RiDMC: an R package for the numerical analysis of dynamical systems

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Dynamical Systems

- Dynamical systems theory is an interdisciplinary field, with major contributions coming from mathematics and physics but also many other fields like population studies and meteorology
- A dynamical system is a mathematical model which formalizes the 'rules' describing the time dependence of a point's position in its ambient space
- The point symbolizes a state of the system, and is usually represented as a d-variate real vector
- Examples of dynamical systems include the description of the swinging of a clock pendulum, the flow of water in a pipe, the number of fish each spring in a lake, the daily rainfall in a city, etc.

RiDMC: the story

- iDMC (the interactive Dynamical Model Calculator) is a stand-alone Java application -with GUI- from which the C library `idmclib` originated as a spin-off (`http://idmc.googlecode.com`)
- `idmclib` is a standard-C library which relies on the LUA library for model code interpretation and on the Gnu Scientific Library (GSL) for computational tasks and random number generation. The `idmclib` is small, self-sufficient, and documented. License: GPL-v2 (`http://idmclib.googlecode.com`)
- RiDMC is a self-contained R package which internally uses the `idmclib` C library for core numerical analyses, and exploits R power for delivering a more complete, interactive and flexible environment to the final user for the numerical analysis of dynamical systems

RiDMC workflow

What is the typical workflow with RiDMC?

- write down the model in the LUA language, save it in a plain text file
- load the model as an R object
- perform analyses by using one or more model *methods*
- plot resulting objects

Writing models

- Models are specified in the interpreted LUA language
- The language is very easy to learn, and many models are already given as examples

Hénon map

$$\begin{cases} x_{t+1} & = a - x_t^2 + by_t \\ y_{t+1} & = x_t \end{cases}$$

```
name = `Henon`
type = `D`
parameters = {`a`, `b`}
variables = {`x`, `y`}
function f(a, b, x, y)
x1 = a - x^2 + b * y
y1 = x
return x1, y1
end
```

Analyzing a model

- Package design is object oriented, and all major analysis functions have been written as (S3) `Model` methods
- To date, the following methods are available:

function	**description**
Trajectory, TrajectoryList	Model trajectories
Basin, BasinMulti	Basins of attraction
Bifurcation	Bifurcation diagram
LyapunovExponents	Lyapunov exponents
cycles	Periodic Cycles

- Each method returns an object which can be directly plotted by the usual `plot` method

Trajectories

Trajectories

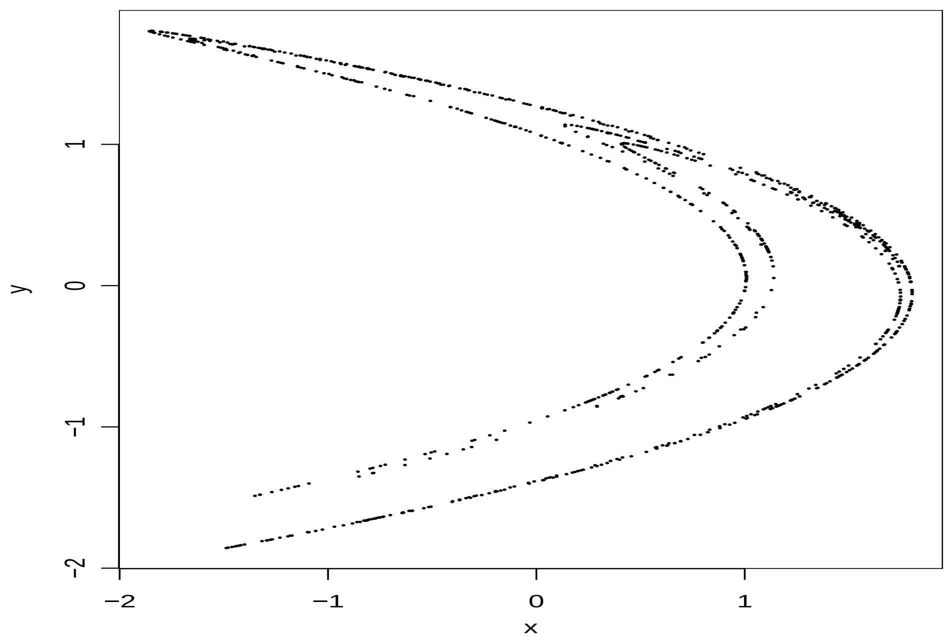
- A first, basic explorative analysis of a dynamical system involves the visual inspection of model trajectories
- Trajectories can be plotted vs time axis or represented in the system state space, where time dimension is lost, but other model features can be appreciated
- With `RiDMC` one can easily compute and plot trajectories for both discrete and continuous time dynamical systems

Trajectories (II)

```
> m <- Model('henon.lua')
> tr <- Trajectory(m, par, var, time, transient)
> tr
= iDMC model discrete trajectory =
model:  Henon
parameter values:  1.42 0.3
starting point:  0 0
transient length:  10000
time span:  1000
```

Trajectories (III)

```
plot(tr)
```



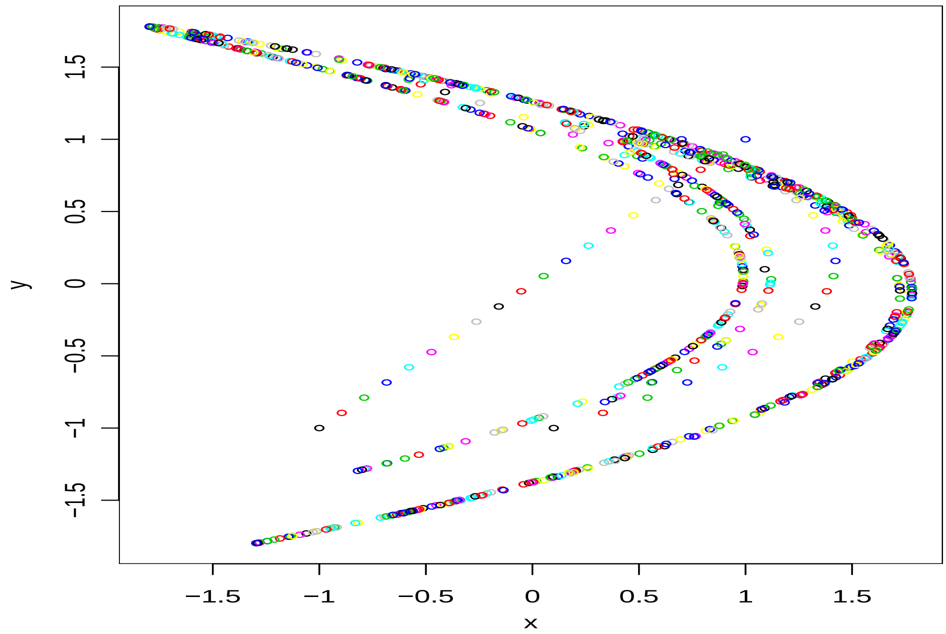
Attractors

Attractors

- A key aspect of a dynamical system is its limit behaviour, i.e. the system's state as time tends to infinity
- As we have already seen, this can be approximated by using the `Trajectory` method and exploiting the `transient` option
- Even more useful in this respect can be the `TrajectoryList` method, which shows multiple trajectories in the same plot, by allowing for variations in starting points and/or parameter values

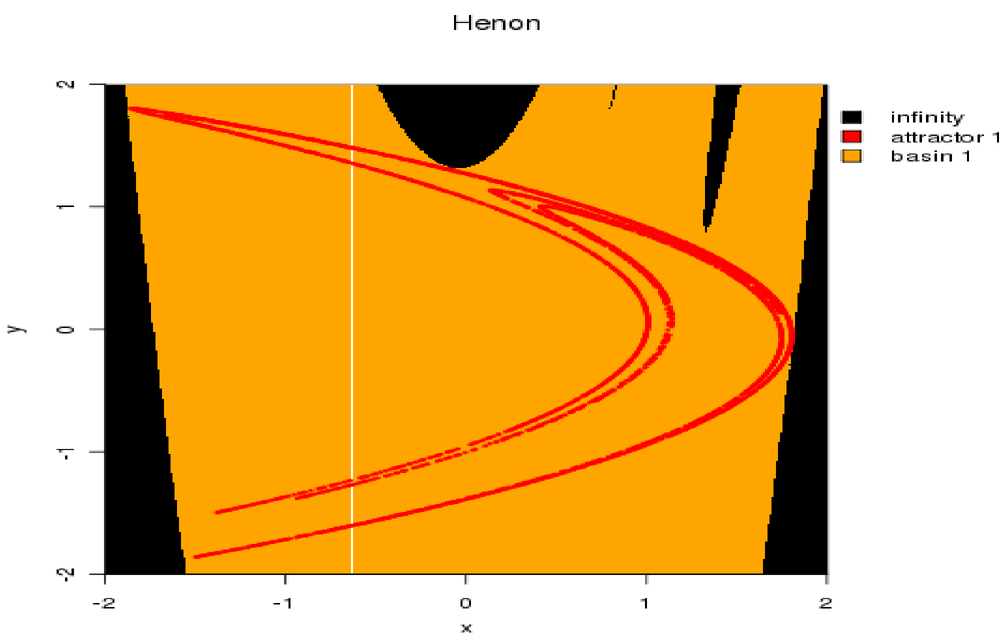
Attractors

```
> par <- c(a = 1.4, b = 0.3)
> var <- list(c(x = -1, y = -1), c(x = 1, y = 1))
> trL <- TrajectoryList(m, n=20, par, var, time=50)
> plot(trL)
```



Basins of attraction

```
> bs <- Basin(m, par, xlim, ylim, transient,
iterations)
```



Sensitive Dependence on Initial Conditions

Lyapunov exponents

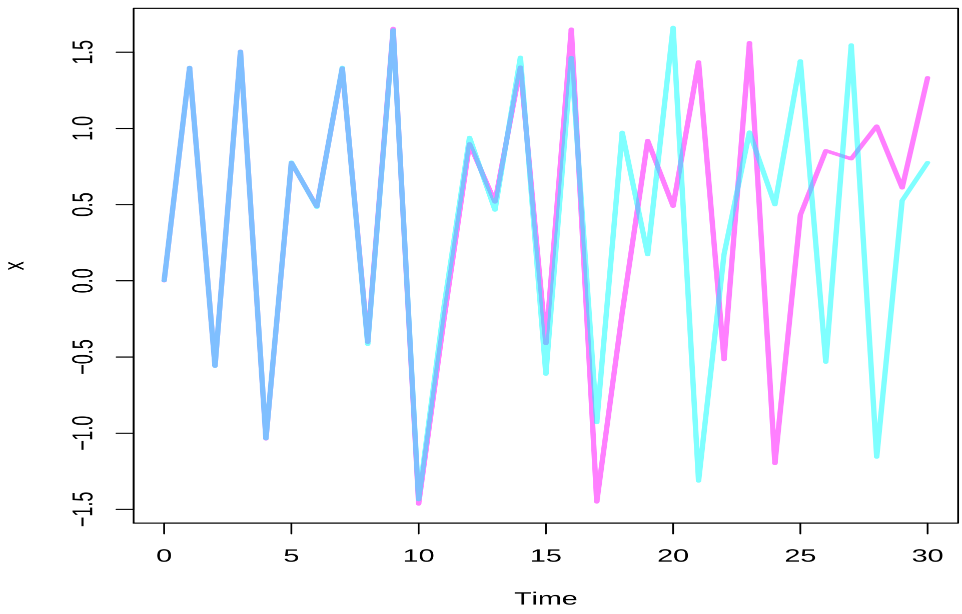
- One of the more interesting possibilities of nonlinear systems is sensitive dependence on initial conditions
- The Lyapunov Exponent (LE) measures the average rate of divergence in time of two nearby trajectories:

$$|\delta \mathbf{x}_t| \simeq e^{\lambda t} |\delta \mathbf{x}_0|$$

- Positive values of λ indicate SDIC and suggest chaotic attractors
- Computing the value of λ can be very hard to do analytically, but numerical approximations can be obtained with `RiDMC`

Sensitive Dependence on Initial Conditions

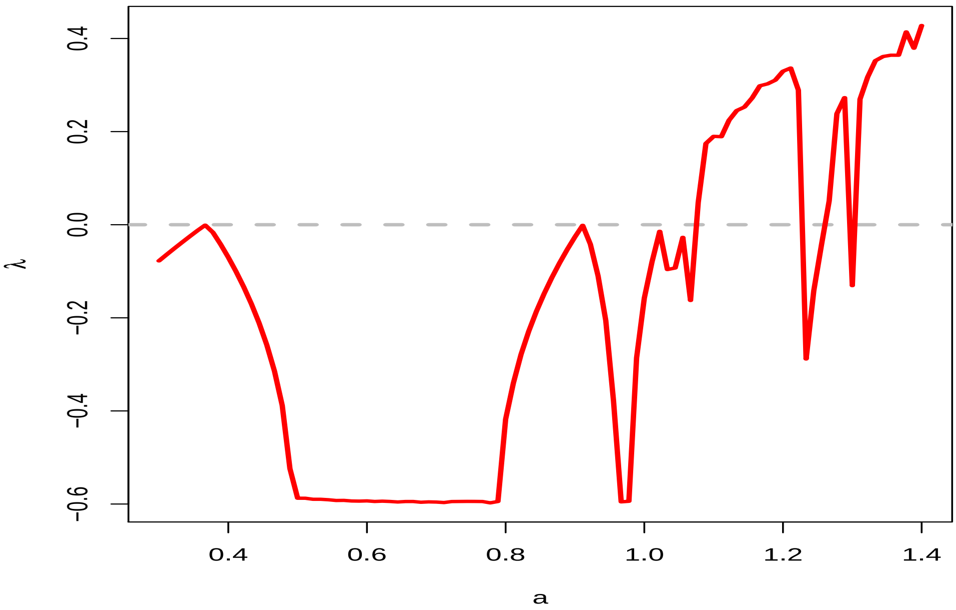
```
> par <- c(a = 1.4, b = 0.3)
> x0 <- c(x = 0, y = 0)
> var <- list(x0, x0 + 0.001)
> trL <- TrajectoryList(m, n = 2, par, var, time = 30)
```



Lyapunov exponents (II)

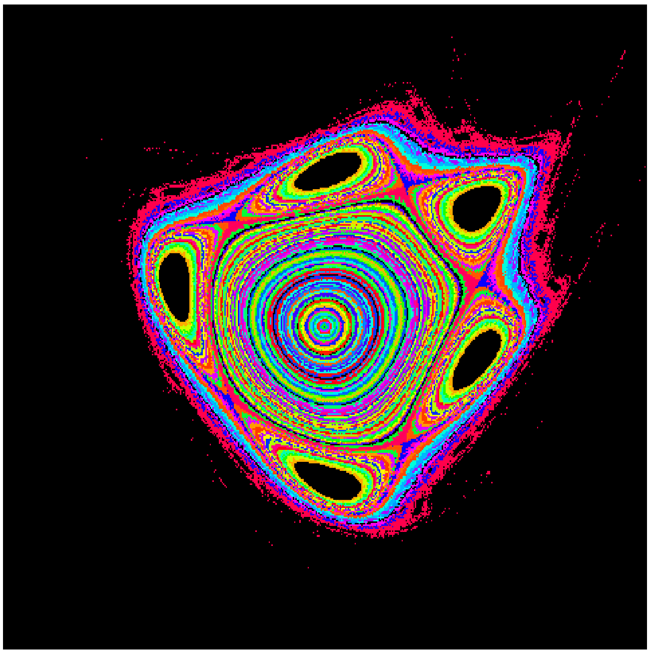
```
> ly <- LyapunovExponents(m, par, var, time, par.min,
     par.max)
> ly
=iDMC Lyapunov exponents diagram=
Model:  Henon
Starting point: x = 0.5, y = 1
Parameter values: a = 1.4, b = 0.3
Varying par.:  a
Varying par. range: [ 0.3, 1.4 ]
MLE range: [ -0.5975,  0.4279 ]
```

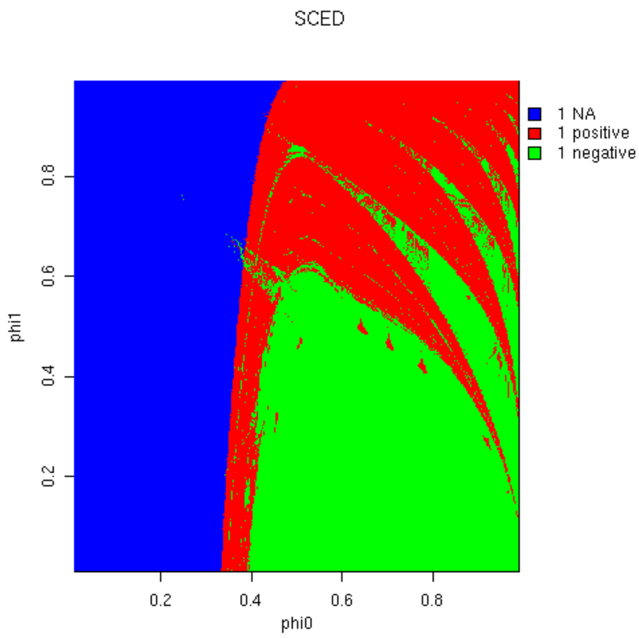
Lyapunov exponents (III)



Note

RiDMC isn't just for toy models...





Current status

- The idmclib C API is quite stable. Currently working on documentation and distribution system
- RiDMC core computing functions are stable too
- The plotting functions (grid-based) may change in the future
- Extract raw data and write your custom plotting functions if you want forward-compatibility of your code!

Perspectives

- fix bugs
- stabilize plotting functions
- add more analysis routines

The end.