Statistical Modeling of Networks using the statnet suite of packages in R

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Joint work with

Martina Morris Steve Goodreau
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Working Papers available at

`http://www.stat.ucla.edu/∼handcock`

`http://statnet.org`

UseR! 2010, July 21 2010

Network modeling from a statistical perspective

- Networks are widely used to represent data on relations between interacting actors or nodes.
- The study of social networks is multi-disciplinary
  - plethora of terminologies
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- The data are of at least three forms:
  - individual-level information on the social entities
  - relational data on pairs of entities
  - population-level data

Deep literatures available

- Social networks community (Heider 1946; Frank 1972; Holland and Leinhardt 1981)
- Statistical Networks Community (Frank and Strauss 1986; Snijders 1997)
- Spatial Statistics Community (Besag 1974)
- Statistical Exponential Family Theory (Barndorff-Nielsen 1978)
- Graphical Modeling Community (Lauritzen and Spiegelhalter 1988, . . .)
- Machine Learning Community (Jordan, Jensen, Xing, ...)
- Physics and Applied Math (Newman, Watts, . . .)

Examples of Friendship Relationships

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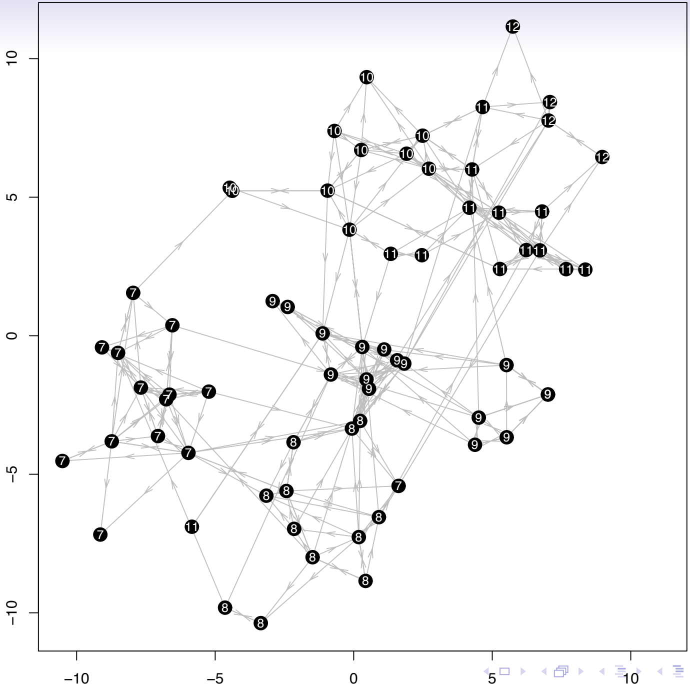
- The National Longitudinal Study of Adolescent Health
  $\Rightarrow$ `www.cpc.unc.edu/projects/addhealth`
  - "Add Health" is a school-based study of the health-related behaviors of adolescents in grades 7 to 12.
- Each nominated up to 5 boys and 5 girls as their friends
- 160 schools: Smallest has 69 adolescents in grades 7–12



10
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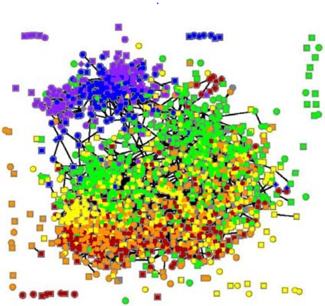
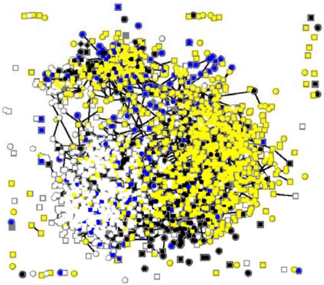
School Community Stratum 44
mutual friendships by Grade



2209 Students

- Grade 7
- Grade 8
- Grade 9
- Grade 10
- Grade 11

School Community Stratum 44
mutual friendships by Race



2209 Students

- **White** (non-Hispanic)
- **Black** (non-Hispanic)
- **Hispanic** (of any race)
- **Asian / Native Am / Other** (non-Hispanic)
- **Race NA**

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  $\Rightarrow$ Lazarsfeld and Merton, 1954; Freeman, 1996; McPherson et al., 2001

  - higher propensity to form ties between actors with similar attributes
    e.g., age, gender, geography, major, social-economic status
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  - people feel comfortable if they agree with others whom they like
- *Context* is important $\Rightarrow$ Simmel (1908)
  - triad, not the dyad, is the fundamental social unit

The Choice of Models depends on the objectives

- Primary interest in the nature of relationships:
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  - spread of HIV and other STDs
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- Tertiary interest in the effect of *interventions* on network structure and processes that develop over a network

Perspectives to keep in mind

- Network-specific versus Population-process
  - *Network-specific*: interest focuses only on the actual network under study
  - *Population-process*: the network is part of a population of networks and the latter is the focus of interest
    - the network is conceptualized as a realization of a social process

The statnet project (2000-present)

- Mission: Develop new statistical methodology for the representation, visualization, analysis and simulation of (social) network data
  - develop computational methods for these statistical methods
  - implement these methods within a coherent suite of user-friendly R packages
  - make them open-source and foster a community outside the developers
- Primary sources of information
  - `http://statnet.org`:
    website, software, manuals, documentation, community
  - `http://www.jstatsoft.org/v24`:
    Special volume of the *Journal of Statistical Software* on statnet

Statnet Commons

Statnet Commons, a collaborative effort among individual statnet developers and their institutions.
The Statnet Commons aims to:

- coordinate development of the Statnet software by contributing organizations
- to manage the resulting work for the advancement of public benefit
- provide for an environment of continuous sharing and collaborative work among individual members
- provide a mechanism for releasing stable versions of the software under GPL at regular intervals.

- Community activities
  - Pedagogical efforts: tutorials, workshops, seminars
  - Make it easy to add new packages that can add functionality to statnet

Statistical Models for Social Networks

Notation

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- call $Y \equiv [Y_{ij}]_{n \times n}$ a *sociomatrix*
  - a $N = n(n-1)$ binary array
- The basic problem of stochastic modeling is to specify a distribution for Y i.e., $P(Y = y)$

A Framework for Network Modeling

Let $\mathcal{Y}$ be the sample space of Y e.g. $\{0, 1\}^N$
Any model-class for the multivariate distribution of Y
can be *parametrized* in the form:

$$P_\eta(Y = y) = \frac{\exp\{\eta \cdot g(y)\}}{\kappa(\eta, \mathcal{Y})} \qquad y \in \mathcal{Y}$$

Besag (1974), Frank and Strauss (1986)

- $\eta \in \Lambda \subset R^q$ q-vector of parameters
- $g(y)$ q-vector of *network statistics*.
  $\Rightarrow$ $g(Y)$ are jointly sufficient for the model
- For a "saturated" model-class $q = |\mathcal{Y}| - 1$ e.g. $2^N - 1$
- $\kappa(\eta, \mathcal{Y})$ distribution normalizing constant

$$\kappa(\eta, \mathcal{Y}) = \sum_{y \in \mathcal{Y}} \exp\{\eta \cdot g(y)\}$$

Simple model-classes for social networks

Homogeneous Bernoulli graph (Erdős-Rényi model)

- Y_{ij} are independent and equally likely
  with log-odds $\eta = \text{logit}[P_\eta(Y_{ij} = 1)]$

$$P_\eta(Y = y) = \frac{e^{\eta \sum_{i,j} y_{ij}}}{\kappa(\eta, \mathcal{Y})} \qquad\qquad y \in \mathcal{Y}$$

  where $q = 1$, $g(y) = \sum_{i,j} y_{ij}$, $\kappa(\eta, \mathcal{Y}) = [1 + \exp(\eta)]^N$
- homogeneity means it is unlikely to be proposed as a model for real phenomena

Dyad-independence models with attributes

- Y_{ij} are independent but depend on dyadic covariates $x_{k,ij}$

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$$\kappa(\eta, \mathcal{Y}) = \prod_{i,j} [1 + \exp(\sum_{k=1}^{q} \eta_k x_{k,ij})]$$

Of course,

$$logit[P_\eta(Y_{ij} = 1)] = \sum_k \eta_k x_{k,ij}$$

Generative Theory for Network Structure

Actor Markov statistics

$\Rightarrow$ Frank and Strauss (1986)

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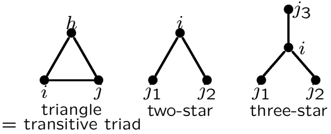
- Degree distribution: $\mathrm{d}_k(y) =$ proportion of actors of degree k in y.
- triangles: $\mathrm{triangle}(y) =$
number of triads that form a complete sub-graph in y.



More General mechanisms motivated by conditional independence

$\Rightarrow$ Pattison and Robins (2002), Butts (2005)
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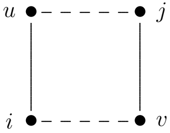

Partial conditional dependence when four-cycle is created

This produces features on configurations of the form:

- edgewise shared partner distribution: $\mathrm{esp}_k(y) =$ proportion of edges between actors with exactly k shared partners $k = 0, 1, \ldots$

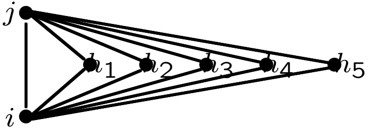

Figure: The actors in the non-directed (i, j) edge have 5 shared partners

- dyadwise shared partner distribution: $\mathrm{dsp}_k(y) =$ proportion of dyads with exactly k shared partners $k = 0, 1, \ldots$

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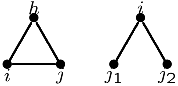
  mean clustering coefficient



triangle
= transitive triad

two-star

Statistical Inference for ERGM parameter η

Base inference on the loglikelihood function,

$$\ell(\eta) = \eta \cdot g(y_{\text{obs}}) - \log \kappa(\eta)$$

$$\kappa(\eta) = \sum_{\substack{\text{all possible}\\ \text{graphs } z}} \exp\{\eta \cdot g(z)\}$$

Approximating the loglikelihood

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- Suppose $Y_1, Y_2, \ldots, Y_m \overset{\text{i.}i.d.}{\sim} P_{\eta_0}(Y = y)$ for some η_0.
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&\equiv \tilde{\ell}(\eta) - \tilde{\ell}(\eta_0).
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- Approximate the MLE $\hat{\eta} = \text{argmax}_\eta \{\tilde{\ell}(\eta) - \tilde{\ell}(\eta_0)\}$ (MC-MLE) $\Rightarrow$ Geyer and Thompson (1992)

How can we tell if a model class is useful?

Many aspects:

- Is the model-class itself able to represent a range of realistic networks?
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- What are the properties of different methods of estimation?
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  - *computational failure*: estimates do not exist for certain observable graphs
- Can we assess the goodness-of-fit of models?
  - appropriate measures and tests (Besag 2000; Hunter, Goodreau, Handcock 2007)

Model Degeneracy

idea: A random graph model is *near degenerate* if the model places almost all its probability mass on a small number of graph configurations in $\mathcal{Y}$.

e.g. empty graph, full graph, an individual graph, no $2-$stars, mono-degree graphs

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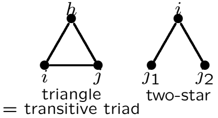
- Example: The *two-star* model

$$P(Y = y) = \frac{\exp\{\eta_1 \mathrm{edge}(y) + \eta_2 \mathrm{two\!-\!star}(y)\}}{c(\eta_1, \eta_2)} \qquad y \in \mathcal{Y}$$

is near-degenerate for most values of $\eta_2 > 0$

$$\mathrm{edge}(y) = \sum_{i<j} y_{ij} \qquad \mathrm{two\!-\!star}(y) = \sum_{i<j<k} y_{ij} y_{ik}$$



Illustrations of models within this model-class

- village-level structure

  – $n = 50$

  – mean clustering coefficient = 15% – degree distribution: Yule with scaling exponent 3.
- larger-level structure

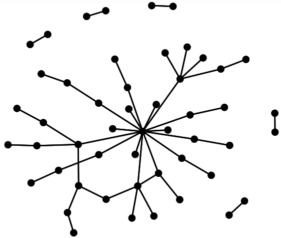
  – $n = 1000$

  – mean clustering coefficient = 15% – degree distribution: Yule with scaling exponent 3.
- Attribute mixing

  – Two-sex populations

  – mean clustering coefficient = 15% – degree distribution: Yule with scaling exponent 3.

Yule with zero clustering coefficient conditional on degree



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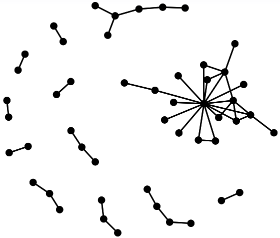

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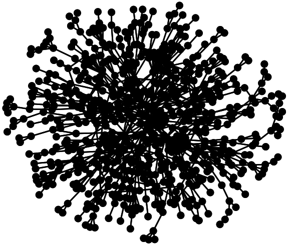

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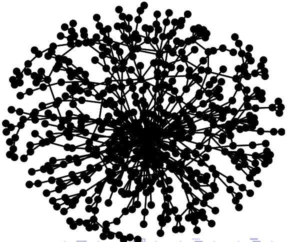
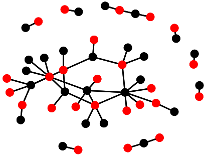
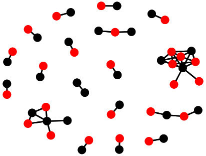


Heterosexual Yule with no correlation



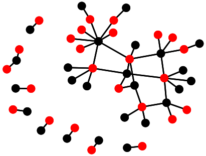
Heterosexual Yule with strong correlation



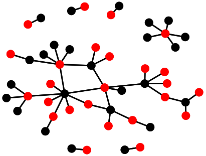
tripercent = 3

tripercent = 60.6

Heterosexual Yule with modest correlation



Heterosexual Yule with negative correlation



Application to a Protein-Protein Interaction Network

- By interact is meant that two amino acid chains were experimentally identified to bind to each other.
- The network is for *E. Coli* and is drawn from the "Database of Interacting Proteins (DIP)" `http://dip.doe-mbi.ucla.edu`
- For simplicity we focus on proteins that interact with themselves and have at least one other interaction
  – 108 proteins and 94 interactions.

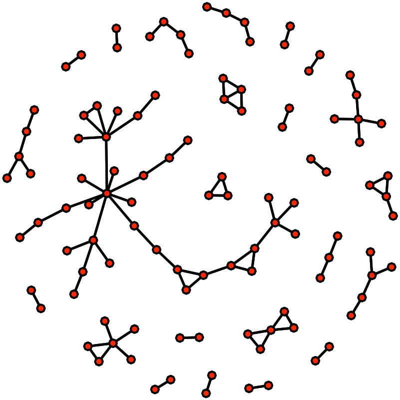


Figure: A protein - protein interaction network for *E. Coli.* The nodes represent proteins and the ties indicate that the two proteins are known to interact with each other.

Statistical Inference and Simulation

- Simulate using a Metropolis-Hastings algorithm (Handcock 2002).
- Here base inference on the likelihood function
- For computational reasons, approximate the likelihood via Markov Chain Monte Carlo (MCMC)
- Use maximum likelihood estimates (Geyer and Thompson 1992)

Parameter	est.	s.e.
Scaling decay rate (ϕ)	3.034	0.3108
Correlation Coefficient (ν)	1.176	0.1457

Table: MCMC maximum likelihood parameter estimates for the protein-protein interaction network.

Clustering and Social Networks

Clustering and Social Networks

- Three types of clustering in social networks:
  - transitivity of relationships
  - homophily of actors with similar *observed* characteristics
  - further clustering that could be due to:
    - homophily on unobserved attributes, or
    - "self-organization" into groups

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- Drawing conclusions about clustering of social actors is often a focus of interest in social network analysis
- But most methods don't address it directly
- Instead conclusions about clustering are often drawn by informally eyeballing results from other methods
- We present a statistical model of social networks that incorporates clustering and allows formal inference about:
    - whether or not there is clustering (beyond transitivity)
    - if so, how many groups there are
    - who is in what group
    - uncertainty about group memberships

Positing Latent Social Structure via Random Effects

- model an underlying latent "social space" of actors
  - Latent space models: Hoff, Raftery and Handcock (2002)
    Hoff (2003, 2004 ,...)
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Specifically:

$$\log \text{odds}(Y_{ij} = 1 | z_i, z_j, x_{ij}, \beta) = \beta^T x_{ij} - |z_i - z_j| + \delta_i + \gamma_j$$

where β denotes parameters to be estimated.

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- captures position, transitivity, homophily on attributes, and clustering
- captures individual propensities to form and receive ties

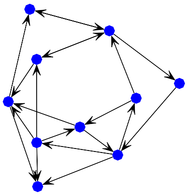
$$\begin{aligned} \delta_i &\overset{\text{i.}i.d.}{\sim} \mathrm{N}(0, \sigma_\delta^2) \quad i = 1, \ldots, n, \\ \gamma_i &\overset{\text{i.}i.d.}{\sim} \mathrm{N}(0, \sigma_\gamma^2) \quad i = 1, \ldots, n, \end{aligned}$$

Summary of latent cluster model

- Model-based clustering of latent positions for social networks provides a formal model of social networks that incorporates clustering
- It permits inference about:
  - whether there is clustering
  - how many groups there are
  - who is in what group
  - uncertainty about group memberships
  - the actors' latent social positions
- It gave reasonable results for two examples
- Software: The R package latentnet, available on CRAN

Partial Observation of Social Networks

- **Sampling Design:** Choose which part to observe:
  "Ask 10% of employees about their collaborations"
  - Egocentric
  - Adaptive
- **Out-of-design Missing Data:**
  "Try to survey the whole company, but someone is out sick"
- **Boundary Specification Problem:**
  "Should a contractor be considered a part of the company?"



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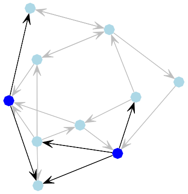

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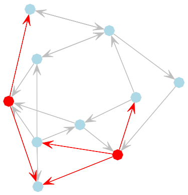

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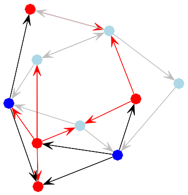

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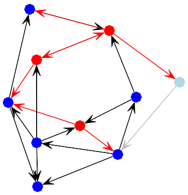

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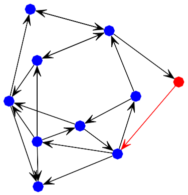

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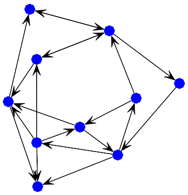

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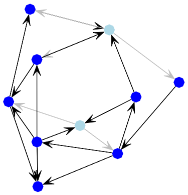

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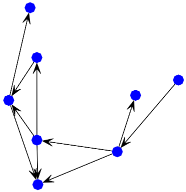

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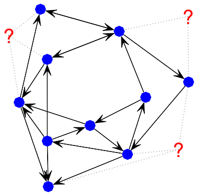

Frameworks for Statistical Analysis

	Network Specific	Population Process
Fully Observed Data	Description	Modeling (Statistical)
Partially Observed Data	Design-Based Inference	Likelihood Inference

statnet capabilities

Required packages: ergm and network available on CRAN

- ergm is a collection of functions to fit, simulate from, plot and evaluate exponential-family random graph models.
  The main functions within the ergm package are:
  - ergm, a function to fit exponential-family random graph models in which the probability of a network is dependent upon a vector of network statistics specified by the user
  - simulate, a function to simulate random networks using an ERGM
  - gof, a function to evaluate the goodness of fit of an ERGM to the data.

  ergm contains many other functions as well.
- network is a package to create, store, modify and plot the data in network objects.
  The network object class, defined in the network package , can represent a range of relational data types and it supports arbitrary vertex / edge / network attributes.
  Data stored as network objects can then be analyzed using all of the component packages in the statnet suite.

statnet capabilities

Optional packages
The optional packages sna, degreenet, latentnet, and networksis are all available on CRAN:

- sna: A set of tools for traditional social network analysis .
- degreenet: This package was developed for the degree distributions of networks. It implements likelihood-based inference, bootstrapping, and model selection, and it includes power-law models such as the Yule and Waring as well as a range of alternative models that have been proposed in the literature. .
- latentnet: A package to fit and evaluate latent position and cluster models for statistical networks.
- networksis: A package to simulate bipartite networks with fixed marginals through sequential importance sampling .

statnet capabilities

Additional optional packages are available on request, as described below.

- dynamicnetwork: A set of tools for visualizing dynamically changing networks .
- netperm: A package for permutation Models for relational data . It provides simulation and inference tools for exponential families of permutation models on relational structures.
- rSoNIA: Provides a set of methods to facilitate exporting data and parameter settings and launching SoNIA, which stands for Social Network Image Animator . SoNIA facilitates interactive browsing of dynamic network data and exporting animations as a QuickTime movies.

Additional capabilities

- statnet can efficiently deal with large networks
  (it handles data natively in edgelist form (within the backend).
- In terms of data representations, it can generally support networks on the order of 10^8 edges and/or nodes.
- missing data on relations are handled
- dynamic models has been developed and coded, but is not yet on CRAN (Krivitsky 2009)

Statistical Challenges and Opportunities

- massive and varied types of data
  - incorporation of these into the model is sometimes difficult
- networks fundamentally relational
  - traditional notions based on independence flawed
- noise in the relations and attributes
- partially observed networks
  - almost always (non-ignorable) missing values

    $\Rightarrow$ Handcock and Gile (2008)
  - often the boundary of the network is endogenous
- measuring goodness-of-fit of network models

  $\Rightarrow$ Hunter, Goodreau and Handcock (2007)
- representing uncertainty in the inference
- visualization of complex models and networks

- In some disciplines the basic question of inference is ignored
- understanding properties of sparse representations
  - e.g., concept of "model degeneracy" $\Rightarrow$ Handcock (2003)
  - MLE, maximum pseudo-likelihood
- improve estimation methods
  - technology transfer of approximate likelihood methods and ideas developed in Genetics and Computer Science
  - Variational methods (Jordan et al 1998, ...)

Opportunities

- Sciences should make better use of network sampling techniques
  - adaptive network designs (e.g., link tracing)
    - $\Rightarrow$ Handcock and Gile (2008)
  - respondent-driven sampling for hard-to-reach populations
    - $\Rightarrow$ Gile and Handcock (2008)
- Dynamic and longitudinal models (harder and easier)
- Most models condition on the number of nodes
  - models "generating" the number of nodes are important

Summary

- Network representations intersect with most sciences
- Sparse models are being used to capture structural properties
- The models must depend on the scientific objective.
- Some seemingly simple models are not so.
- The inclusion of attributes is very important
  - actor attributes
  - dyad attributes e.g. homophily, race, location
  - structural terms e.g. transitive homophily