Cross-sectional and Spatial Dependence in Panels

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2 sides to the talk:

Robustness features against XS correlation

- XS-dependence without any explicit spatial characteristic (e.g., due to the presence of common factors)
- OLS/FE/RE estimates are still consistent but for valid inference we need robust covariance matrices

(to be included in the `plm` package)

Spatial models characterizing XS dependence in a parametric way

- explicitly taking distance into account
- distance matrix is exogenous and time-invariant (although it needn't be *geographic* distance)
- the estimation framework is ML

(forthcoming in an *ad hoc* package)

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1. Robust linear restriction testing in `plm`
2. General cross-sectional correlation robustness features
3. Diagnostics for *global* cross-sectional dependence
4. Diagnostics for *local* cross-sectional dependence
5. ML estimators and ML-based tests for spatial panels

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Robustness features for panel models

The `plm` package for panel data Econometrics (Croissant and Millo):

- version 1.0-0 now on CRAN
- paper just appeared in Econometrics Special Issue of the JSS (27/2)

implements the general framework of robust restriction testing (see package `sandwich`, Zeileis, JSS 2004) based upon

- correspondence between conceptual and software tools in

$$W = (R\beta - r)'[R' vcov(\beta) R]^{-1}(R\beta - r)$$

- White (-Eicker-Huber) robust vcov, a.k.a. the *sandwich* estimator

The `plm` version of robust covariance estimator (`pvcovHC()`) is based on White's formula and (partial) demeaning

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Outline of the talk

Robust diagnostic testing under XSD

So we need a vcov estimator robust vs. XS correlation. 3 possibilities: 2 based on the general framework

$$vcov(\beta) = (X'X)^{-1} \sum_i X_i E_i X_i' (X'X)^{-1}$$

- White cross-section: $E_i = e_i e_i'$ is robust w.r.t. arbitrary heteroskedasticity and XS-correlation; depends on T-asymptotics
- Beck & Katz unconditional XS-correlation (a.k.a. PCSE): $E_i = \frac{\epsilon_i' \epsilon_i}{N_i}$

or the Driscoll and Kraay (RES 1998) estimator, robust vs. time-space correlation decreasing in time ...

... and the trick of robust diagnostic testing is done! Just supply the relevant *vcov* to `coeftest{lmtest}` or `linear.hypothesis{car}`

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Testing for XS dependence

The CD test 'family' (Breusch-Pagan 1980, Pesaran 2004) is based on transformations of the product-moment correlation coefficient of a model's residuals, defined as

$$\hat{\rho}_{ij} = \frac{\sum_{t=1}^{T} \hat{u}_{it}\hat{u}_{jt}}{(\sum_{t=1}^{T} \hat{u}_{it}^2)^{1/2}(\sum_{t=1}^{T} \hat{u}_{jt}^2)^{1/2}}$$

and comes in different flavours appropriate in N-, NT- and T- asymptotic settings:

$$CD = \sqrt{\frac{2T}{N(N-1)}}(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \hat{\rho}_{ij})$$

$$LM = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} T_{ij}\hat{\rho}_{ij}^2$$

$$SCLM = \sqrt{\frac{1}{N(N-1)}}(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \sqrt{T_{ij}\hat{\rho}_{ij}^2})$$

Friedman's (1928) rank test and Frees' (1995) test substitute Spearman's rank coefficient for ρ

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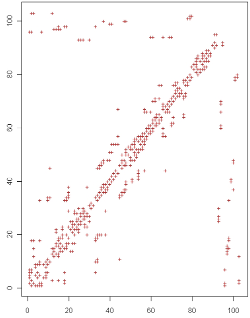
Introducing georeferentiation: the local CD tests (1)

Restricting the test to *neighbouring* observations: meet the W matrix!

Figure: Proximity matrix for Italy's NUTS2 regions



The local CD tests (2)

The CD(p) test is *CD* restricted to neighbouring observations

$$CD = \sqrt{\frac{T}{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w(p)_{ij}}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}[w(p)]_{ij}\hat{\rho}_{ij}\right)$$

where $[w(p)]_{ij}$ is the (i,j)-th element of the p-th order proximity matrix, so that if h, k are not neighbours, $[w(p)]_{hk} = 0$ and $\hat{\rho}_{hk}$ gets "killed"; W is employed here as a binary selector: any matrix coercible to boolean will do

`pcdtest(..., w=W)` will compute the local test. Else if `w=NULL` the global one.

Only CD(p) is documented, but in principle any of the above tests (LM, SCLM, Friedman, Frees) can be restricted.

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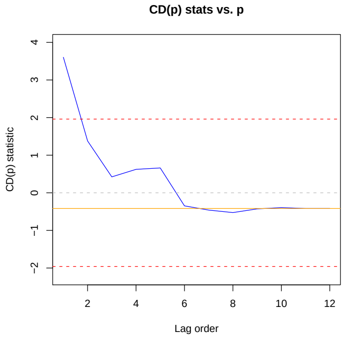
Recursive CD plots

The CD test, seen as a descriptive statistic, can provide an informal assessment of the degree of 'localness' of the dependence: let the neighbourhood order p grow until $CD(p) \rightarrow CD$



forthcoming as `cdplot()`

Outline of the talk

A recap on spatial models

Spatial econometric models have either a spatially lagged dependent variable or error (or both, or worse. . .)

The two standard specifications:

- Spatial Lag (SAR): $y = \psi W_1 y + X\beta + \epsilon$
- Spatial Error (SEM): $y = X\beta + u; u = \lambda W_2 u + \epsilon$

The general model (Anselin 1988):

$$y = \psi W_1 y + X\beta + u; u = \lambda W_2 u + \epsilon; E[\epsilon\epsilon'] = \Omega$$

Hence, if $A = I - \psi W_1$ and $B = I - \lambda W_2$, the general log-likelihood is

$$logL = -\frac{N}{2} ln\pi - \frac{1}{2} ln|\Omega| + ln|A| + ln|B| - \frac{1}{2} e'e$$

The general estimation framework (Anselin 1988)

The likelihood is thus a function of β, ψ, λ and parameters in Ω. The overall errors' covariance can be scaled as $B'\Omega B = \sigma_e^2 \Sigma$. This likelihood can be concentrated w.r.t. β and σ_e^2 substituting $e = [\hat{\sigma}_e^2 \Sigma]^{-\frac{1}{2}}(Ay - X\hat{\beta})$

$$logL = -\frac{N}{2} ln\pi - \frac{N}{2} \hat{\sigma_e^2} - \frac{1}{2} ln|\Sigma| + ln|B| + ln|A| - \frac{1}{2\hat{\sigma_e^2}}(Ay - X\hat{\beta})'\Sigma^{-1}(Ay - X\hat{\beta})$$

and a closed-form GLS solution for β and σ_e^2 is available for any given set of spatial parameters ψ, λ and scaled covariance matrix Σ

$$\begin{aligned} \hat{\beta} &= (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Ay \\ \hat{\sigma_e^2} &= \frac{(Ay - X\hat{\beta})'\Sigma^{-1}(Ay - X\hat{\beta})}{N} \end{aligned} \tag{1}$$

so that a two-step procedure is possible which alternates optimization of the concentrated likelihood and GLS estimation.

Operationalizing the general estimation method

The general estimation method can be made operational for specific Σs parameterized as $\Sigma(\theta)$ by plugging in the relevant Σ, Σ^{-1} and $|\Sigma|$ into the log-likelihood and then optimizing by a two-step procedure, alternating:

$$GLS : \beta = (X'[\Sigma(\hat{\theta})^{-1}]X)^{-1}X'[\Sigma(\hat{\theta})^{-1}]Ay \rightarrow \hat{\beta}$$

$$ML : max ll(\theta|\hat{\beta}) \rightarrow \hat{\theta}$$

until convergence

The computational problem: $\Sigma = \Sigma(\theta, \lambda)$ *and* $A = A(\psi)$ *so all inverses and determinants are to be recomputed at every optimization loop*

Anselin (ibid.) gives efficient procedures for estimating the "simple" cross-sectional SAR and SEM specifications: see package `spdep` by Roger Bivand for very fast R versions. There are few software implementations for more general models (notably, Matlab routines by Elhorst (IRSR 2003) for FE/RE SAR/SEM panels).

A slightly less general (panel) model

In this general framework, the availability of estimators is limited by that of computationally tractable (inverses and determinants of-) error covariances.

Let us consider a panel model within a more specific, yet quite general setting, allowing for a spatially lagged response and the following features of the composite error term (i.e., parameters describing Σ):

- random effects ($\phi = \sigma^2_\mu / \sigma^2_\epsilon$)
- spatial correlation in the idiosyncratic error term (λ)
- serial correlation in the idiosyncratic error term (ρ)

$$
\begin{aligned}
y &= \psi(I_T \otimes W_1)y + X\beta + u \\
u &= (\imath_T \otimes \mu) + \epsilon \\
\epsilon &= \lambda(I_T \otimes W_2)\epsilon + \nu \\
\nu_t &= \rho \nu_{t-1} + e_t
\end{aligned}
$$

Available models and performance

Lag and error models can be mixed up, giving rise to the following possibilities:

par $\neq 0$	$\mu\lambda\rho$	$\mu\lambda$	$\mu\rho$	$\lambda\rho$	λ	ρ	μ	(none)
ψ	SAREMSRRE	SAREMRE	SARSRRE	SAREMSR	SAREM	SARSR	SARRE	**SAR**
(none)	SEMSRRE	SEMRE	*SRRE*	SEMSR	**SEM**	*SR*	*RE*	*OLS*

where SARRE, SEMRE are the 'usual' random effects spatial panels and SAR, SEM the standard spatial models (here, pooling with $W = I_T \otimes w$)

My very naive, modular and high-level implementation of the estimation theory looks like working! (thanks to the power of R and many simplifications taken from Baltagi, Song, Jung and Koh, 2007). Computing times on Munnell's (1990) data (48 US states over 17 years) are 43" for the SAREMRE and 160" for the full SAREMSRRE model. Furter optimizaton for speed is on the agenda.

Baltagi et al.'s LM testing framework

Most applications concentrate on the error model. In this setting, Baltagi et al. (2007) derive conditional LM tests for

- $\lambda|\rho, \mu$ (needs SRRE estimates of $\hat{u}$)
- $\rho|\lambda, \mu$ (needs SEMRE estimates of $\hat{u}$)
- $\mu|\lambda, \rho$ (needs SEMSR estimates of $\hat{u}$)

So a viable and computationally parsimonious strategy for the error model can well be to test in the three directions by means of conditional LM tests and see whether one can estimate a simpler model than the general one.

An asymptotically equivalent test, much heavier on the machine, is the Wald test implicit in the diagnostics of the general model. The lag specification can be tested for only the second way (the covariance is based on the numerical estimate of the Hessian).

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What's next?

The CD, LM and SCLM tests are already in plm-1.0.0 currently on CRAN. Expect

- the Friedman and Frees tests and the XS robust `pvcov()` functions in the next release
- the spatial ML estimators and tests in a separate package based on `plm` and `spdep`, to come on CRAN in the next months

(but you can get betas from me if you are interested: just email me at `giovanni_millo@generali.com`)

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Thanks

In alphabetical order,

- Roger Bivand
- Yves Croissant
- Achim Zeileis
- ...
- ...and you, for your attention