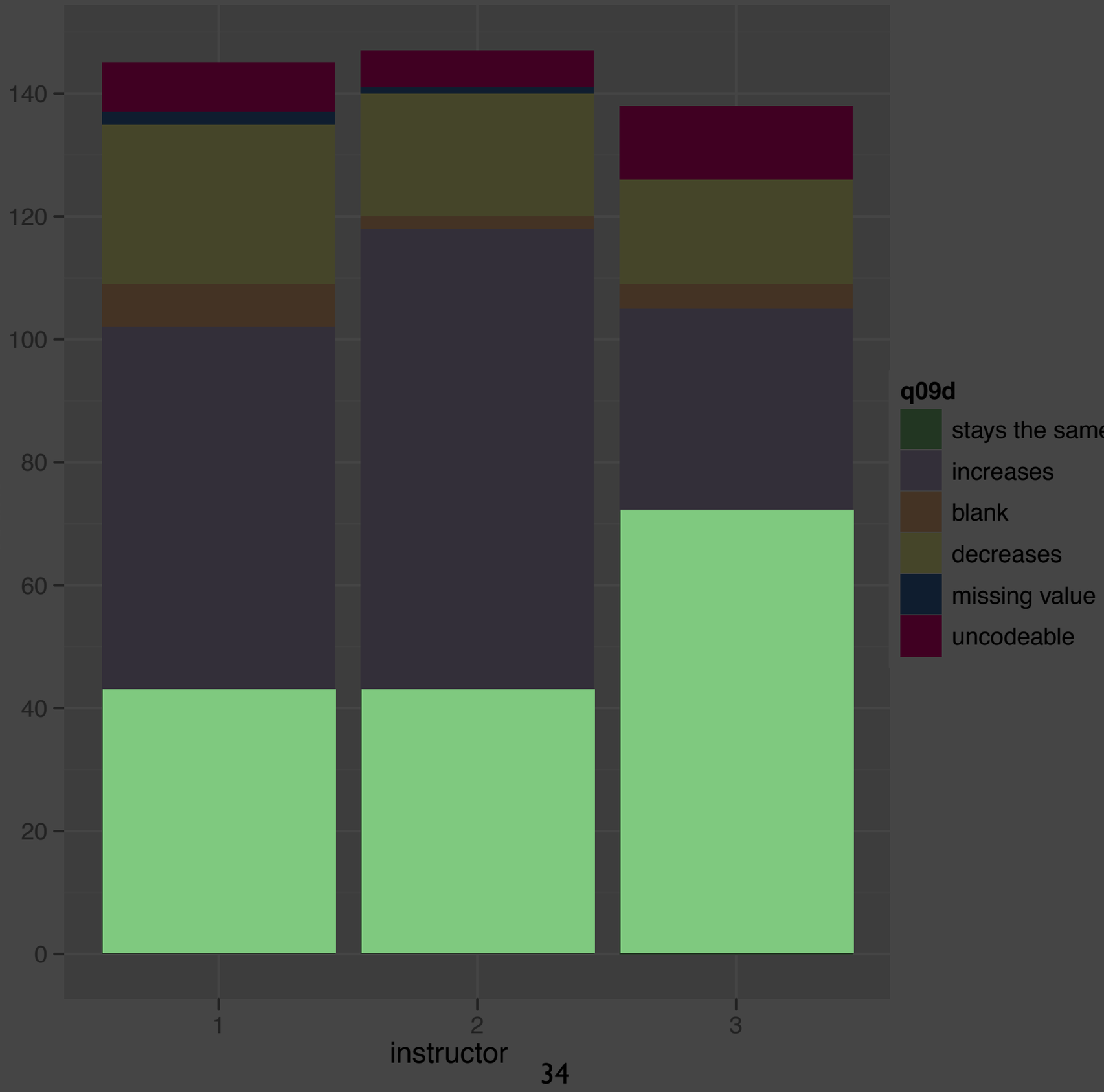
Exploring Free-Response Data

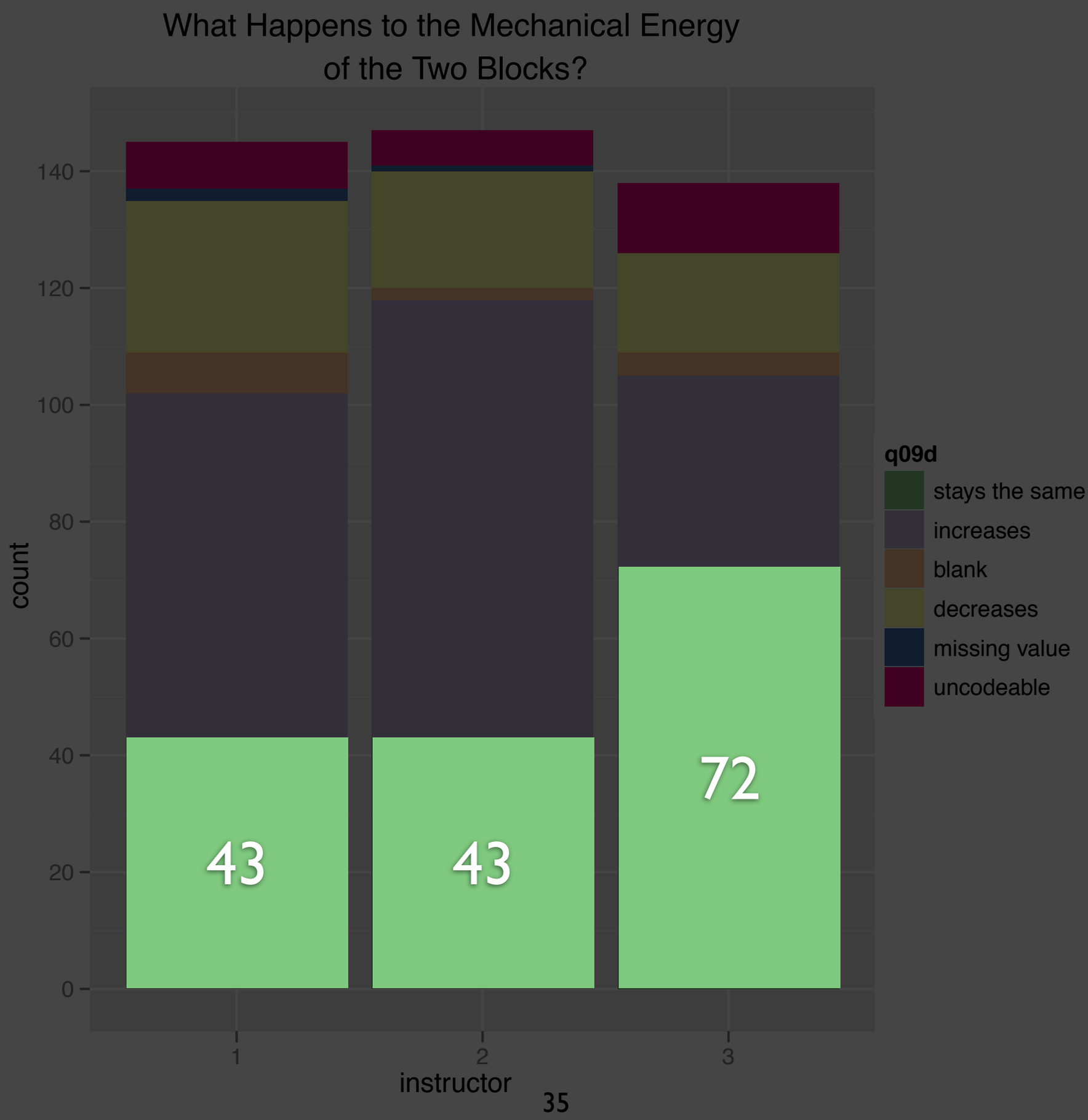
What happens to mechanical energy after the explosion?



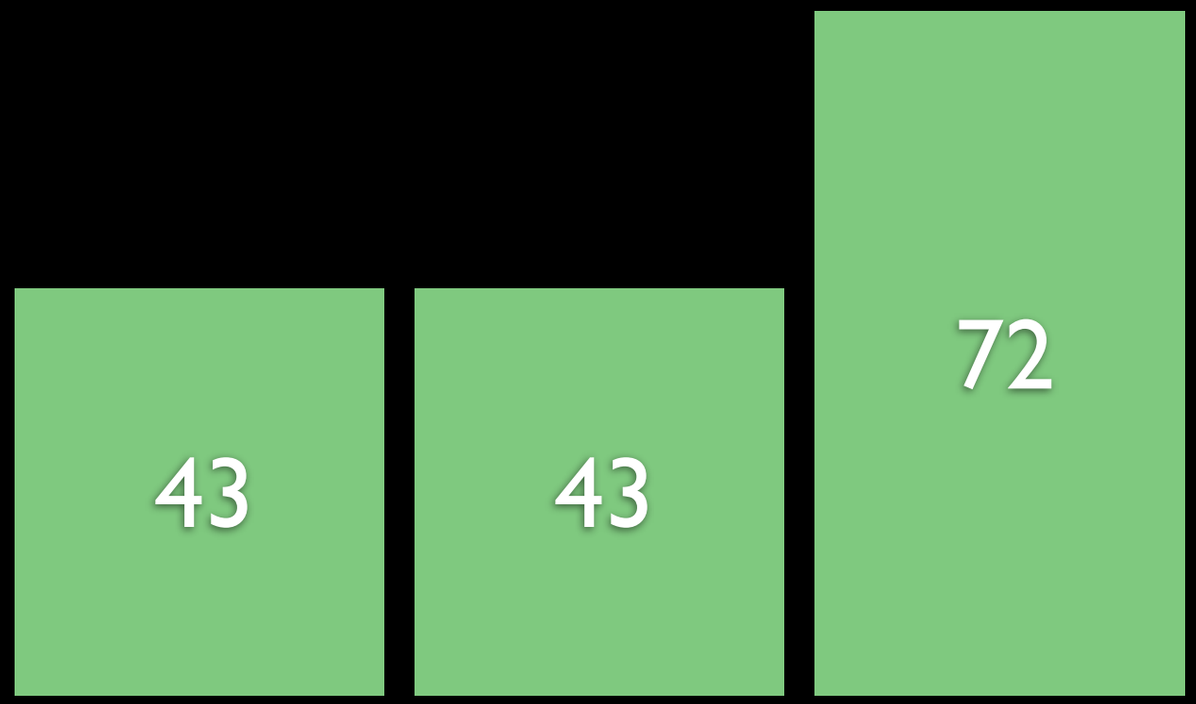

What Happens to the Mechanical Energy
of the Two Blocks?
count
140
120
100
80
60
40
20
0
1
2
3
instructor
q09d
stays the same
increases
blank
decreases
missing value
uncodeable

What Happens to the Mechanical Energy of the Two Blocks?

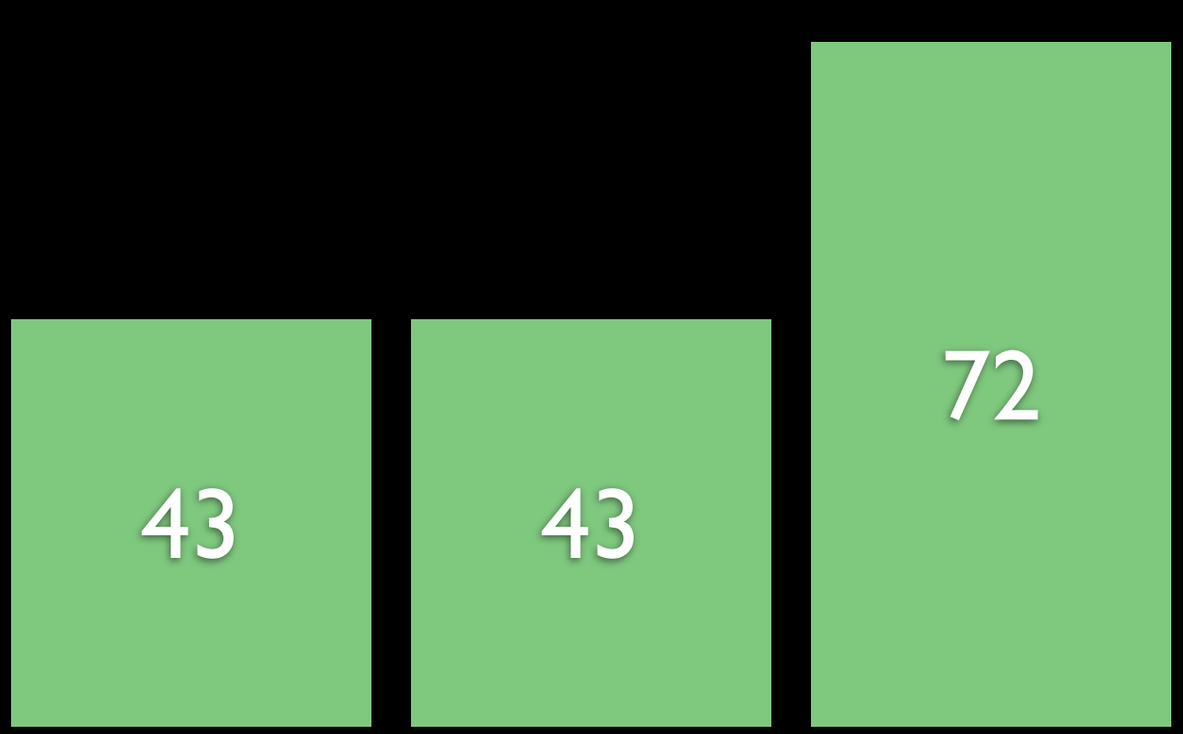


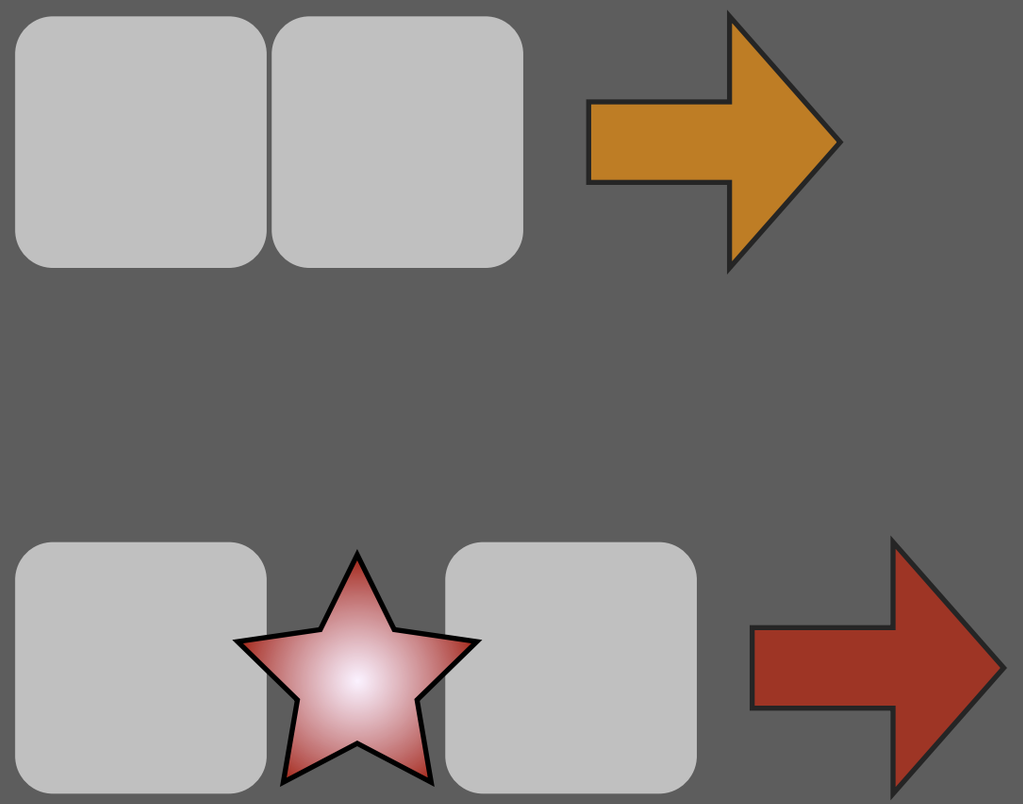

What Happens to the Mechanical Energy of the Two Blocks?
count
140
120
100
80
60
40
20
0
43
43
72
1
2
3
instructor
q09d
stays the same
increases
blank
decreases
missing value
uncodeable

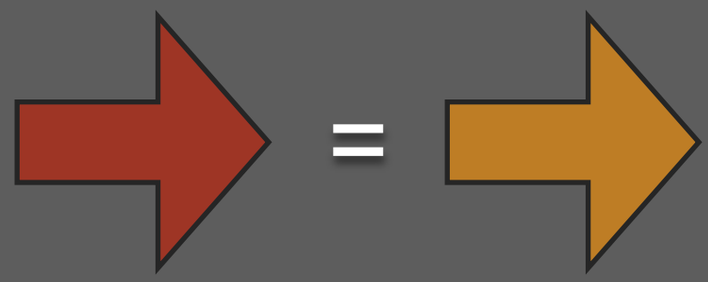
Mechanical Energy Stays the Same



Mechanical Energy Stays the Same







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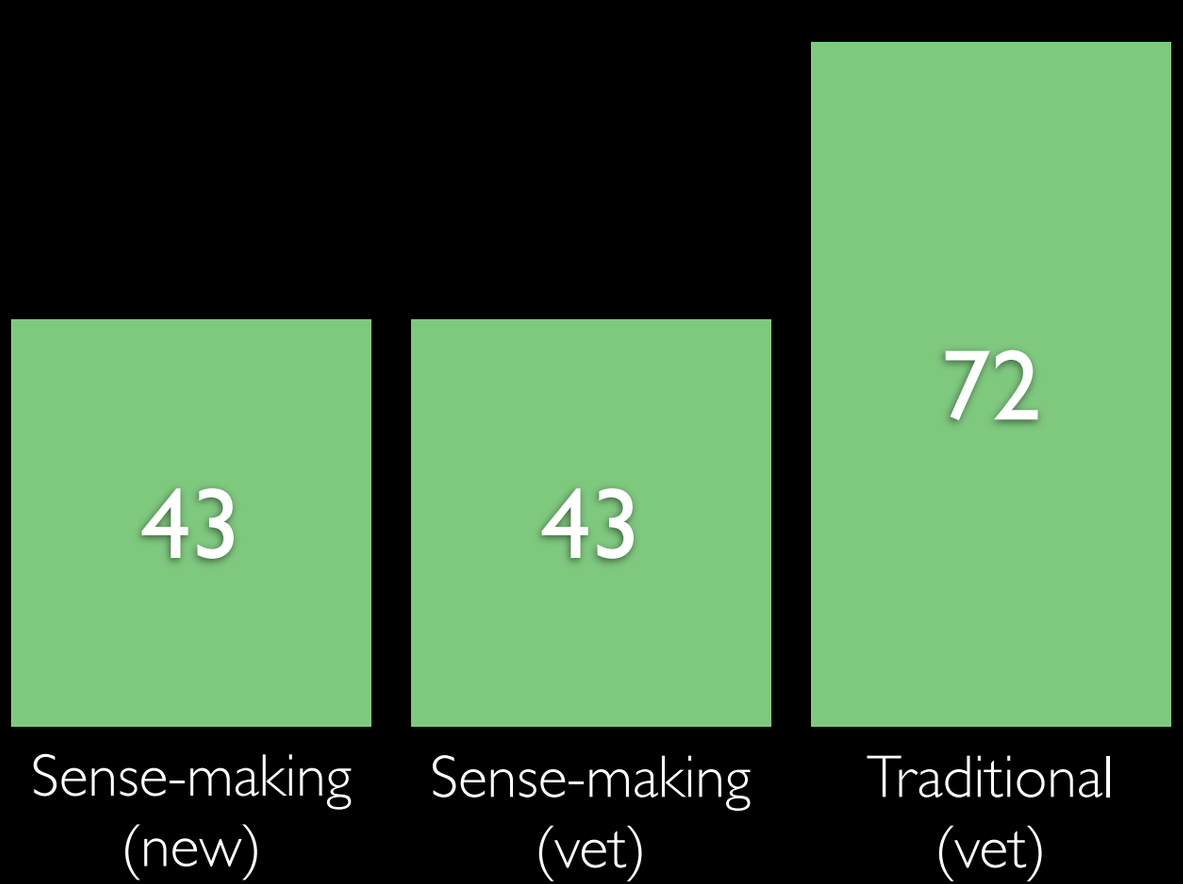
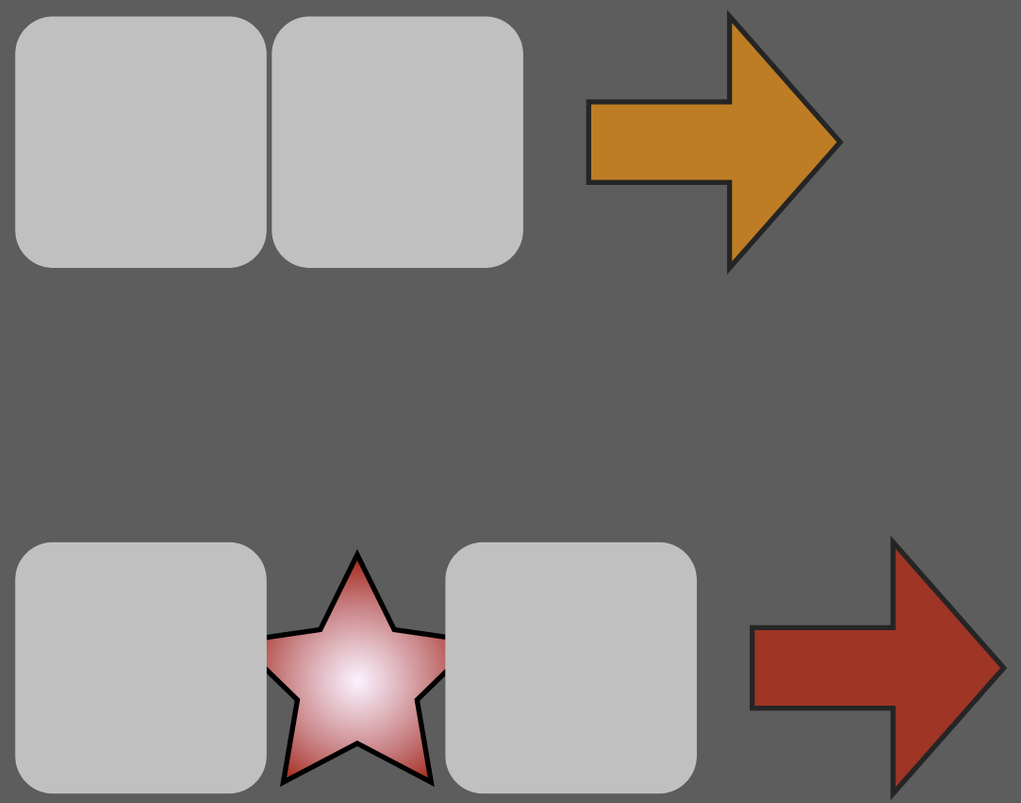
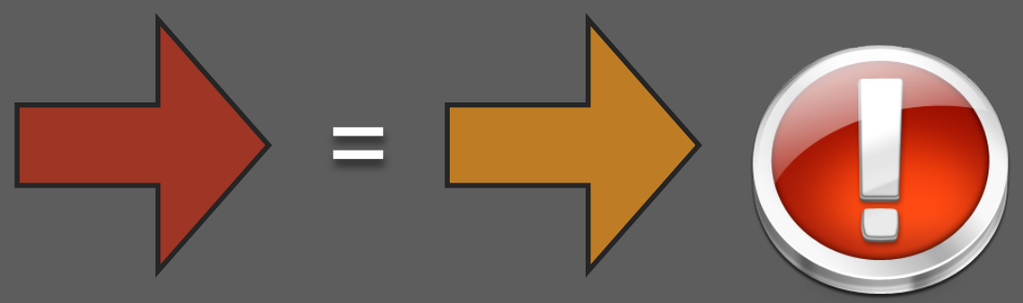
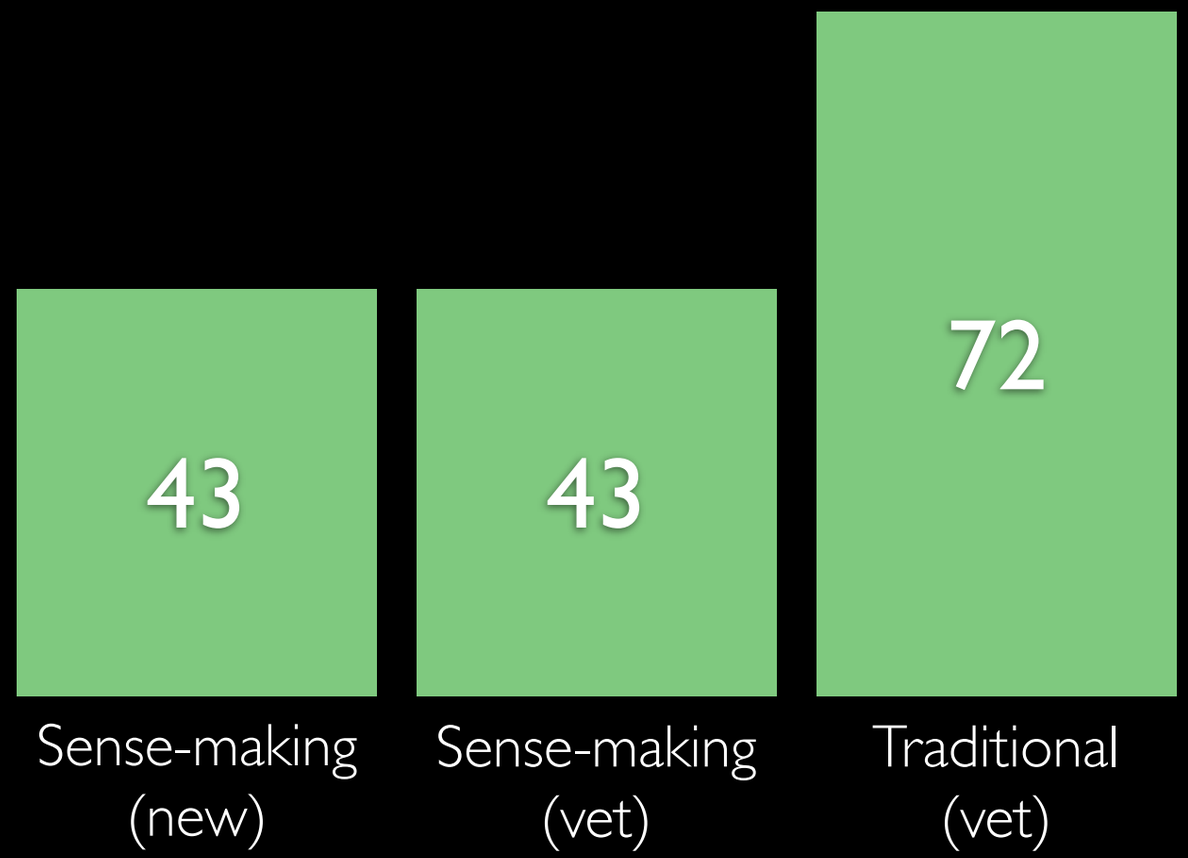


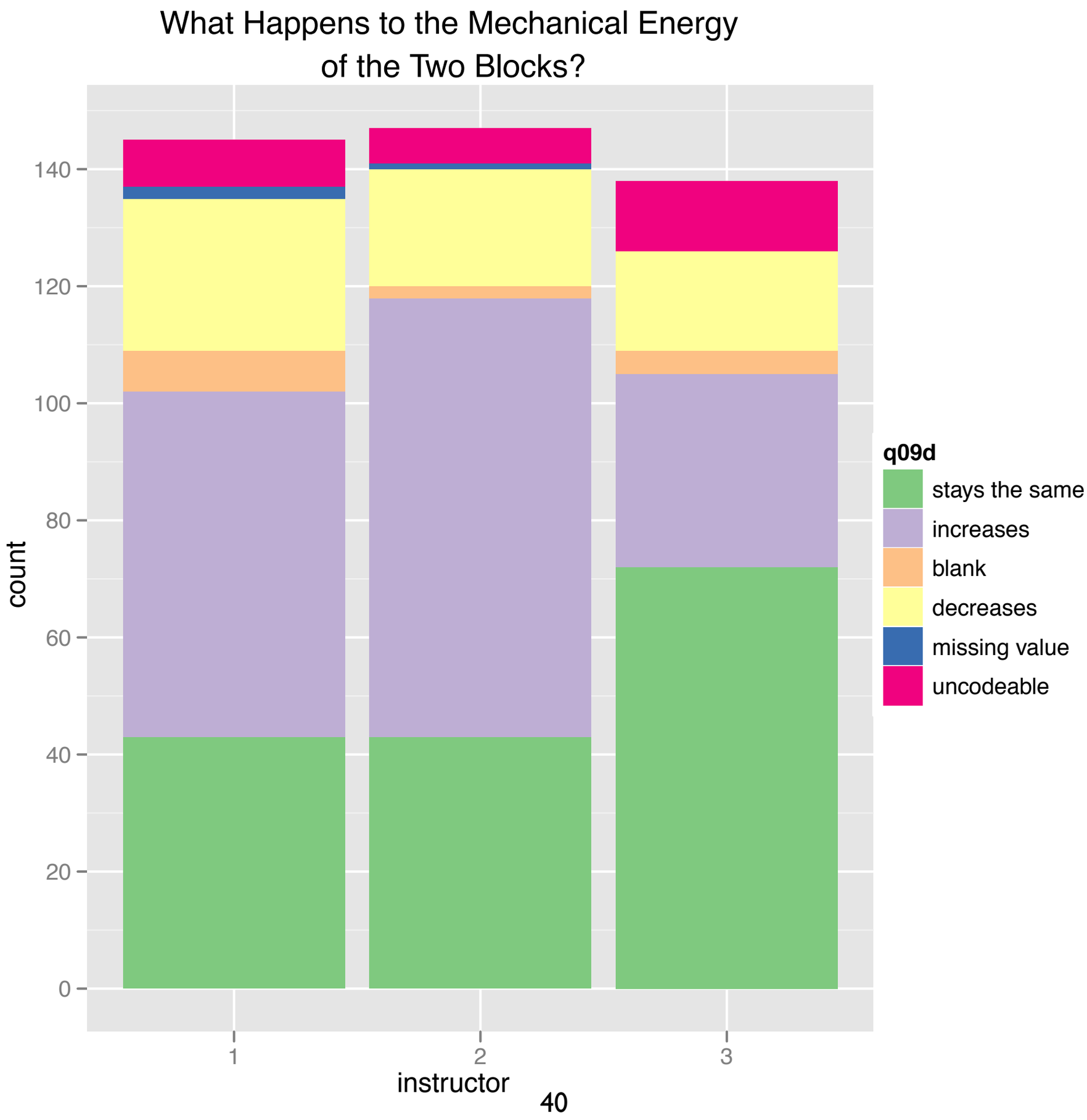




Result 3

When considering energy, more traditional students asserted something **counter-intuitive**

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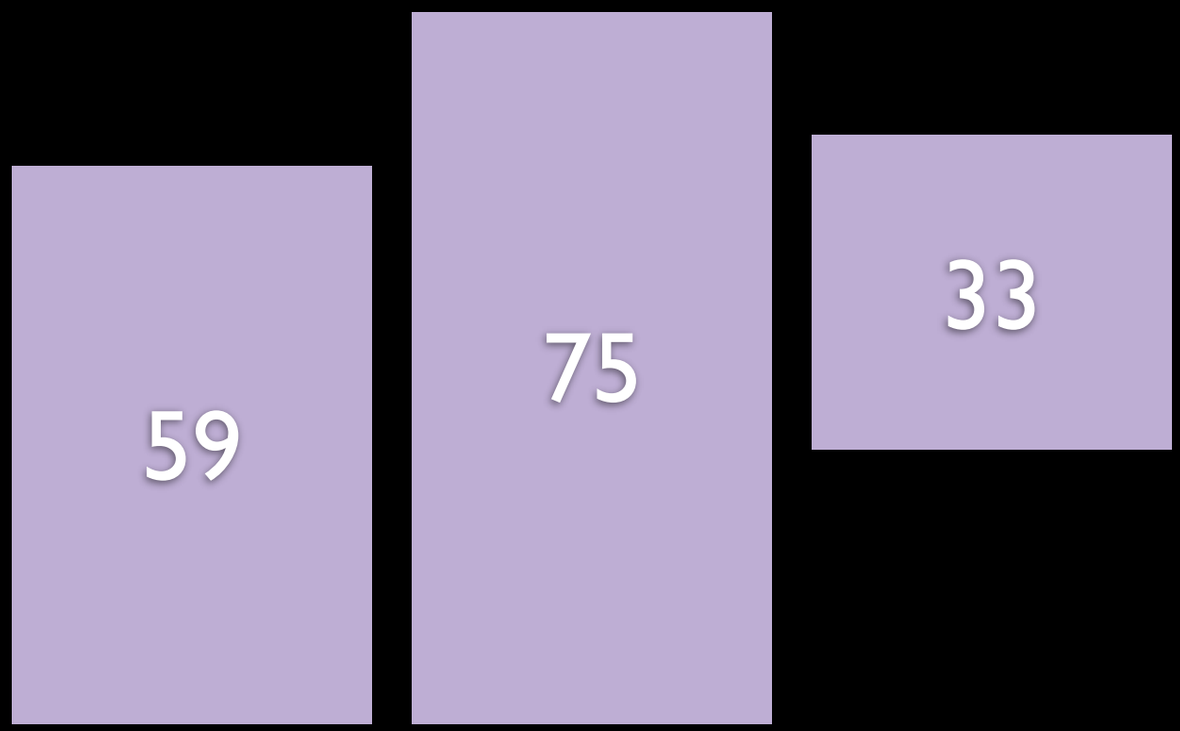
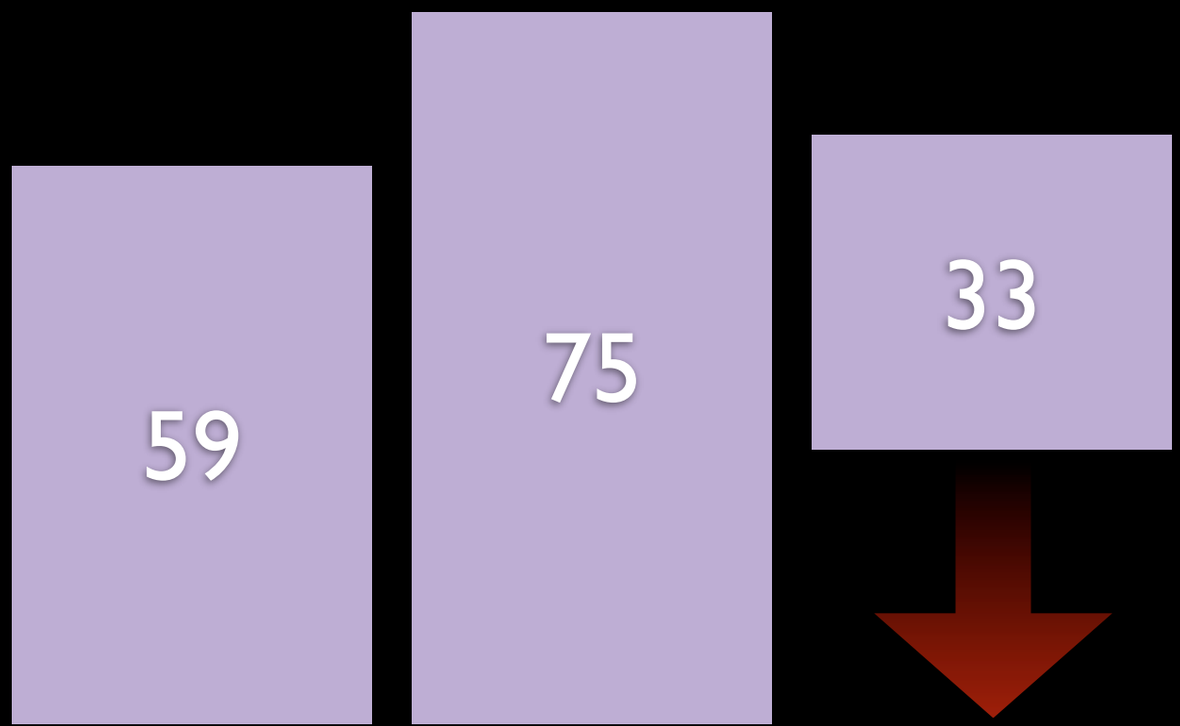
60
40
20
0
1
2
3
instructor
q09d
stays the same
increases
blank
decreases
missing value
uncodeable

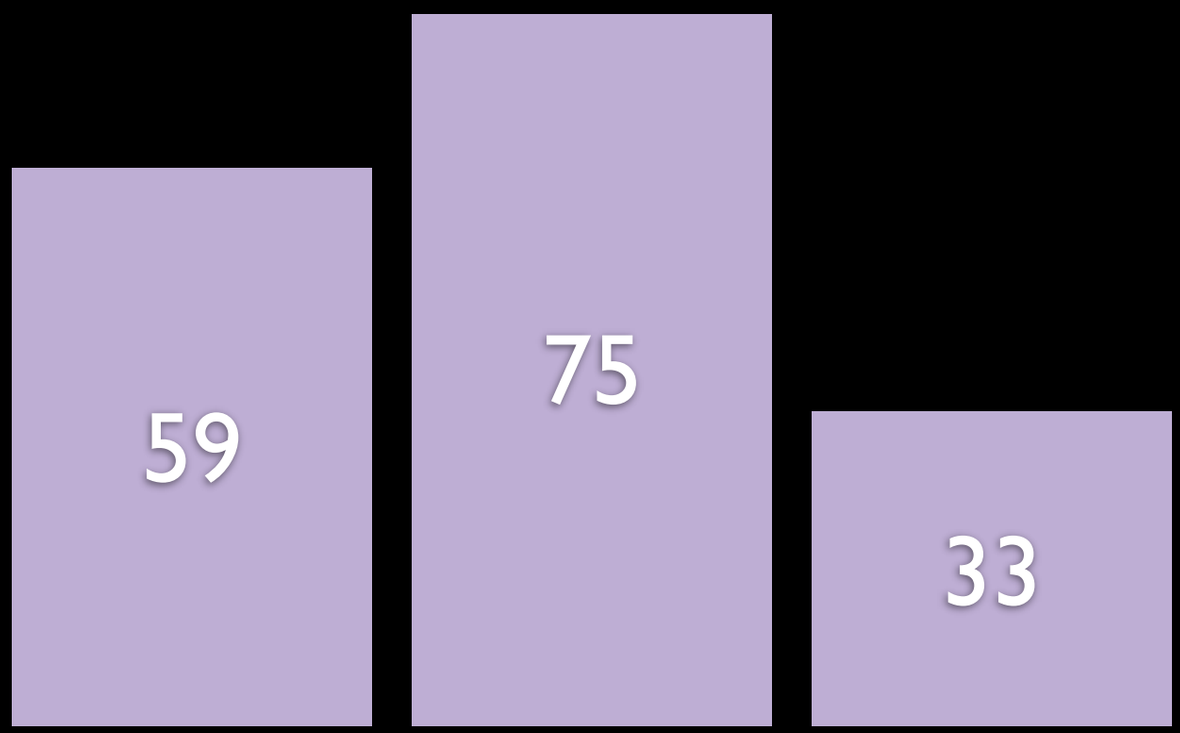
Mechanical Energy Increases



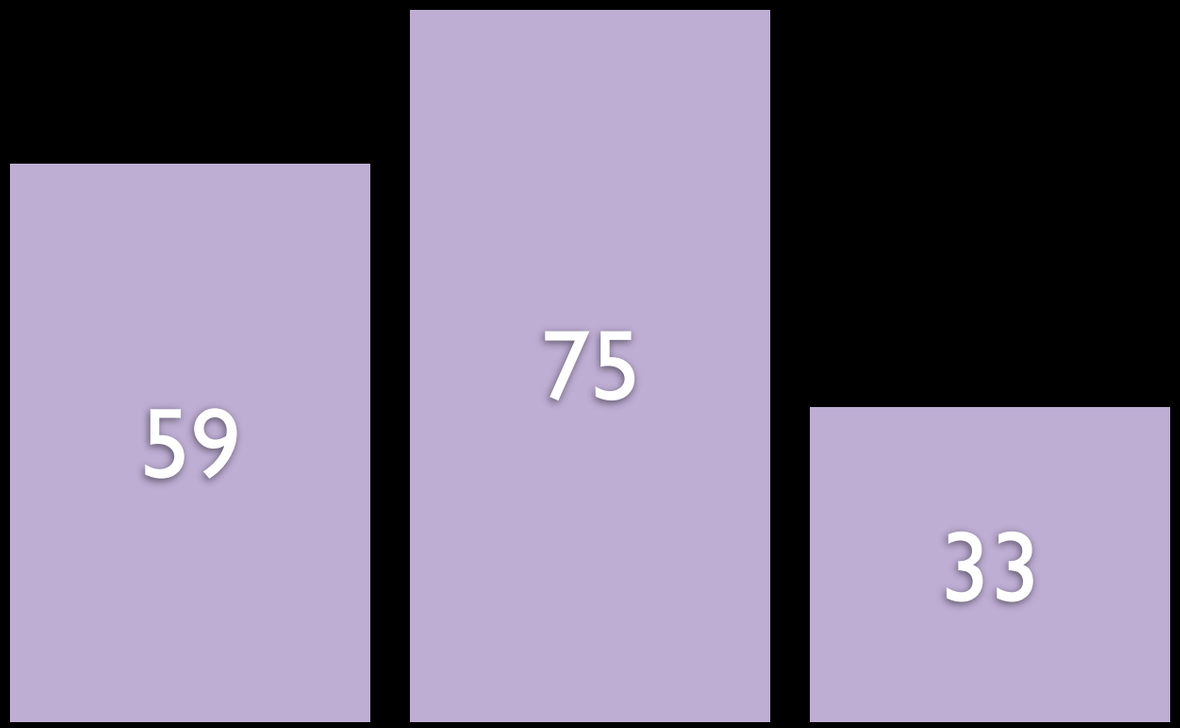
Mechanical Energy Increases

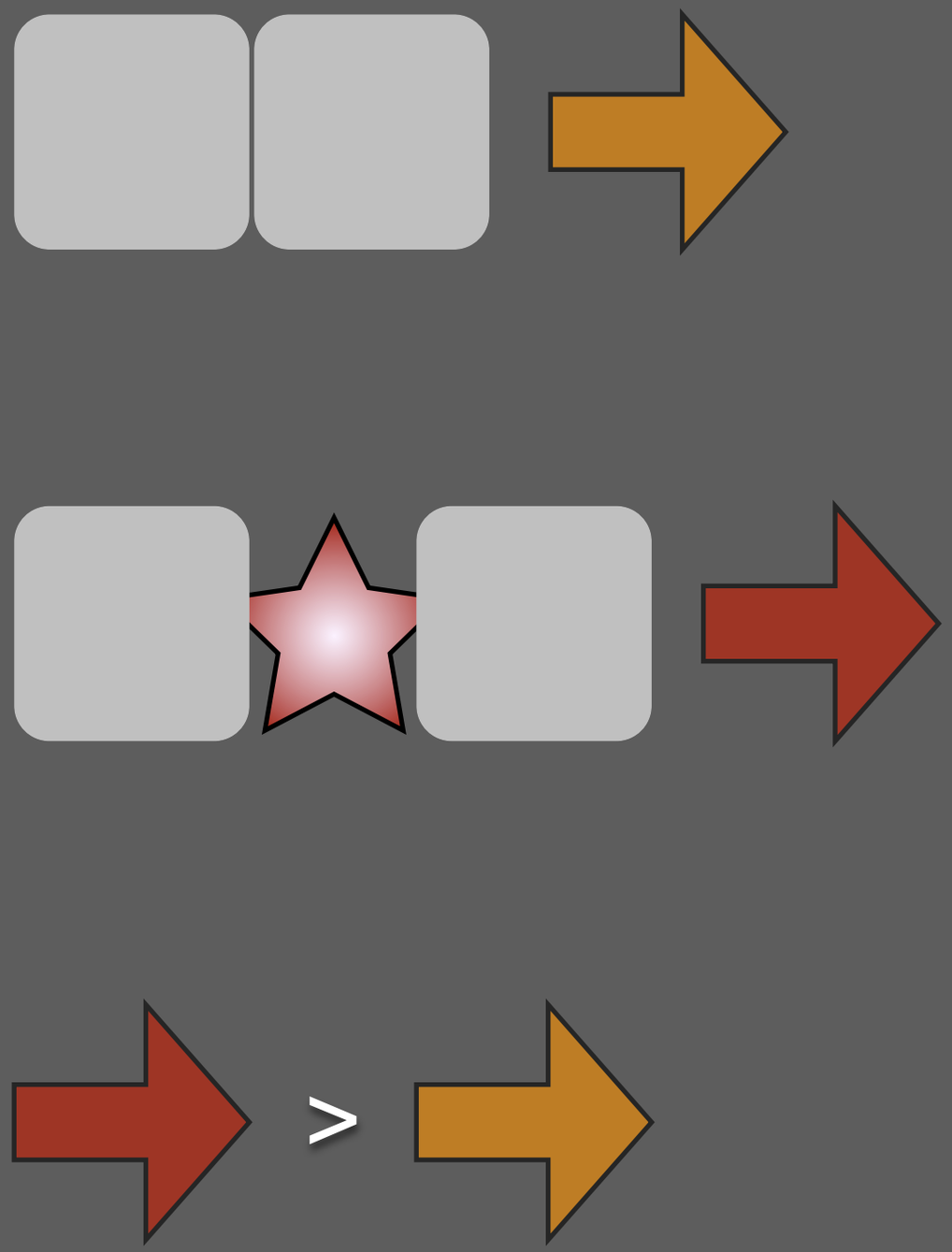


Mechanical Energy Increases

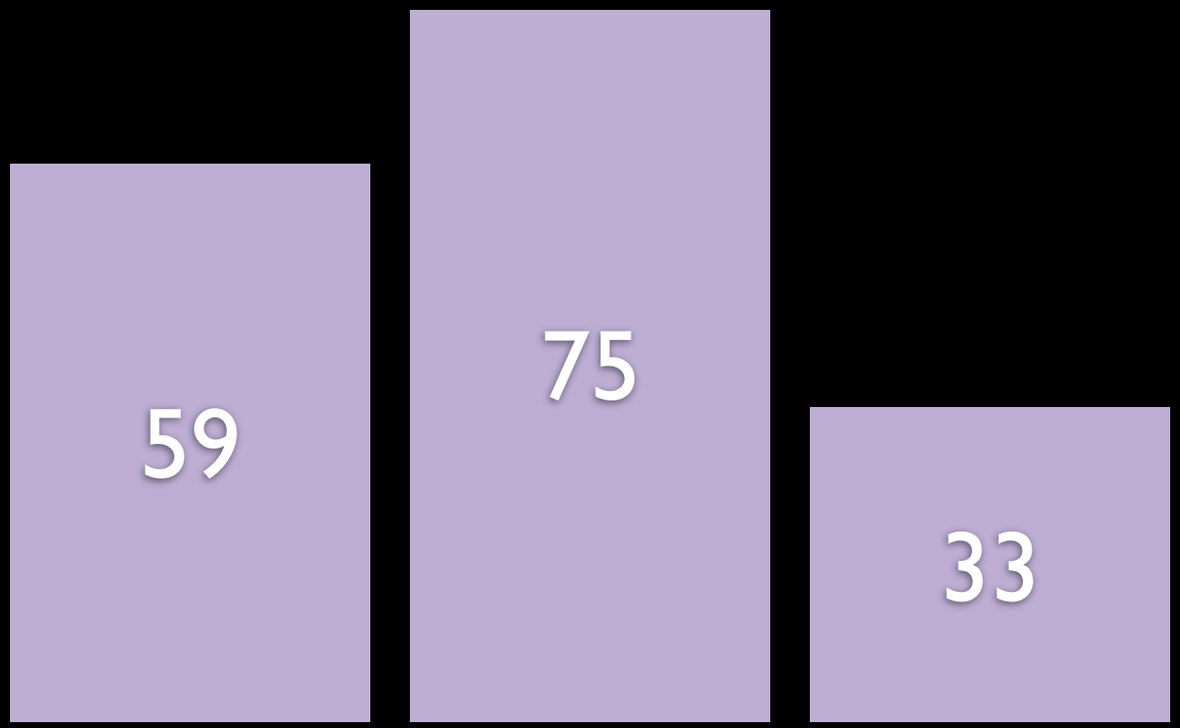


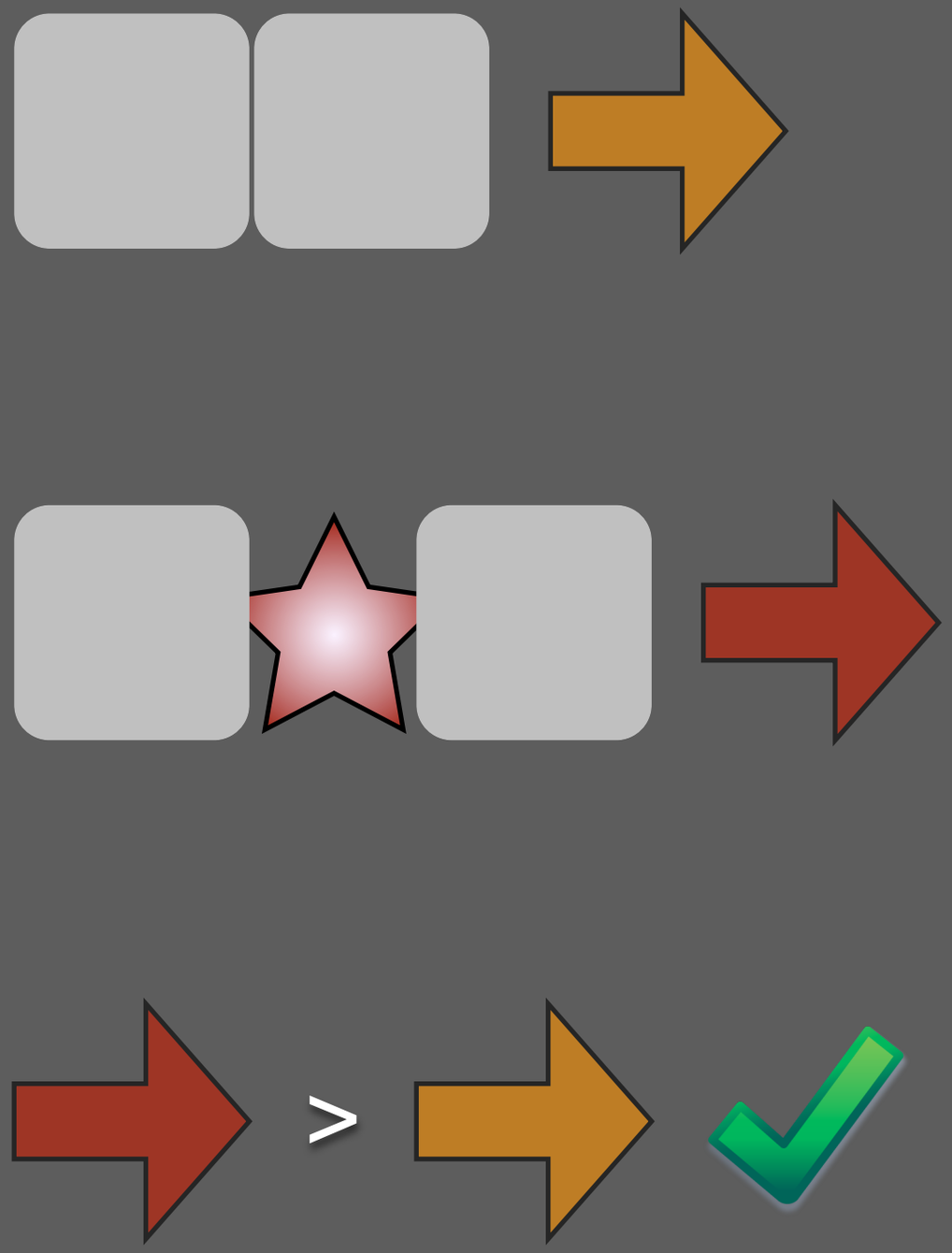
Mechanical Energy Increases





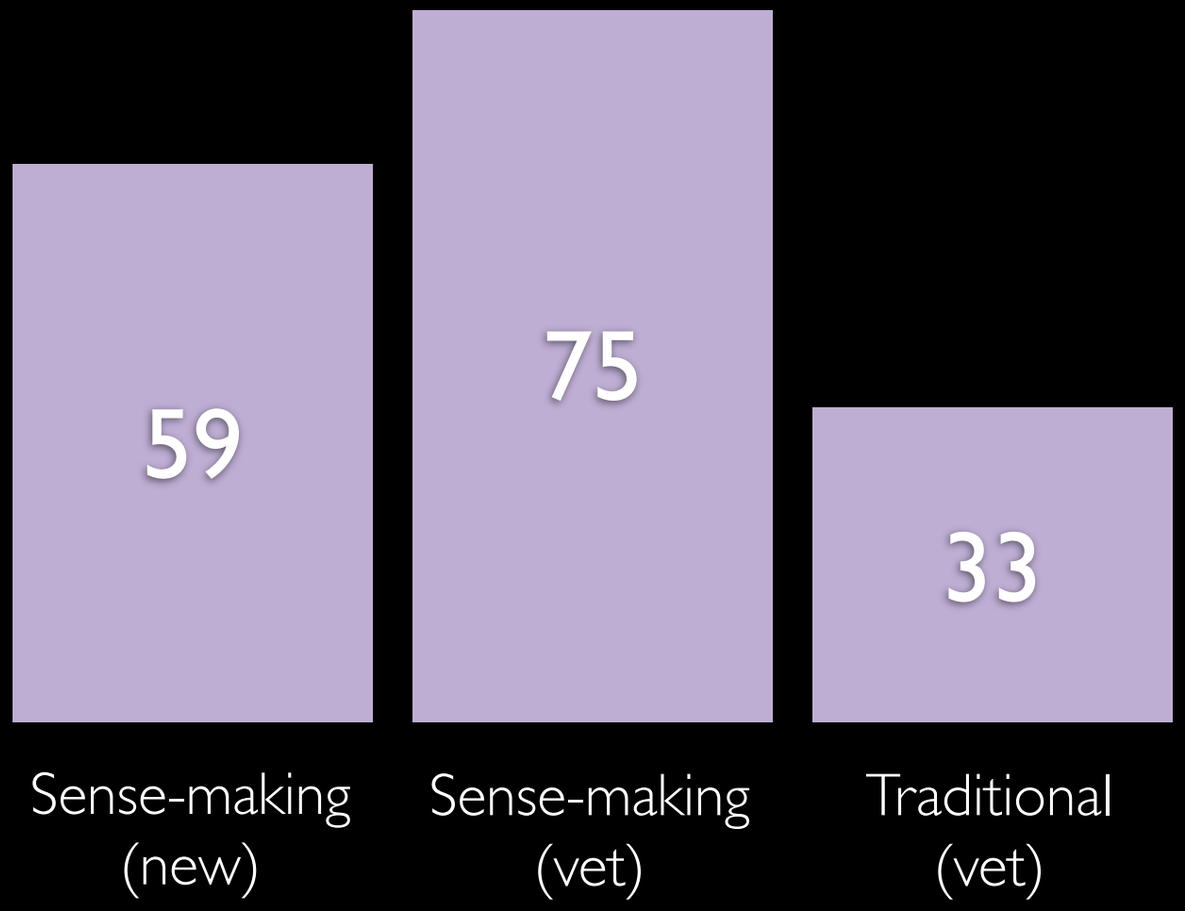
Mechanical Energy Increases





Result 4

When considering energy, more sense-making students were **consistent** with simple intuition.



Ahead,

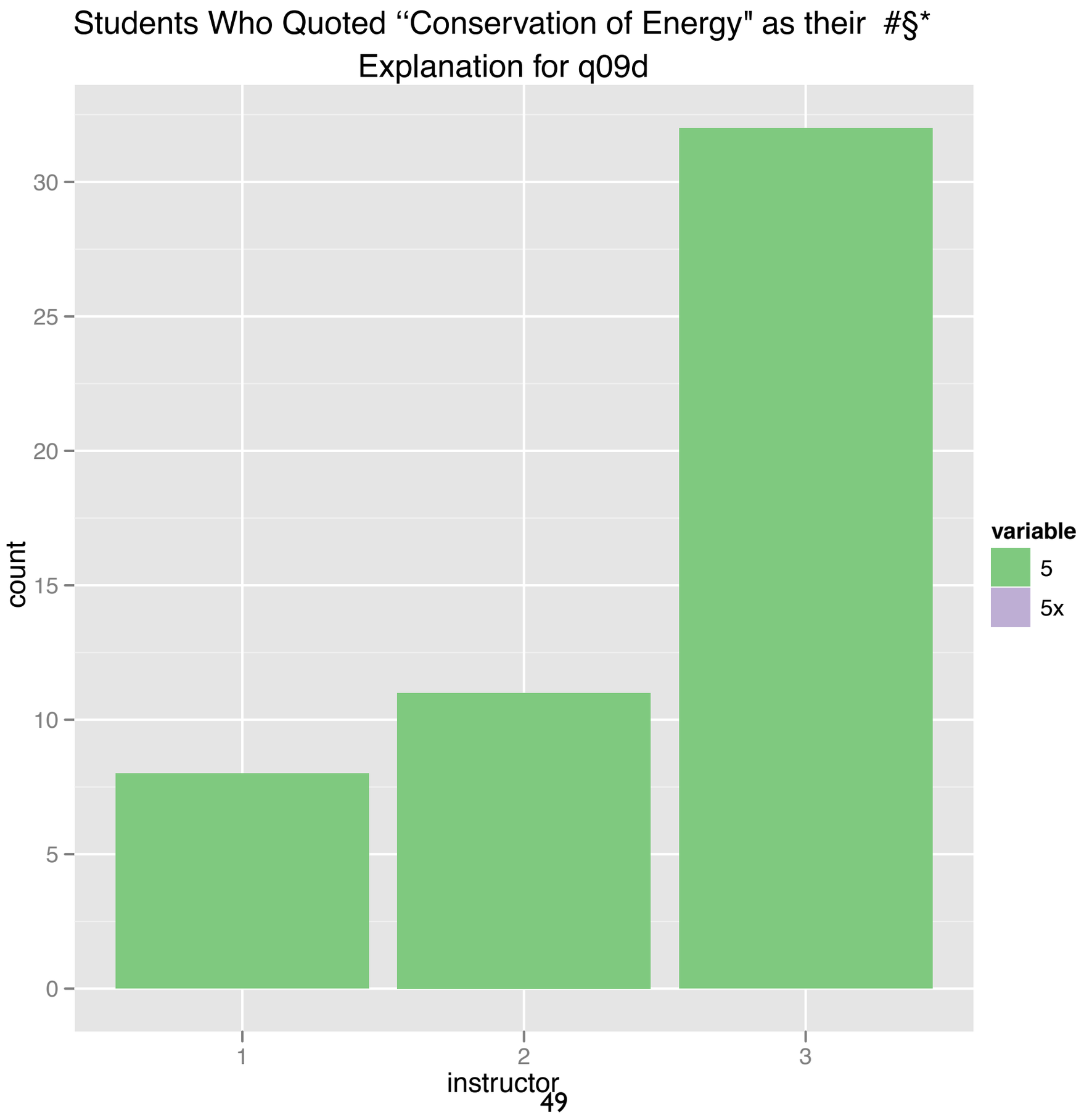
thar be...

DRAGONS!

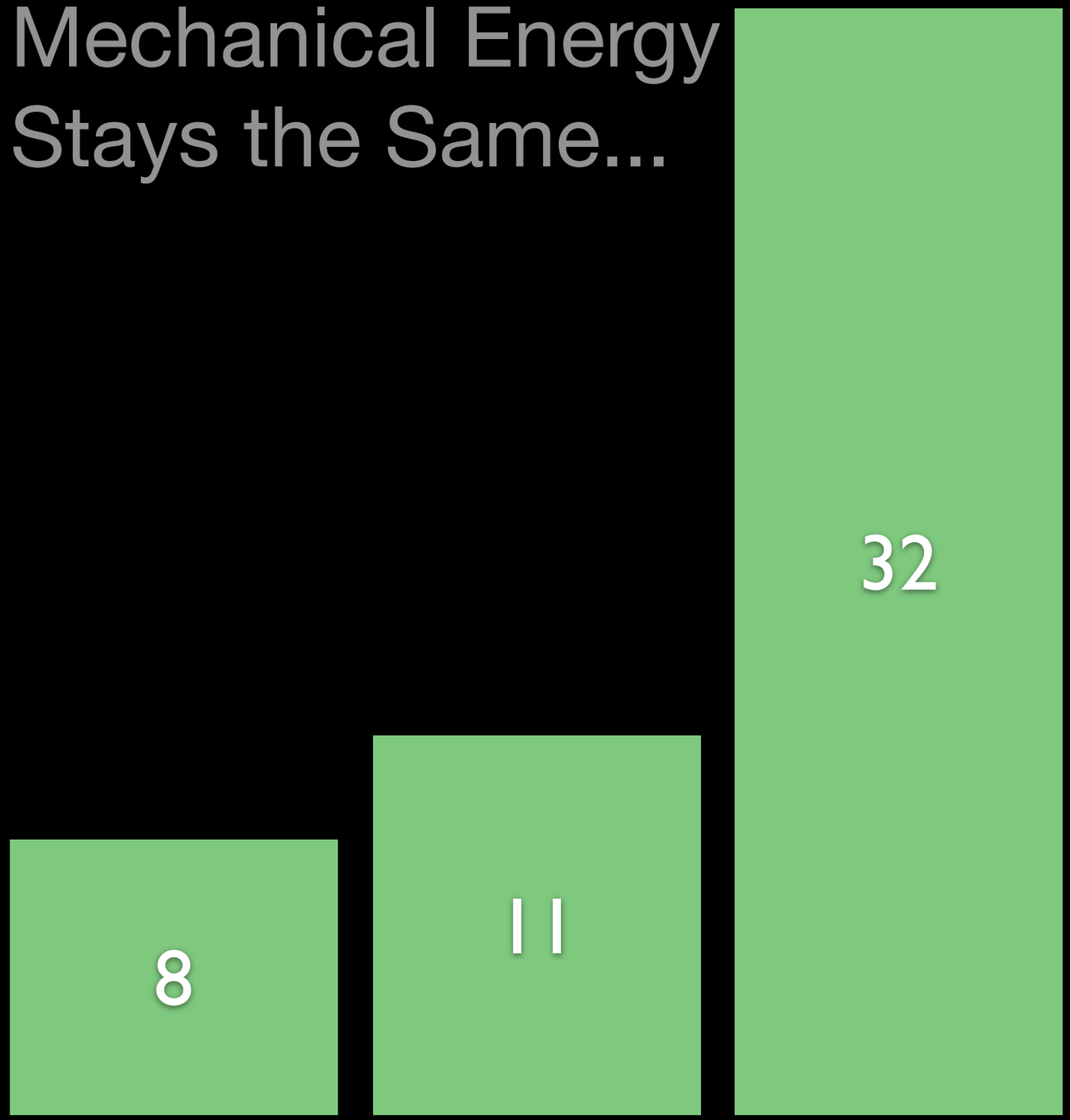
http://www.flickr.com/photos/aztlek/2421800480/in/set-72157605762980035/

This stuff's in beta.

http://www.flickr.com/photos/aztlek/2421800480/in/set-72157605762980035/

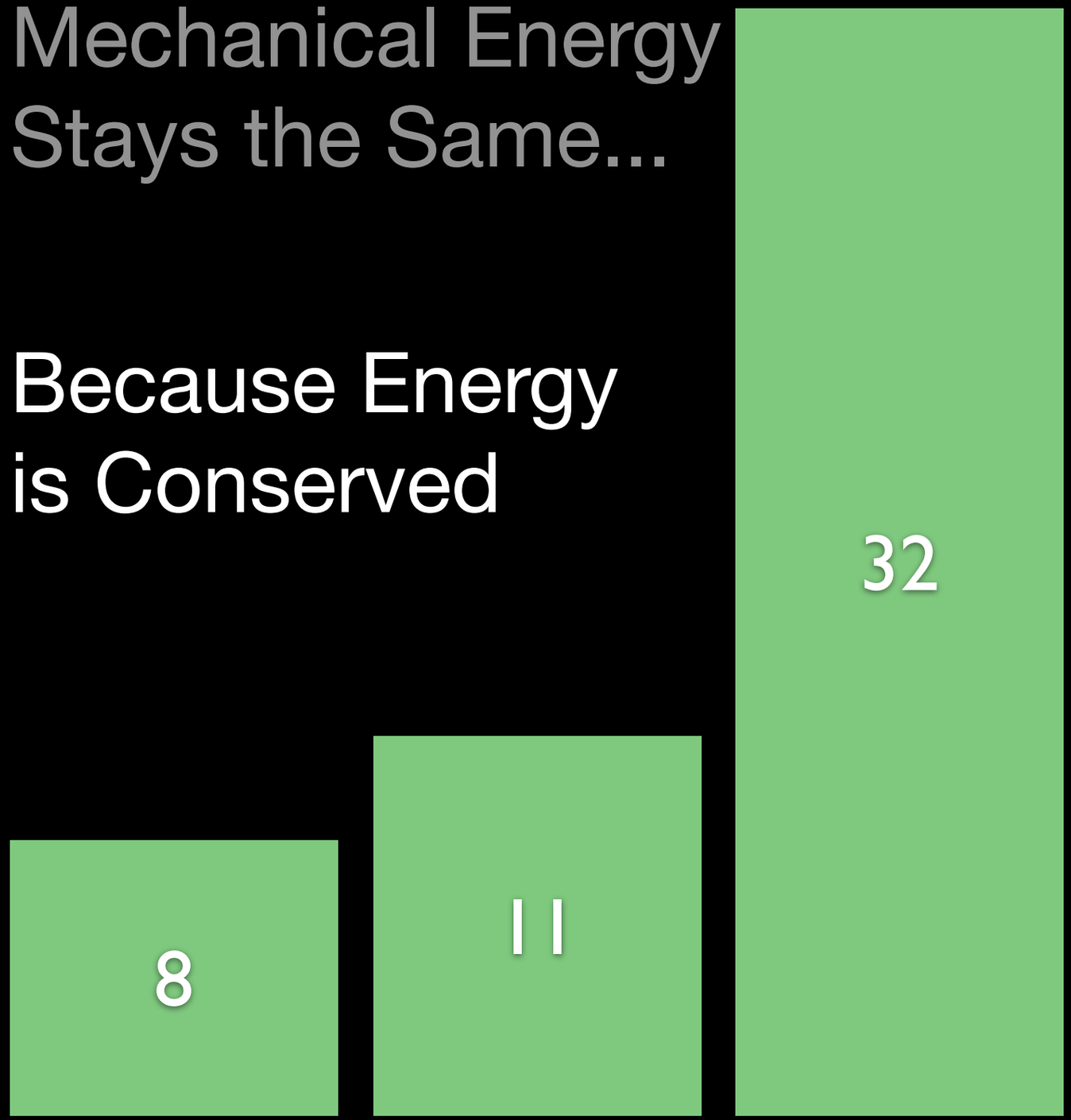

Students Who Quoted ‘‘Conservation of Energy" as their #§*
Explanation for q09d
count
30
25
20
15
10
5
0
1
2
3
instructor
variable
5
5x

Mechanical Energy Stays the Same...



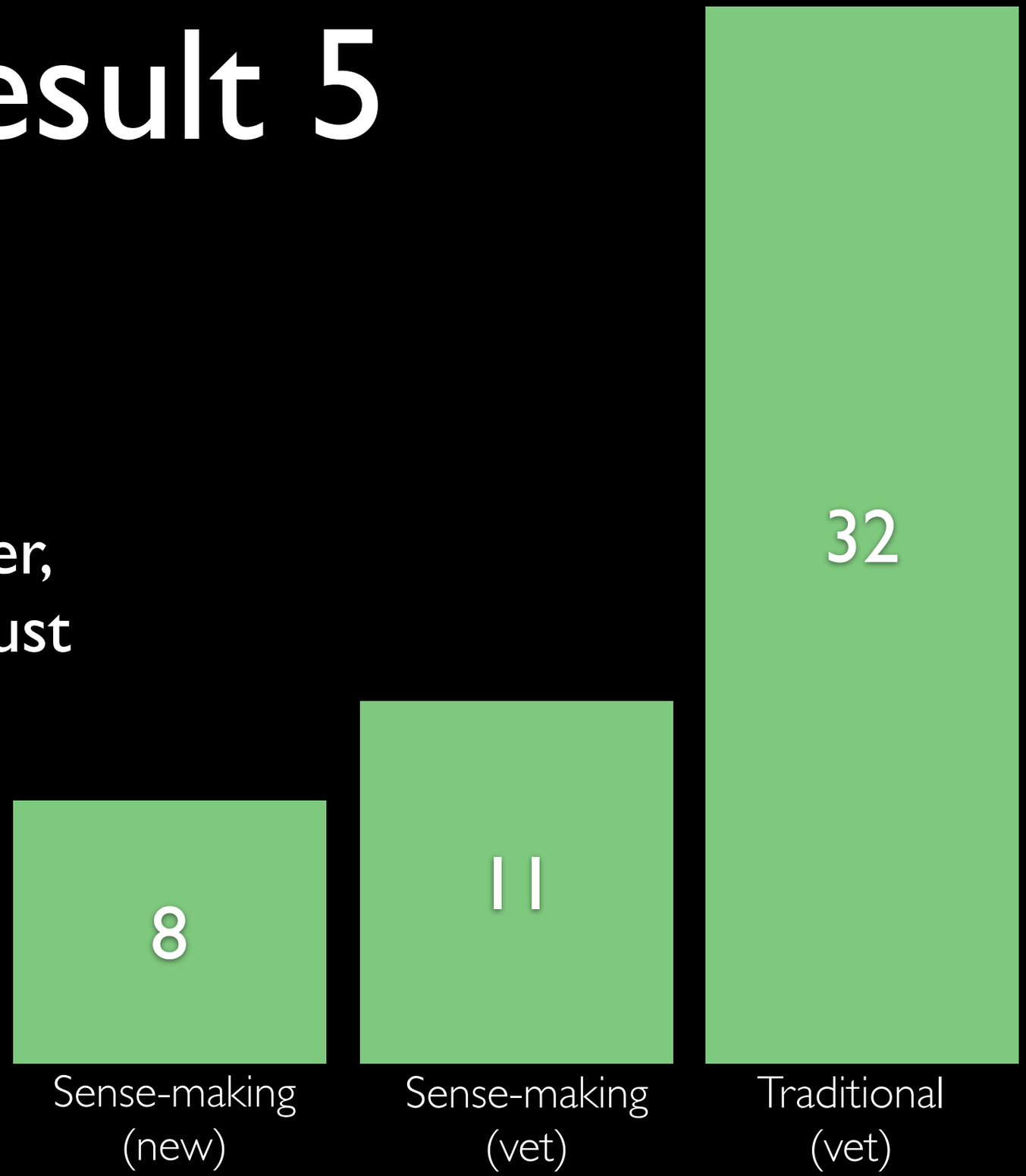
Mechanical Energy
Stays the Same...

Because Energy
is Conserved

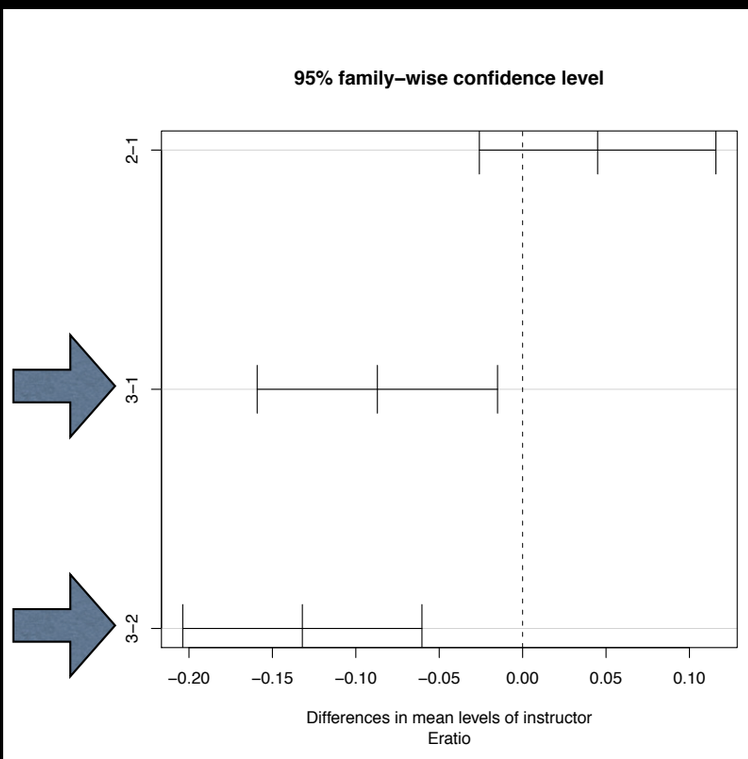


Result 5

When justifying their answer, more traditional students just **quoted a rule**, ignoring that it was tautological *and* inaccurate.



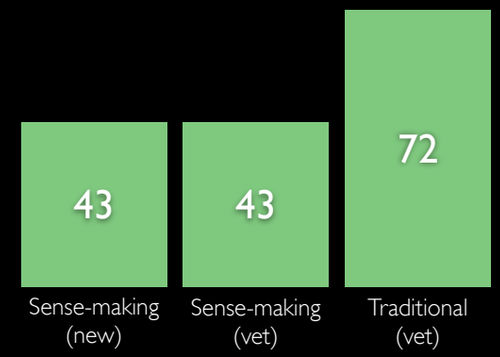
Summary



29

Result 3

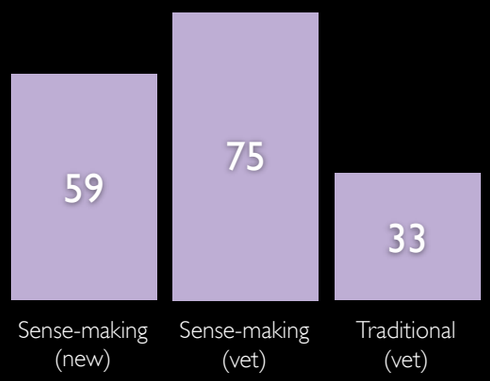
When considering energy, more traditional students asserted something **counter-intuitive**



39

Result 4

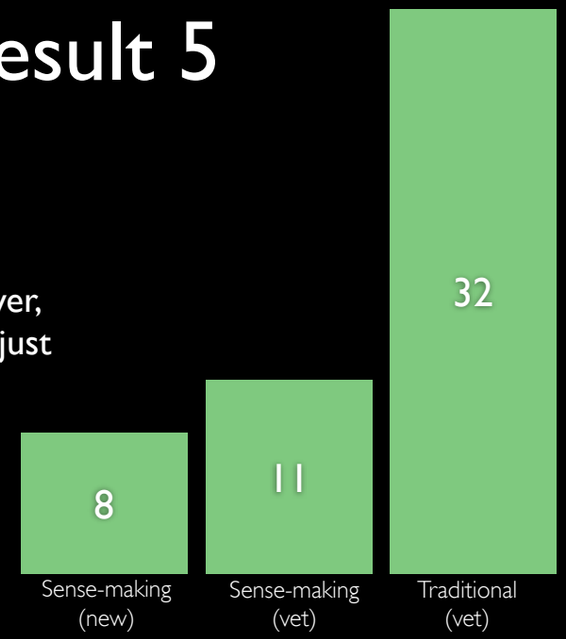
When considering energy, more sense-making students were **consistent** with simple intuition.



46

Result 5

When justifying their answer, more traditional students just **quoted a rule**, ignoring that it was tautological *and* inaccurate.



52

http://github.com/briandk/user2010public

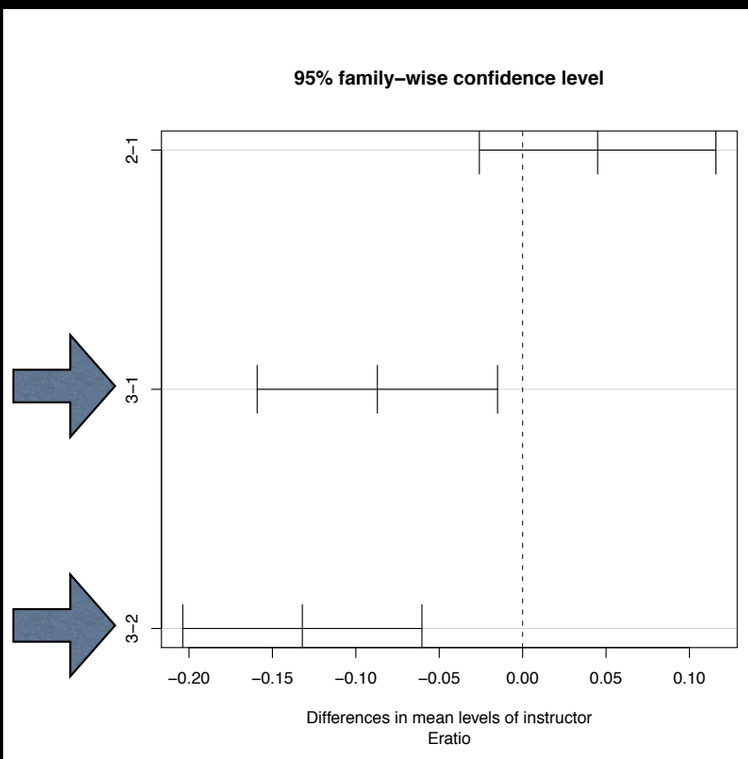
Thanks!

Project Directors

Graduate Assistants

Many Thanks

NSF-DRL grant # 0733613
NSF-EEC grant # 0835880



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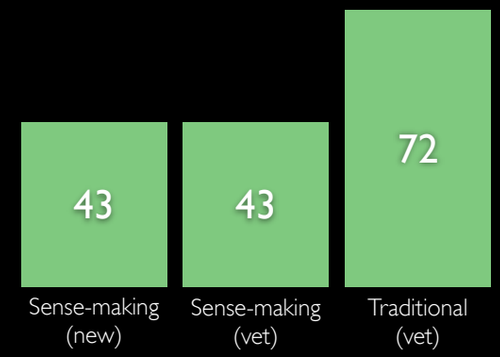

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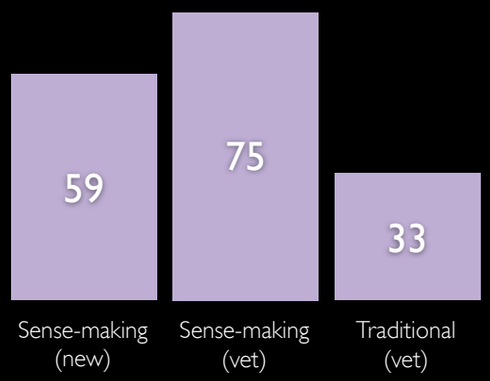

46

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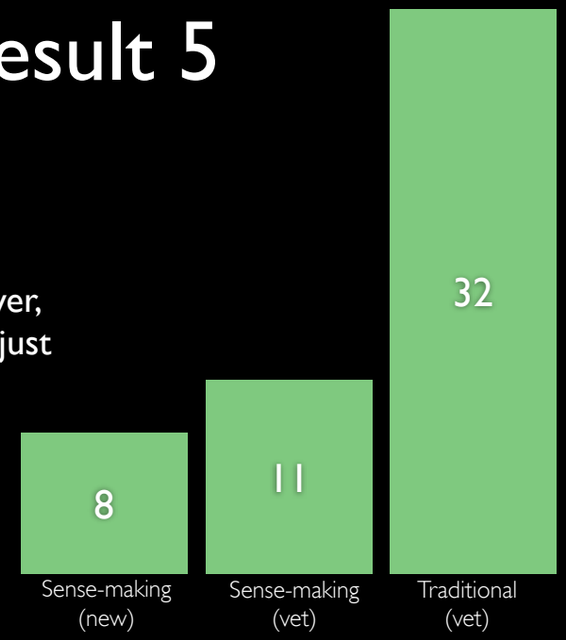
Result 5

When justifying their answer, more traditional students just **quoted a rule**, ignoring that it was tautological *and* inaccurate.



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