Determining Extreme Returns on Stock Indices

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Abstract

Consider a time series of daily returns on a stock index. How can we tell if the return on an arbitrary day is an unexpectedly high gain (or loss)? There is a number of reasons why the answer to this question is important. For example, it serves as the basis for an investigation if investors overreact or underreact in the short run — for example, if, in the given series, a huge gain (loss) is usually followed by a significant loss (gain), the investors may be suspected to overreact, which might even lead to a point where the market is no longer efficient.

It is not meaningful to regard exceedances of a quantile of the entire return series (say, those days with returns above the 95% quantile or below the 5% quantile) as unexpectedly high gains or losses. Obviously, there are good reasons to formulate investors' expectations about tomorrow's returns in terms of a time series model fitted to the series up to today, without using future data. This necessitates the estimation of a sequence of time series models, whose parameters are allowed to evolve in order to reflect changing investor experience and expectation.

If parameter estimation is entirely carried out in the statistical computing language R, we will be faced with the problem of long computing time. This is mainly due to the evaluation of the likelihood function, which has to be computed recursively. Therefore, we propose to call a C routine to compute the likelihood and to do everything else in R. Results are presented for several models, including the usual GARCH, a threshold GARCH, and an exponential GARCH.

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