Estimation of Theoretically Consistent Stochastic Frontier Functions in R

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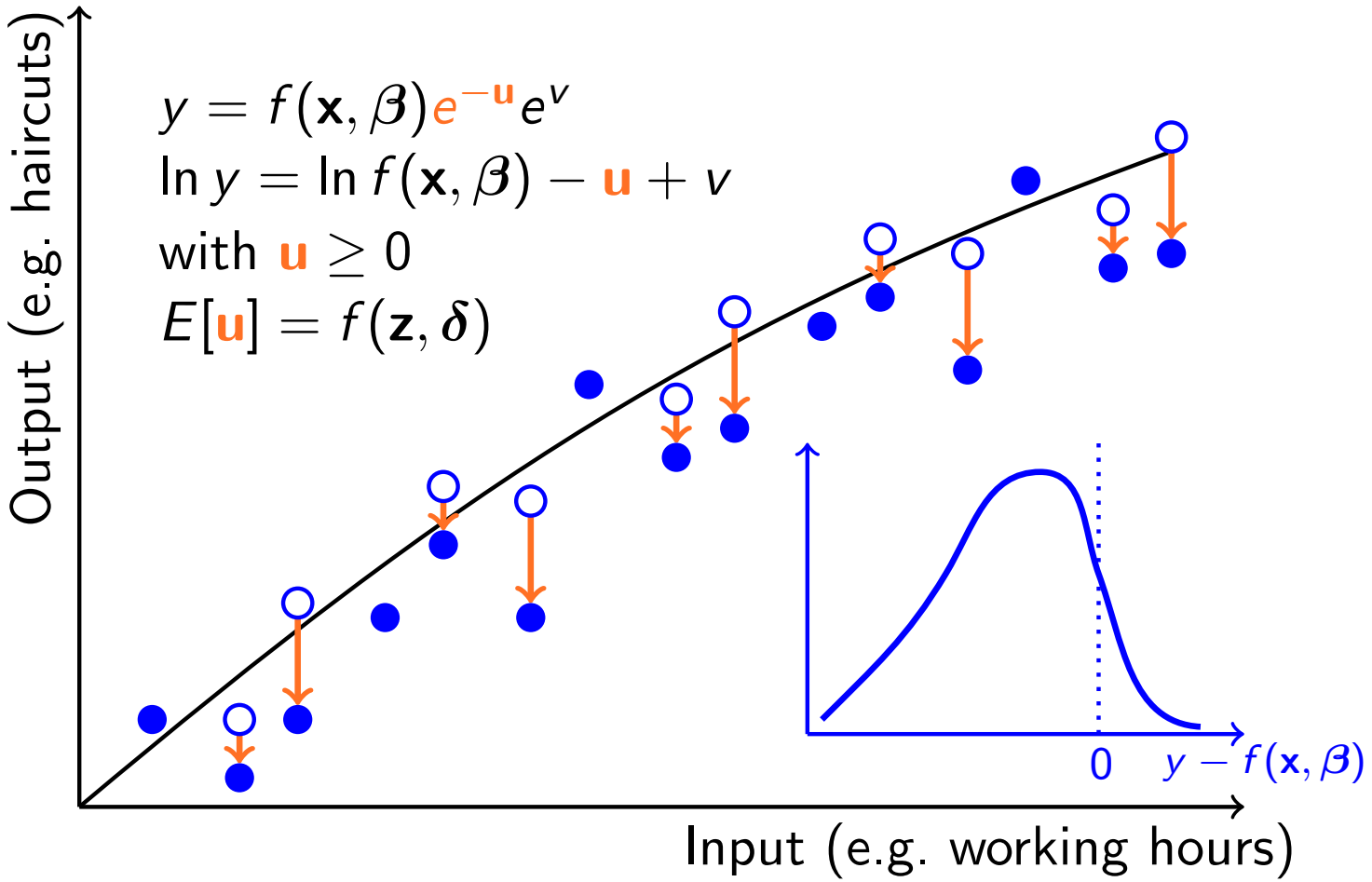
Outline

- Stochastic Frontier Analysis
- Theoretical Consistency
- Restricted Estimation of Frontier Functions
- (Empirical Example)
- Summary and Outlook

Stochastic Frontier Analysis

- Production economics
- Assumption of traditional empirical analyses:
  all producers always manage to optimize their production process
  - $\Rightarrow$ All departures from the optimum
    $=$ random statistical noise
  - $\Rightarrow$ $y = f(\mathbf{x}, \beta) + v$, e.g. with $v \sim N(0, \sigma^2)$
- Practice: producers do not always succeed in optimizing their production
- Stochastic Frontier Analysis (SFA) accounts for failures in optimization (Meeusen & van den Broeck 1977; Aigner, Lovell & Schmidt 1977)

Stochastic Frontier Analysis



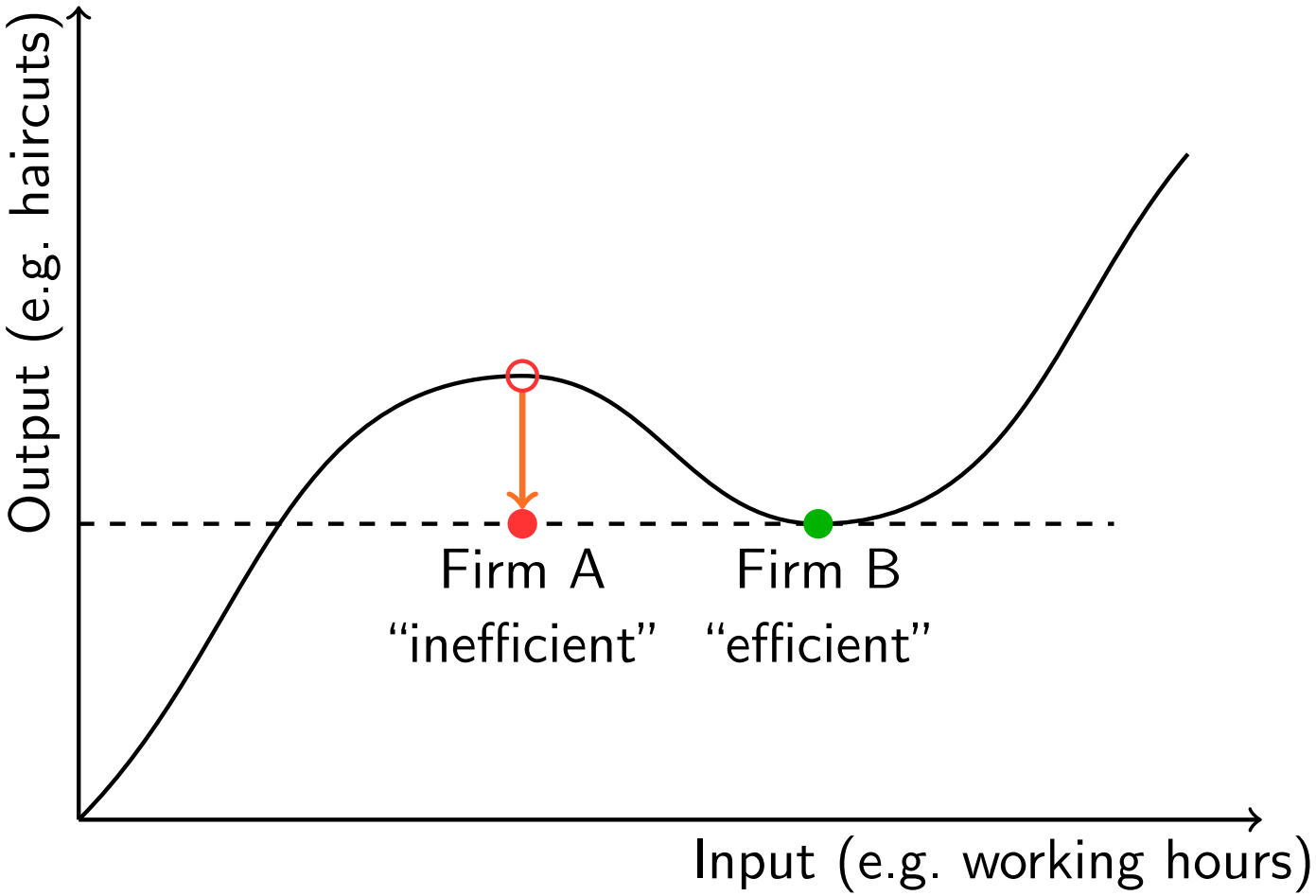
Software for Stochastic Frontier Analysis

- LIMDEP
- STATA
- FRONTIER (Version 4.1)
  - ⇒ Tim Coelli (CEPA, Univ. of Queensland, Brisbane)
  - ⇒ freely available for download (including FORTRAN source)
  - ⇒ but not really free (no license specified)
  - ⇒ command line interface / "instruction file"
  - ⇒ THE software for SFA for a long time
  - ⇒ development stopped in 1996
  - ⇒ LIMDEP and STATA have more features today, but FRONTIER is still widely used

Theoretical Consistency

- Microeconomic theory requires several properties of a production function $y = f(\mathbf{x}, \boldsymbol{\beta})$
- Most important: "monotonicity"
  - $\Rightarrow$ $f(.)$ monotonically increasing in inputs
  - $\Rightarrow$ all marginal products $\partial f / \partial x_i$ are non-negative
- Monotonicity even more important in Stochastic Frontier Analysis (SFA)

Non-monotone Production Frontier



Restricted Estimation of Frontier Functions

- Not available in standard software packages
- Econometric approaches for restricted estimations
  - $\Rightarrow$ ML estimation with restrictions imposed at the sample mean (e.g. Bokusheva and Hockmann: *Production Risk and Technical Inefficiency in Russian Agriculture*, ERAE, 2006)
  - $\Rightarrow$ MCMC estimation with restrictions imposed at all data points (O'Donnell & Coelli: *A Bayesian Approach to Imposing Curvature on Distance Functions*, JE, 2005)
  - $\Rightarrow$ Three-Step Estimation with monotonicity imposed at all data points (Henningsen & Henning: *Estimation of Theoretically Consistent Stochastic Frontier Functions with a Simple Three-Step Procedure*, unpublished, 2008)

Three-Step Estimation

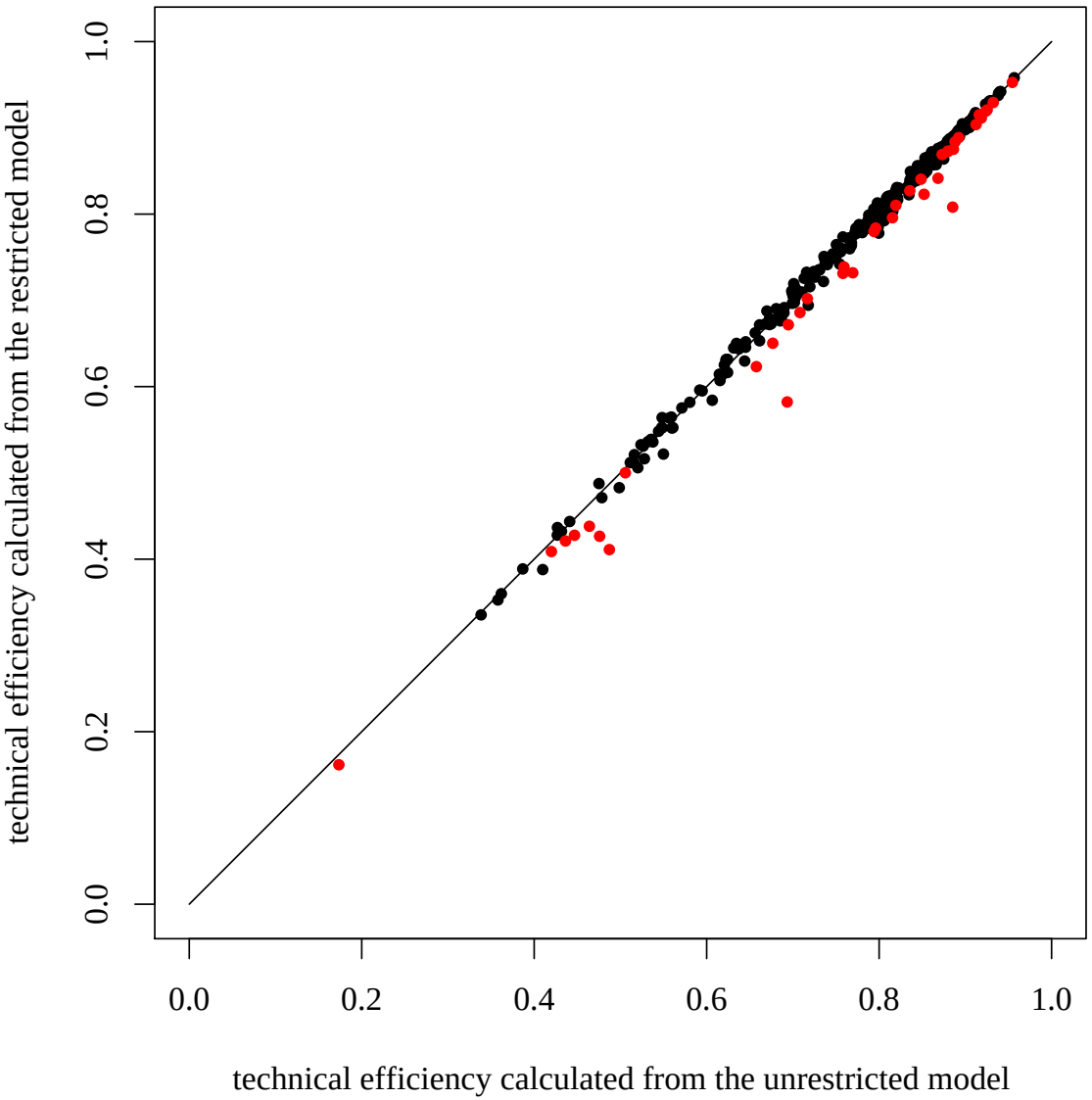
- based on Koebel, Falk & Laisney: *Imposing and Testing Curvature Conditions on a Box-Cox Cost Function*, JBES, 2003

1. Unrestricted frontier estimation (FRONTIER, R:micEcon)
   $\ln y = \ln f(\mathbf{x}, \beta) - u + v,\ E[u] = \mathbf{z}'\delta$
   $\Rightarrow$ unrestricted parameters $\hat{\beta}$, their covariance matrix $\hat{\Sigma}_\beta$
2. Minimum distance estimation (R:constrOptim|solve.QP|optim)
   $\hat{\beta}^0 = \operatorname{argmin} \left[\left(\hat{\beta}^0 - \hat{\beta} \right) \hat{\Sigma}_\beta^{-1} \left(\hat{\beta}^0 - \hat{\beta} \right) \right]$ |nlm|Rdonlp2)
   s.t. $f(\mathbf{x}, \hat{\beta}^0)$ satisfies theoretical conditions
   $\Rightarrow$ restricted param. $\hat{\beta}^0$, "frontier" output $y^{\max} = f(\mathbf{x}, \hat{\beta}^0)$
3. Final frontier estimation (FRONTIER, R:micEcon)
   $\ln y = \alpha_0 + \alpha_1 \ln y^{\max} - u + v,\ E[u] = \mathbf{z}'\delta^0$
   $\Rightarrow$ $y^{\max} = \hat{\alpha}_0 f(\mathbf{x}, \hat{\beta}^0)^{\hat{\alpha}_1}$, $E[e^{-u}]$, $\hat{\delta}^0$

Empirical Example

- rice production in the Philippines
- translog production function
- 1 output (rice), 3 inputs (labour, land, fertiliser)
- 2 variables explaining efficiency (education, upland fields)
- 43 rice producers, 8 years
- unrestricted frontier estimation
  - $\Rightarrow$ monotonicity violated at 39 observation
  - $\Rightarrow$ quasiconcavity violated at 4 observation
- minimum distance estimation
  - $\Rightarrow$ monotonicity and quasiconcavity fulfilled at all observation
- second frontier estimation
  - $\Rightarrow$ virtually no adjustment: $\alpha_0 = 0.0005$, $\alpha_1 = 0.9999$
  - $\Rightarrow$ efficiency estimates ...

Efficiency Estimates



correlation:
Pearson: 0.996
Spearman: 0.995
Kendall: 0.954

Summary and Outlook

Summary

- SFA is an important tool in production/firm analysis
- Theoretical consistency is important especially for frontier functions.
- Imposing restrictions by a three-step estimation procedure
  - $\Rightarrow$ relatively simple compared to other restricted frontier estimations
  - $\Rightarrow$ can be done easily in R (using also FRONTIER)

Outlook

- Integrating FRONTIER into an R package
- Adding further functions for SFA (e.g. MCMC estimation)
- Coworkers and contributors are welcome!