Using R for small area estimation in the Norwegian National Forest Inventory

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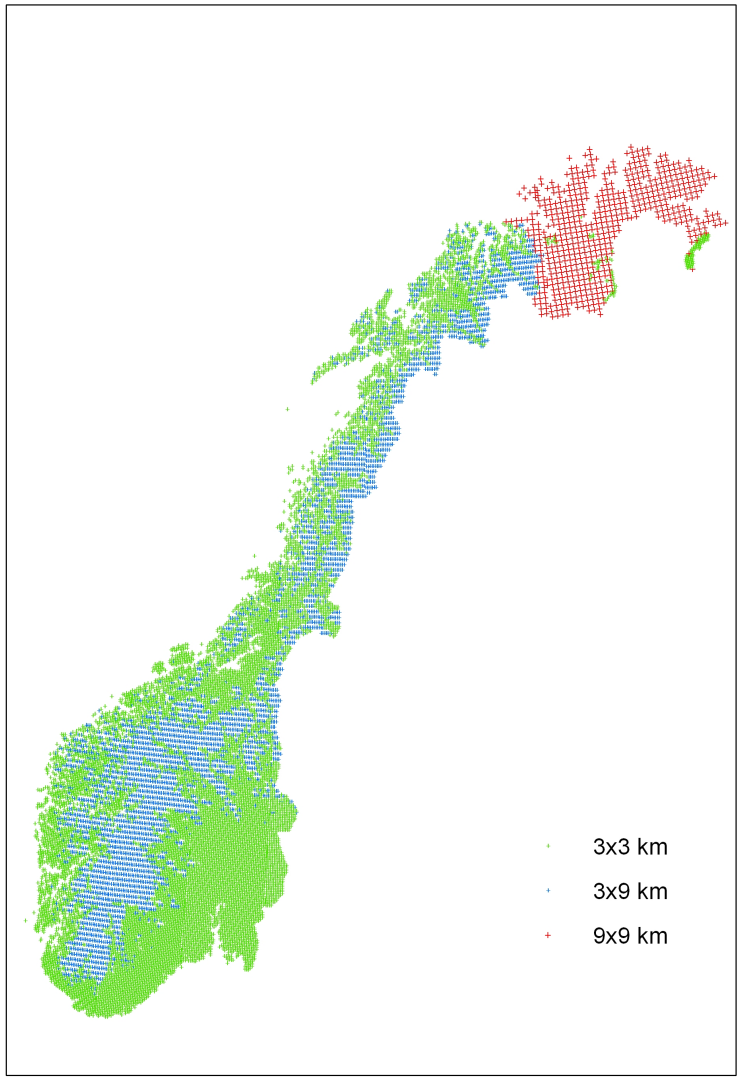
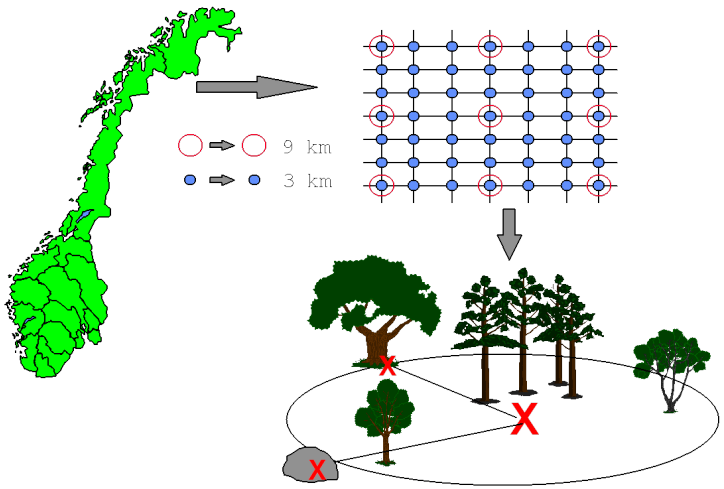
The Norwegian National Forest Inventory

- Since 1919
- >22.000 sample plots
- 1/5th of plots measured every year



The Norwegian National Forest Inventory

- Circular sample plots, 250 m^2
- Species, diameter, location, height
- Ecological parameters, harvests
- Timber volume, biomass
- Ca 150 different parameters



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Why NFI?

- Political decision processes
- National estimates of forest parameters
- Reporting requirements (e.g. Kyoto protocol, FAO)
- Strategic planning

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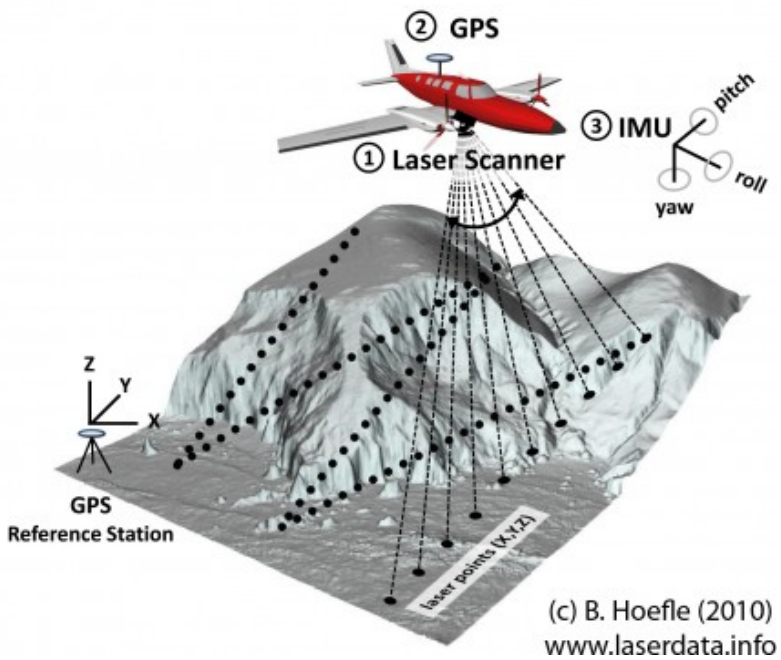
Small area estimation

- Too few NFI plots for a reliable estimate
- Estimates on stand level $\rightarrow$ operational forest management decisions
- Combination of field plots with RS
- Design-based $\rightarrow$ model-based



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Aim

- Overview of synthetic estimators
- Empirical coverage of CIs
- Implementation: JoSAE package

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Regression synthetic estimator – mean

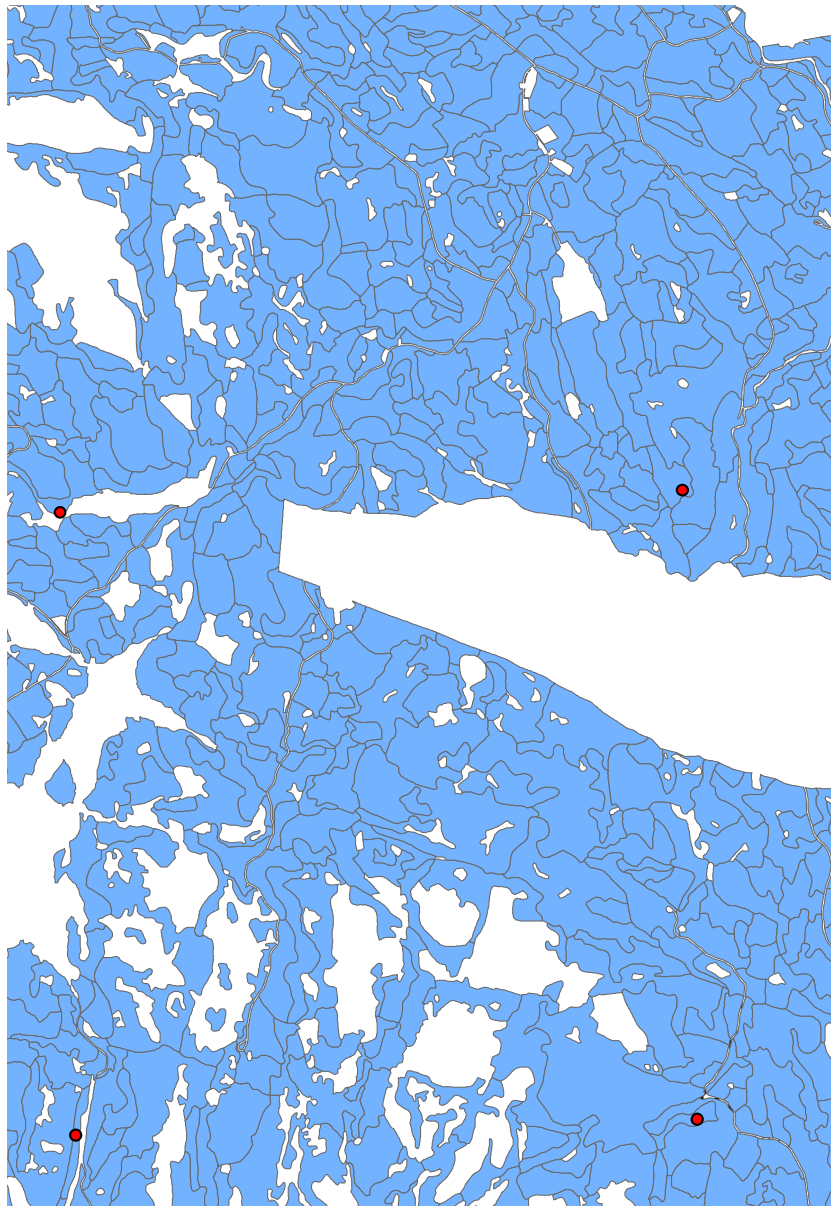
- Systematic sample
- Linking model

$$\boldsymbol{y} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim N(\mathbf{0}, \sigma_\varepsilon^2)$$

- Mean of the model predictions for the small area m

$$\hat{\bar{Y}}_m = \frac{1}{N_m} \sum \mathbf{x}_m^T \hat{\beta} = \bar{\mathbf{x}}_m^T \hat{\beta}$$

- Biased – but useful



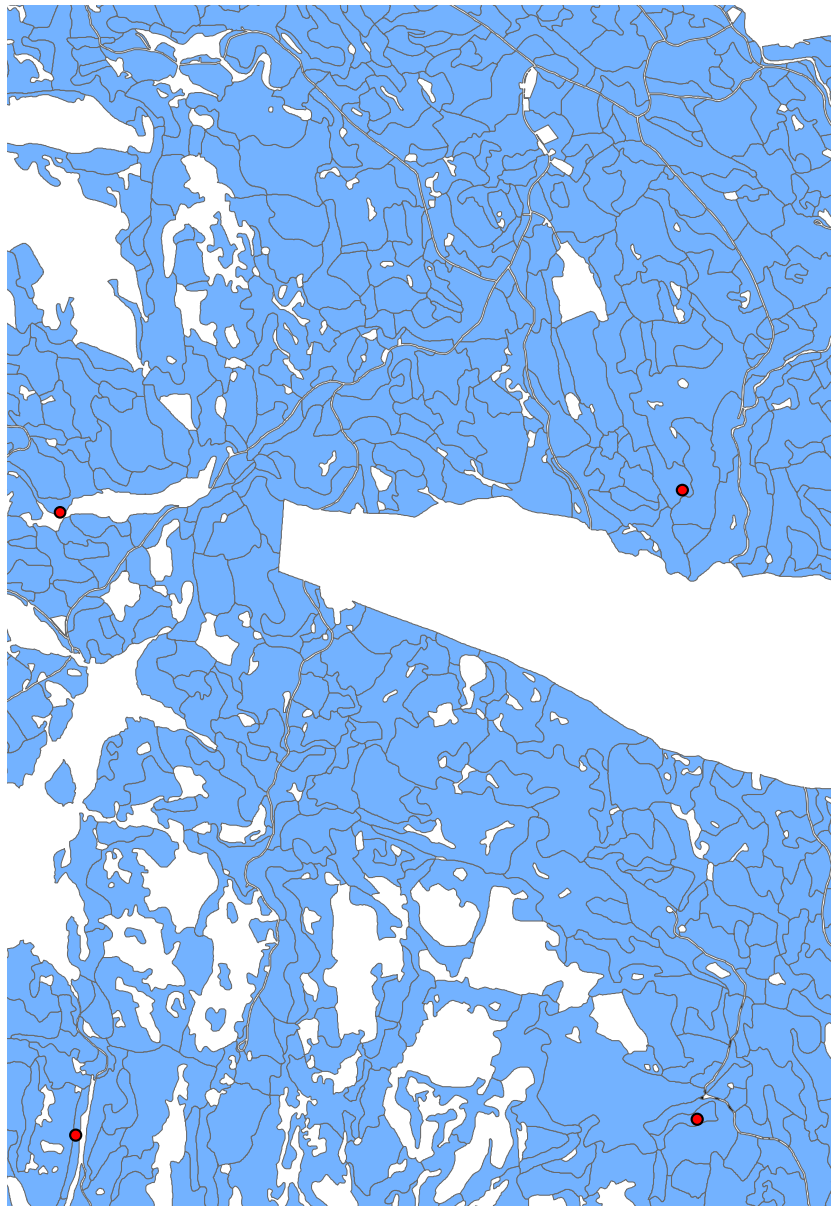
Regression synthetic estimator – Var I

"...easily obtained..."

Est. of superpopulation parameters

$$\widehat{Var}_S(\hat{\bar{Y}}_m) = \bar{\mathbf{X}}_m^T \widehat{Cov}(\hat{\beta}) \bar{\mathbf{X}}_m$$

- Independent of stand size
- Residual error ignored since $\varepsilon \sim N(\mathbf{0}, \sigma_\varepsilon^2)$



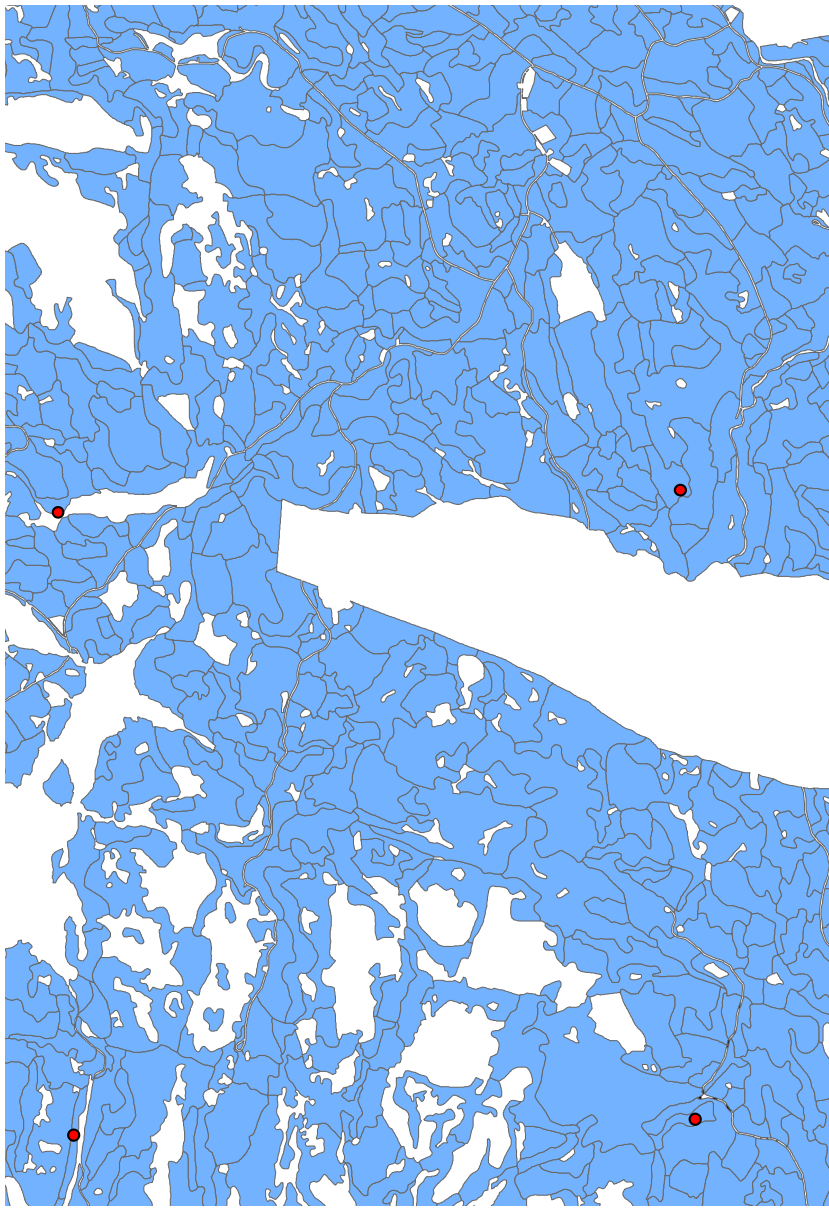
Regression synthetic estimator – Var II

For small stands

Pred. finite population elements

Including heteroskedasticity

$$\widehat{Var}_{SH}(\hat{\bar{Y}}_m) = \widehat{Var}_S(\hat{\bar{Y}}_m) + \frac{1}{N_m^2} \sum_i^{N_m} \hat{\sigma}_i^2$$

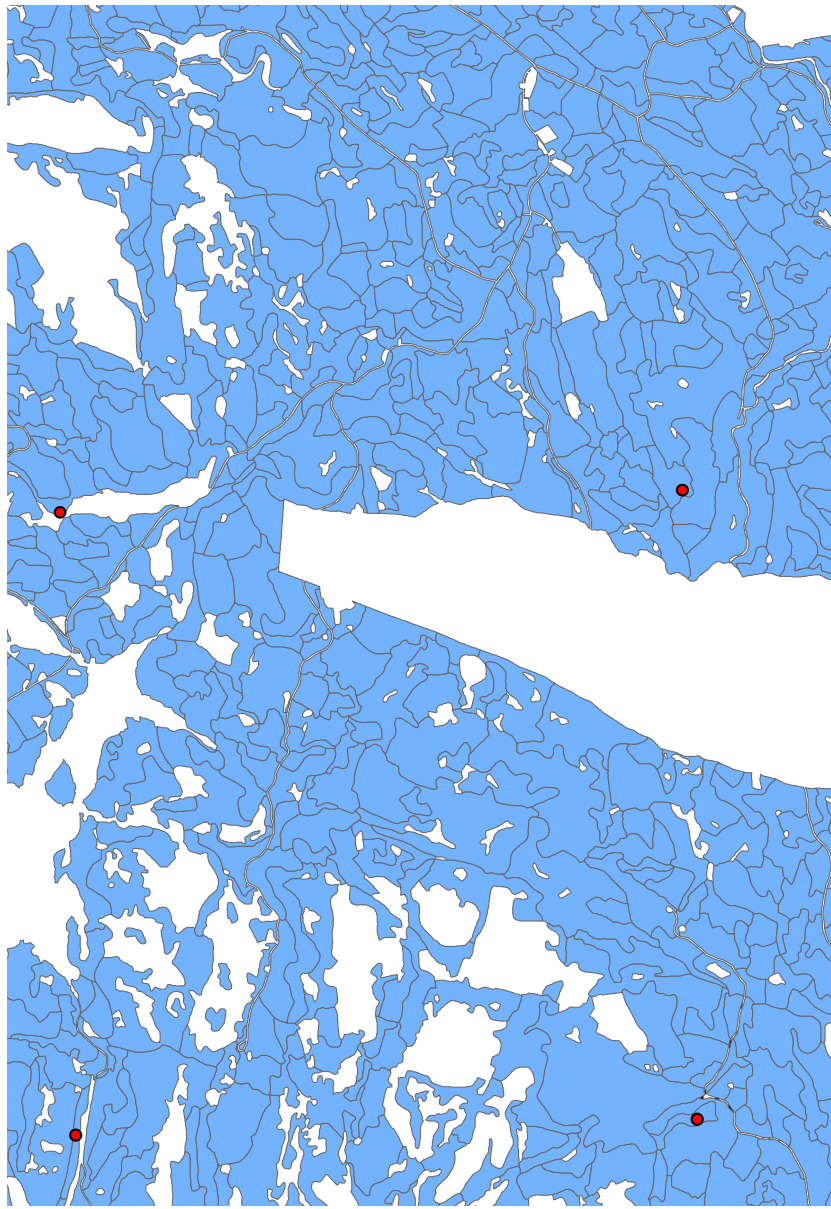


Regression synthetic estimator – Var III

Including spatial autocorrelation

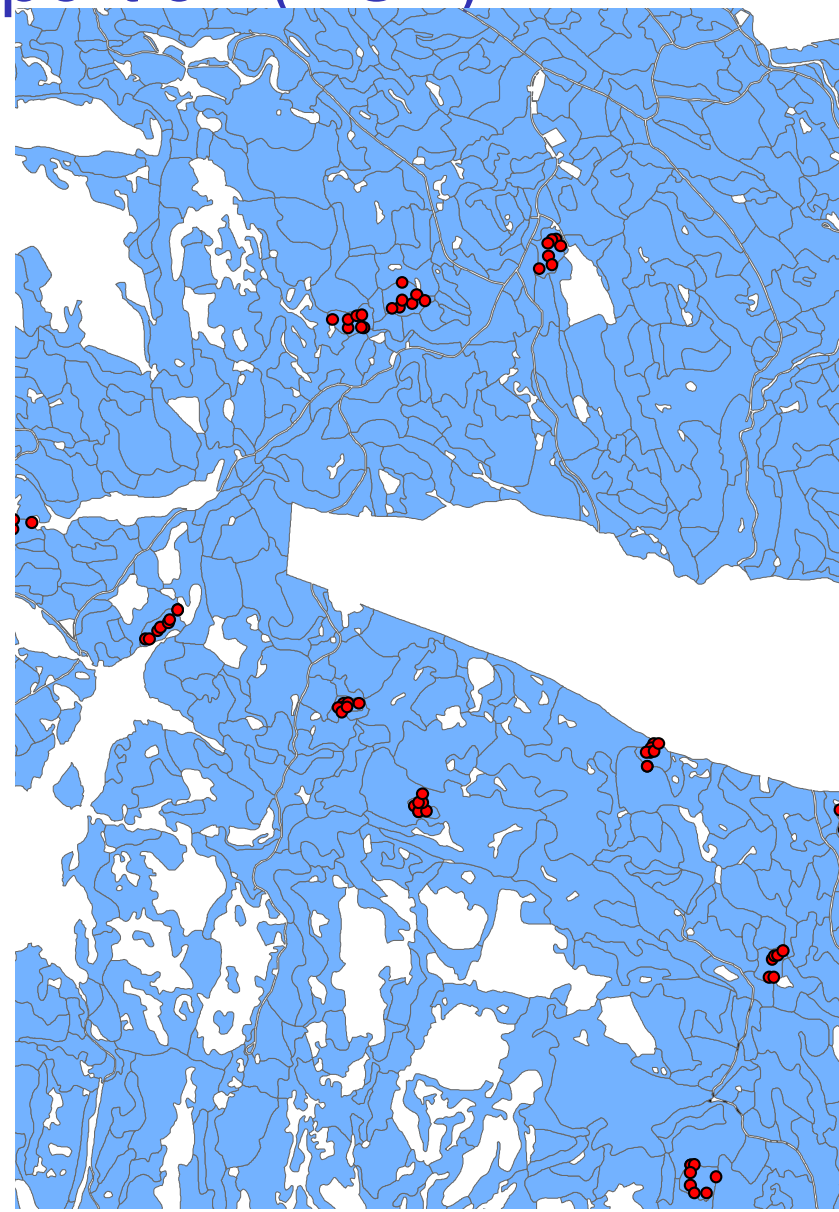
$$\widehat{Var}_{SSS}(\hat{\bar{Y}}_m) = \widehat{Var}_S(\hat{\bar{Y}}_m) + \frac{1}{N_m^2} \sum_{i}^{N_m} \sum_{j}^{N_m} \hat{\sigma}_i \hat{\sigma}_j \hat{\varrho}_{ij}$$

- Requires an estimate of spatial autocorrelation



Empirical coverage proportion (ECP)

- Sample plots clustered in stands
- Info on spatial autocorrelation



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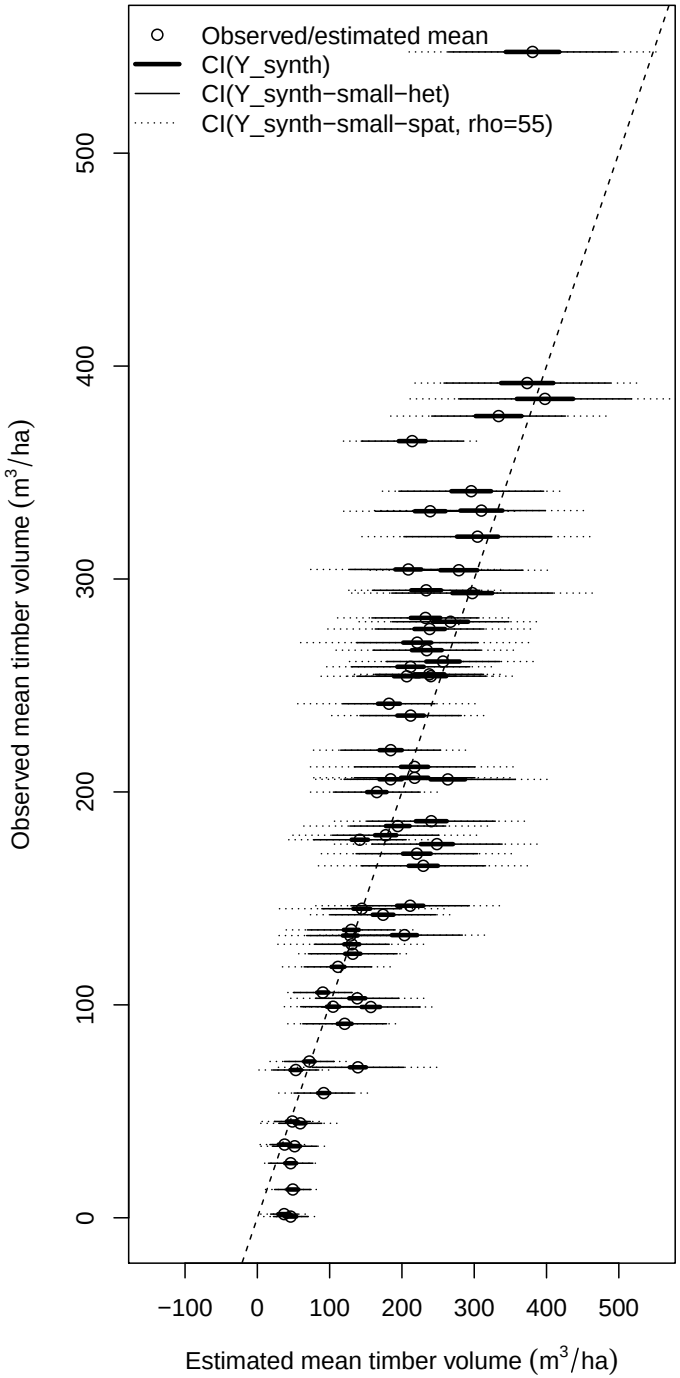
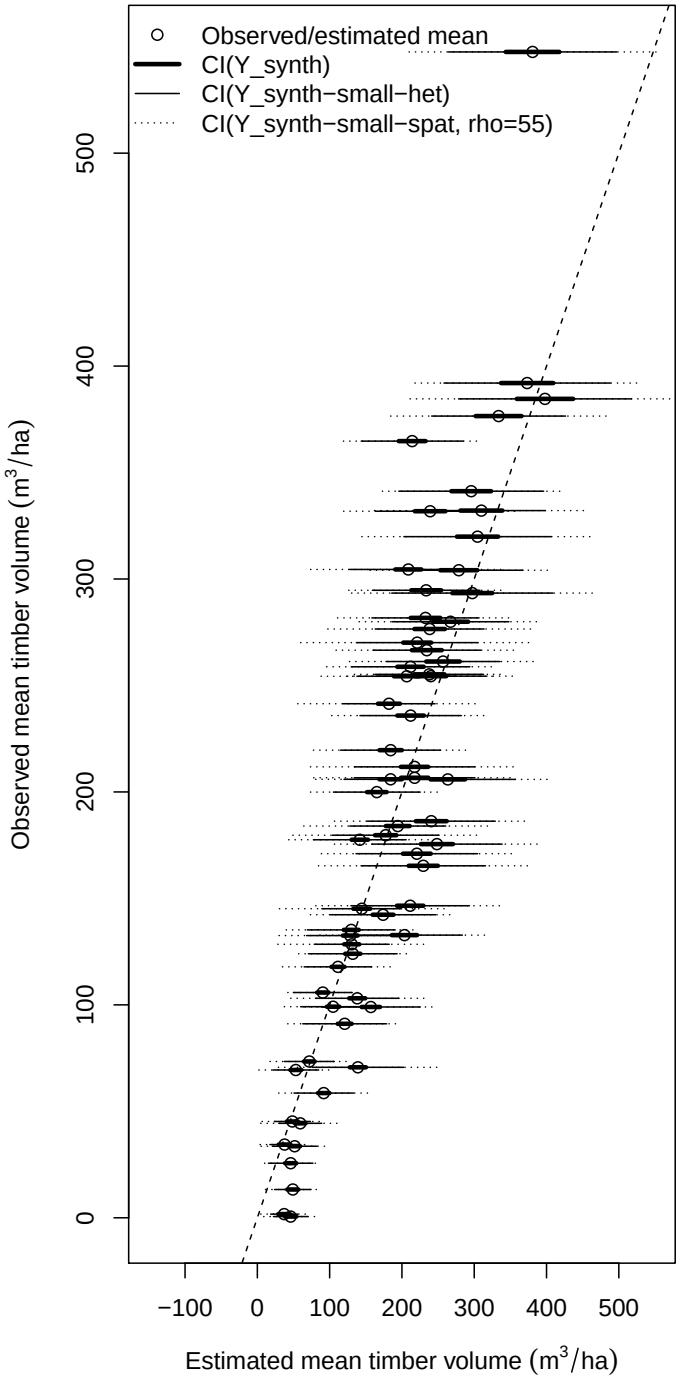

Observed/estimated mean
CI(Y_synth)
CI(Y_synth−small−het)
CI(Y_synth−small−spat, rho=55)
Observed mean timber volume (m3/ha)
Estimated mean timber volume (m3/ha)
0
100
200
300
400
500
−100
0
100
200
300
400
500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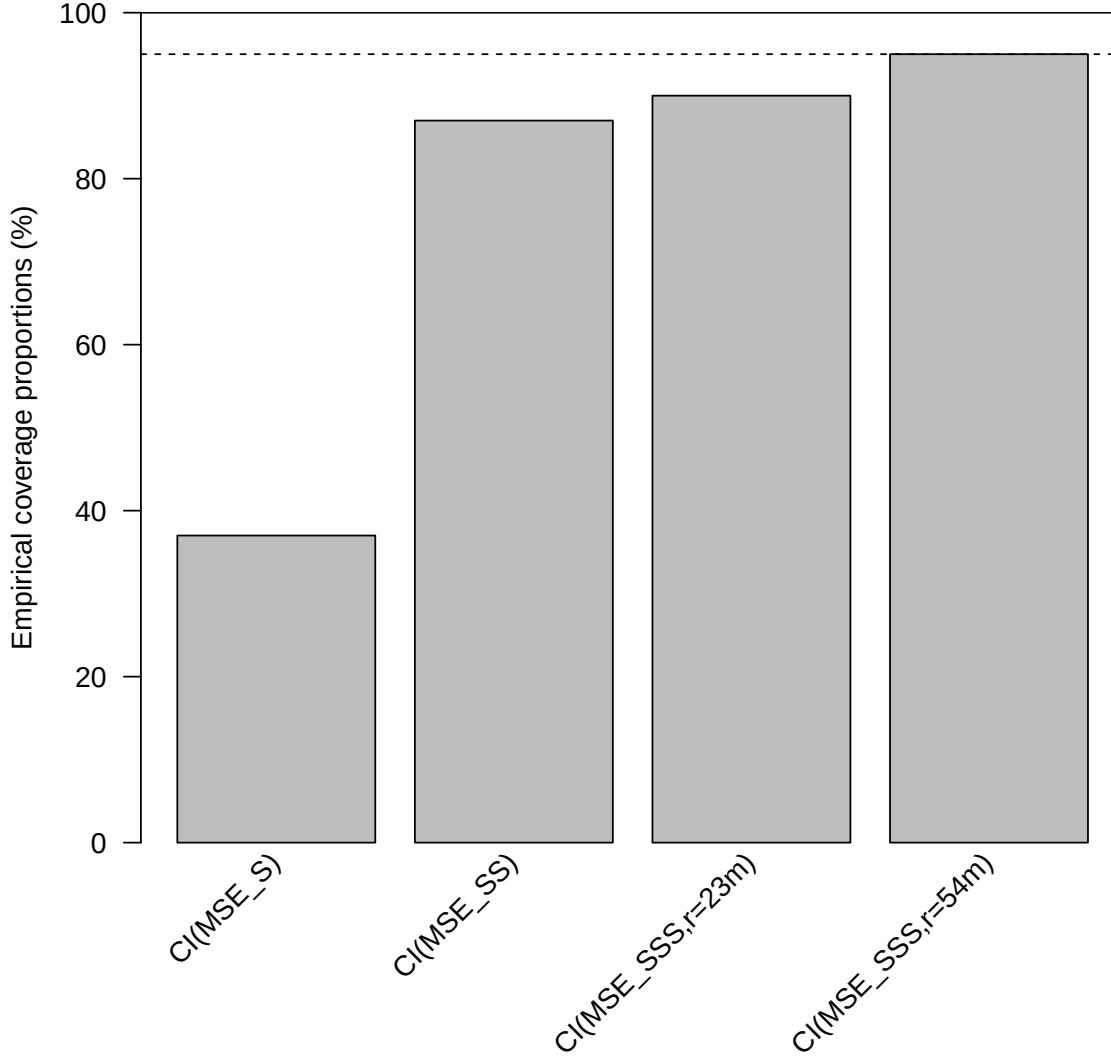
Empirical coverage proportions (%)
0
20
40
60
80
100
CI(MSE_S)
CI(MSE_SS)
CI(MSE_SSS,r=23m)
CI(MSE_SSS,r=54m)

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Summary

$$\widehat{Var}_S(\hat{\bar{Y}}_m) = \bar{\mathbf{X}}_m^T \widehat{Cov}(\hat{\beta}) \bar{\mathbf{X}}_m$$

$$\widehat{Var}_{SS}(\hat{\bar{Y}}_m) = \widehat{Var}_S(\hat{\bar{Y}}_m) + \frac{\hat{\sigma}_\varepsilon^2}{N_m}$$

$$\widehat{Var}_{SSS}(\hat{\bar{Y}}_m) = \widehat{Var}_S(\hat{\bar{Y}}_m) + \frac{1}{N_m^2} \sum_i^{N_m} \sum_j^{N_m} \hat{\sigma}_i \hat{\sigma}_j \hat{\varrho}_{ij}$$

- Random effects on stand level

$\Rightarrow$ Careful interpretation!

- R package JoSAE will be updated (paper accepted), other SAE packages available!

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Thank you for listening!

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