ArDec: Autoregressive-based time series decomposition in R

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Time series decomposition

Approaches:

- **non-parametric**: filtering / smoothing (eg STL, discrete wavelet transform, ...)
- **model-based**: regression, structural models, ...

Goals:

- remove "known" (non-stationary) components
- describe components of interest (seasonal, trend, ...)

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Trend & seasonality

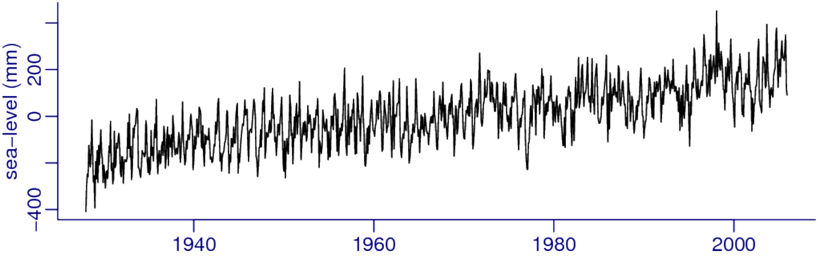
- *“The essential idea of trend is that it is smooth.”*
- *“A trend is a consistent pattern over time.*
- *“A trend is a long-term movement in time series data after other components have been accounted for. “*
- *“A trend is a trend, is a trend, is a trend, ...”*

Trend & seasonality

- *"the characteristics of a time series giving rise to spectral peaks at seasonal frequencies"* [Nerlove 1964]
- *"the intra-year pattern of variation which is repeated constantly or in an evolving fashion from year to year"* [Shiskin et al. 1967]
- *"periodic fluctuations that recur with about the same intensity each year"* [Hillmer and Tiao 1982].

Problem

How to retrieve physically-relevant components?



Problem

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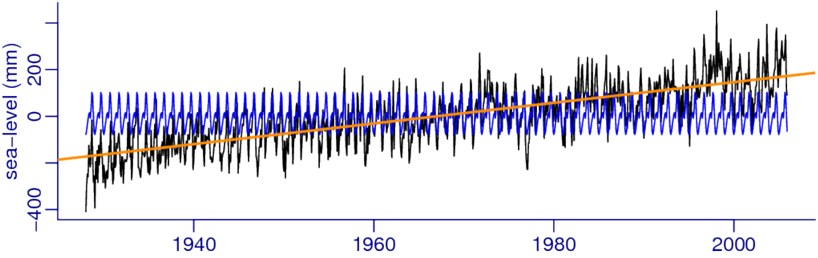


Method

M. West 1997

(Time series decomposition. *Biometrika* 84)

Basic concept:

$$X_t = \sum_{j=1}^{p} \phi_j X_{t-j} + \varepsilon_t \Longrightarrow X_t = \sum_{j=1}^{p} \gamma_t^j$$

State-space representation of AR(p) process

$$X_t = F^T Z_t$$
$$Z_t = G Z_{t-1} + \varepsilon_t$$

with

$$F^T = [1\ 0\ \ldots\ 0]$$

$$Z_t^T = [X_t\ X_{t-1}\ \ldots\ X_{t-p+1}]$$

$$G = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_p \\ 1 & 0 & \ldots & 0 \\ \vdots & 1 & \ddots & \vdots \\ 0 & & & \end{bmatrix}$$

State-space representation of AR(p) process

$$X_t = F^T Z_t$$
$$Z_t = GZ_{t-1} + \varepsilon_t$$

$$G = EAE^{-1} \longrightarrow r_j e^{\pm i w_j}$$

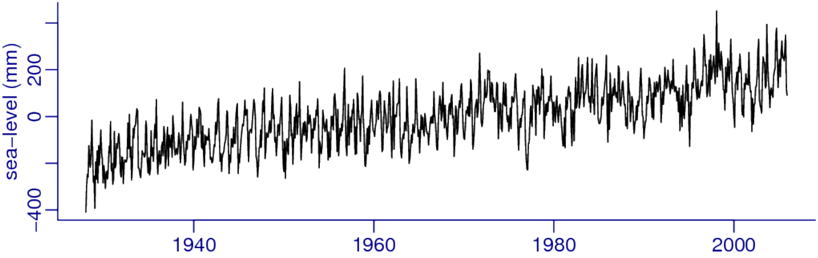
$$a = E^T F \ , \ b_t = E^{-1} Z_t$$

$$X_t = \sum_{j=1}^{p} \gamma_t^j \ , \ \gamma_t^j = a^j b_t^j$$

$$w_j = 0 \longrightarrow \gamma_t^j = r_j \gamma_{t-j} + \nu_t$$

$$w_j \neq 0 \longrightarrow \gamma_t^j = 2 r_j cos(w_j) \gamma_{t-1}^j + r_j^2 \gamma_{t-2}^j + \eta_t$$

Decomposition of sea-level records with ArDec



Decomposition of sea-level records with ArDec

```
> library(ArDec)
> coef=ardec.lm(dat)$coefficients

> coef
```

```
X1          X2          X3          X4          X5          X6
0.386186446 0.050007536 0.088643459 0.002730004 0.045915250 -0.009539645
X7          X8          X9         X10         X11         X12
0.058137588 0.032015897 -0.075378709 0.064847440 0.117705959 0.169322783
X13         X14         X15         X16         X17         X18
0.060288889 -0.077621640 -0.074880590 -0.012911223 0.010869043 -0.009742732
X19         X20         X21         X22         X23         X24
-0.048499636 -0.002643505 -0.044422722 0.054698372 0.052529147 0.139849769
X25         X26
0.072803836 -0.081687104
```

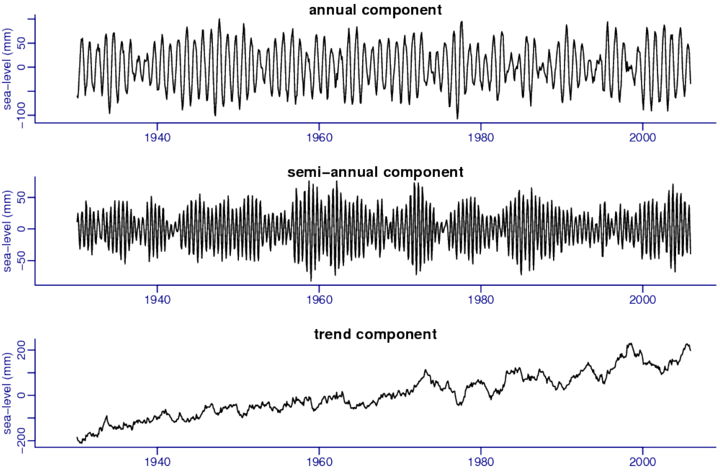
Decomposition of sea-level records with ArDec

```
> ardec(dat,coef)

period      damping
1 "trend"   "0.996"
2 "12.089"  "0.986"
3 "6.000"   "0.982"
```

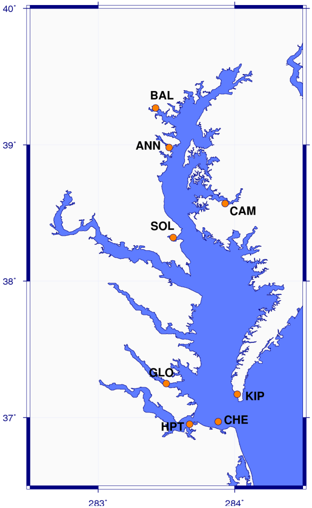
Decomposition of sea-level records with ArDec

```
> str(ardec.components(ardec.out))

List of 2
$ periodcomps:List of 2
..$ periods: num [1:2] 12.1  6.0
..$ comps  : mts [1:936, 1:2] NA NA NA NA NA NA NA NA NA ...
.. ..- attr(*, "dimnames")=List of 2
.. .. ..$ : NULL
.. .. ..$ : chr [1:2] "Series 1" "Series 2"
.. ..- attr(*, "tsp")= num [1:3] 1928 2006   12
.. ..- attr(*, "class")= chr [1:2] "mts" "ts"
$ trendcomp  : Time-Series [1:936] from 1928 to 2006: NA NA ...
```

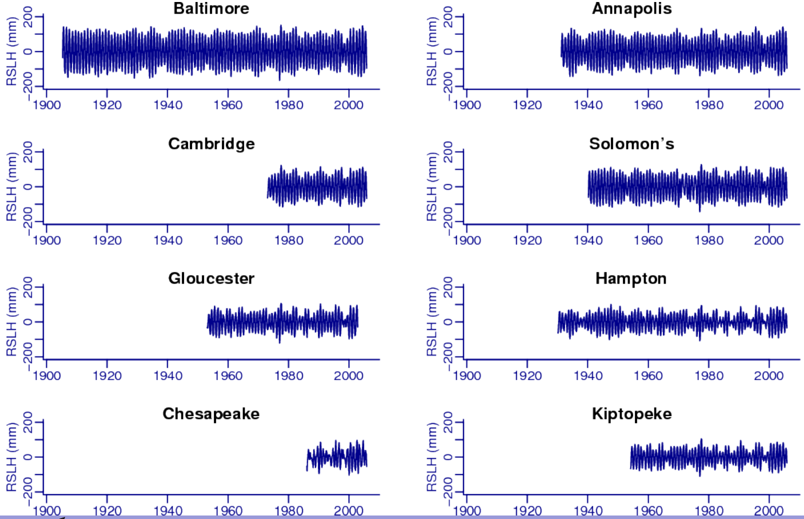
Decomposition of sea-level records with ArDec



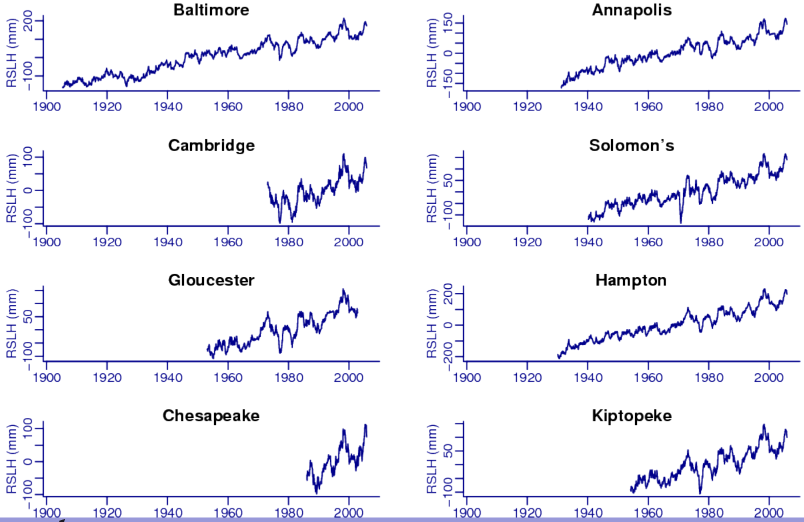
Chesapeake bay



Chesapeake bay: annual components



Chesapeake bay: trend components



Package ArDec

- implements autoregressive-based time series decomposition
- model-based, additive decomposition
- yields periods of physically-relevant (non-damped) components
- extracts flexible, time-varying estimates of such components
- option of Bayesian framework for autoregressive estimation

Thanks!