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Scienze della Comunicazione
e dell'Economia

The BayHaz package for Bayesian estimation of smooth hazard rates in R

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Smooth hazard rate estimation

CPP and BPS priors

Prior elicitation

Posterior computation

Directions for future work

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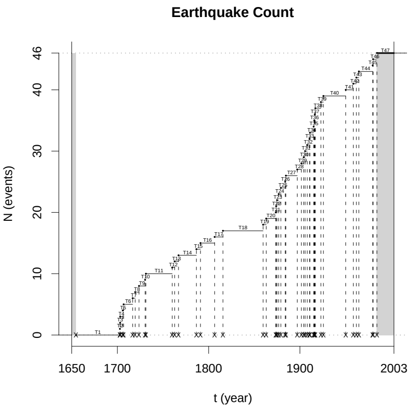
Suppose we observe either $\{T_i = t_i\}$ or $\{T_i > t_i\}$ for $i = 1, \ldots, n$, where

$$T_1, \ldots, T_n | \rho \overset{i.i.d.}{\sim} \rho(t) \exp\left\{-\int_0^t \rho(s) ds\right\} dt$$

are survival times with unknown (non-defective) hazard rate ρ, that is,

$$\rho \geq 0, \qquad \exists t > 0 : \int_0^t \rho(s) ds < \infty, \qquad \int_0^\infty \rho(s) ds = \infty.$$

We want to learn the shape of ρ from data (non-parametric approach) but we know that ρ is smooth.



Events with moment magnitude greater than 5.1 in a very active Italian seismogenic zone...

...the inter-event times can be considered exchangeable; they are available as a data set of `BayHaz` [La Rocca, 2007]:

```
library(BayHaz)
data(earthquakes)
```

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A compound Poisson process (CPP) prior hazard rate [La Rocca, 2008] is defined by

$$\rho(t) \quad = \quad \xi_0 k_0(t) + \sum_{j=1}^{\infty} \xi_j k(t - \sigma_j), \qquad t \geq 0,$$

where $\sigma_j, j \geq 1$, are the jump-times of a CPP process with gamma distributed jump-sizes $\xi_j, j \geq 1$, while k is a zero-mean Gaussian density (kernel), ξ_0 is an independent random variable with the same distribution as any jump-time ξ_j, and k_0 is a suitable function such that the mean of $\rho(t)$ does not depend on t.

A first-order autoregressive Bayesian penalized spline (BPS) prior hazard rate, based on [Hennerfeind *et al.*, 2006], is defined by

$$\rho(t) = \exp\left\{\sum_{j=1}^{G+k-2} \eta_j B_j(t)\right\}, \quad 0 \le t \le T_\infty,$$

where η is a normal first order autoregressive stationary process, while $B_j(t)$ is the j-th B-spline basis function of order k, evaluated at t, defined on a grid of $G + 2k - 2$ equispaced knots with first internal knot at 0 and last internal knot at T_∞ (time-horizon of interest); there are G internal nodes, and B-spline basis functions sum to one within them.

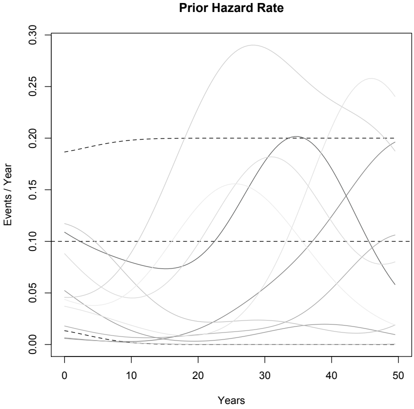
For CPP priors, a time-scale equivariant elicitation procedure is available to assign a constant prior expected hazard rate while controlling prior variability, based on the following quantities:

- `r0` prior mean hazard rate (r_0);
- `H` corresponding (asymptotic) coefficient of variation;
- `T00` time-horizon of interest (T_∞);
- `M00` number of extremes within the time-horizon in a "typical" hazard rate trajectory (M_∞).

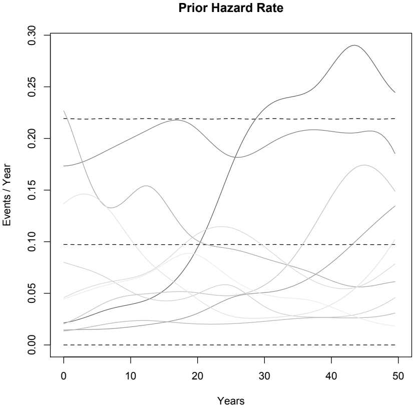
There is a technical issue (disregarded in these slides) concerning the number of CPP jumps needed to cover the time-horizon of interest.

A procedure to find a matching BPS prior is also available.

Prior elicitation




```
hypCPP <- CPPpriorElicit(r0 = 0.1, H = 1,
                         T00 = 50, M00 = 2)
priorCPP <- CPPpriorSample(ss = 10,
                           hyp = hypCPP)
CPPplotHR(priorCPP, tu = "Year")
```




```
hypBPS <- BPSpriorElicit(r0 = 0.1, H = 1,
                         T00 = 50, G = 9)
priorBPS <- BPSpriorSample(ss = 10,
                           hyp = hypBPS)
BPSplotHR(priorBPS, tu = "Year")
```

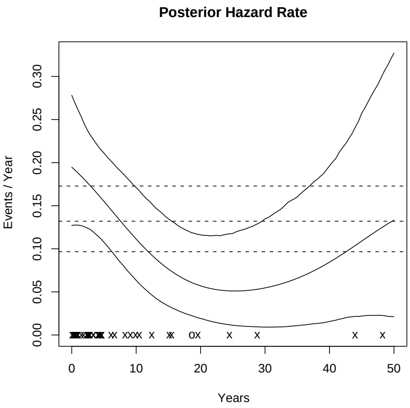
Markov chain Monte Carlo (MCMC) posterior approximation:

- Gibbs-type sampler for CPP posteriors, introducing a latent label per exact observation ⇒ hazard-driven probabilistic clustering;
- tailored proposal density Metropolis-Hastings sampler for BPS posteriors;

```
# CPP posterior sample (about three quarters of an hour on my MacBook2,1)
postCPP <- CPPpostSample(hypCPP, times = earthquakes$ti, obs = earthquakes$ob,
                         mclen = 10000, burnin = 50000, thin = 20, lab = FALSE)
# BPS posterior sample (about one fourth of the time on the same machine)
postBPS <- BPSpostSample(hypBPS, times = earthquakes$ti, obs = earthquakes$ob,
                         mclen = 10000)
```

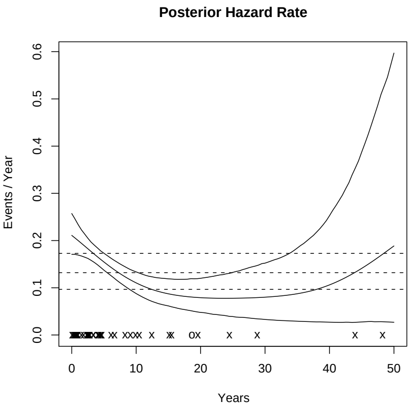
Interface to package `coda` [Plummer et al., 2007] for output diagnostics:

```
MCMCpostCPP <- CPPpost2mcmc(postCPP) # package 'coda' is automatically loaded
MCMCpostBPS <- BPSpost2mcmc(postBPS) # and an MCMC object is created
```



Pointwise posterior mean and equal tail 95% credible band: solid lines refer to the CPP posterior; dashed lines refer to the posterior obtained by means of a constant hazard rate model (using a conjugate gamma prior and letting its shape and rate parameters tend to zero). Exact observations are marked with "x", whereas censored observations are marked with "o".

```
CPPplotHR(postCPP, tu = "Year")
```



Pointwise posterior mean and equal tail 95% credible band: solid lines refer to the BPS posterior; dashed lines refer to the posterior obtained by means of a constant hazard rate model (using a conjugate gamma prior and letting its shape and rate parameters tend to zero). Exact observations are marked with "x", whereas censored observations are marked with "o".

```
BPSplotHR(postBPS, tu = "Year")
```

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Interesting directions for future work include:

- dealing with semiparametric models, e.g., using CPP priors at least for the single binary covariate proportional hazards model [LaRocca, 2004];
- implementing other prior hazard rates;
- revising `R` code and documentation, possibling using `C` code for posterior sampling.

Needless to say, suggestions are welcome... thank you!

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Geoadditive survival models.
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