Computing Weighted χ^2 Distributions and Related Quantities

Christine Choirat, Raffaello Seri*

It is well known that the asymptotic distribution of degenerate $U-$ and $V-$statistics is, in general, an (infinite) weighted sum of χ^2 random variables. The behavior of the statistic in terms of asymptotic distribution and power is strictly linked to the eigenvalues and the eigenfunctions of an integral operator. We provide an algorithm for the numerical approximation of these quantities, and of the cdf of a weighted sum of χ^2 random variables. The algorithm can be used to approximate (as precisely as needed) the power of the test statistics, and to build several measures of performance for tests based on $U-$ and $V-$statistics. The algorithm uses the Wielandt-Nyström method for the approximation of the solution of integral operators.

The algorithm previously exposed has been implemented in an R package. The computation of the eigenvalues and of the eigenfunctions can be performed using a Monte-Carlo method, a quasi-Monte Carlo method based on the Halton sequence or on the Hammersley point set, the trapezium rule, the Gauss-Legendre quadrature rule and the Clenshaw-Curtis quadrature rule. The approximation of the cdf uses a routine recently written by Robert B. Davies in C (indeed, a new version, available from the Internet at `http://www.robertnz.net/ftp/qf.tar.gz` or `qf.zip`, of the 1980 program that was originally written in Algol). The performance of the method can be analyzed through a Berry-Esséen bound. On the basis of extensive experimentation, we advocate the use of the Wielandt-Nyström method based on the Clenshaw-Curtis quadrature rule.

*Università degli Studi dell'Insubria, Dipartimento di Economia, via Monte Generoso 71, 21100 Varese, Italy. E-mails: `{cchoirat, rseri}@eco.uninsubria.it`. Homepages: `http://eco.uninsubria.it/webdocenti/{cchoirat, rseri}`.