Calculating the autocovariances of fractional ARIMA model

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Abstract

Consider a stationary and invertible process $\{Y_t\}$ following a (long memory) fractional ARIMA(p,d,q) model,

$$\phi(B)(1-B)^d Y_t \;=\; \theta(B)\varepsilon_t,$$

where $\{\varepsilon_t\}$ is a white noise process with zero mean and finite variance σ_ε^2. The AR and MA operators are respectively $\phi(B) = 1 - \sum_{j=1}^{p} \phi_j B^j$ and $\theta(B) = 1 - \sum_{j=1}^{q} \theta_j B^j$. Given a sample of T observations, for ML and REML estimation [e.g., Cheang and Reinsel (2003)] to be implemented efficiently for large T, we need efficient computation of the autocovariances $\gamma(l) = \mathrm{Cov}(Y_t, Y_{t-l})$. Sowell (1992) derived from the spectral density a formula for computing $\gamma(l)$ when the AR polynomial $\phi(B)$ has distinct zeros. Recently, Doornik and Ooms (2003) implemented some refinements to this formula for numerically stable evaluation of the autocovariances. In this note we provide an alternate derivation of $\gamma(l)$, leading to recursive relations that allow $\gamma(l)$ to be evaluated accurately and efficiently. We will see how these autocovariances can be programmed in R for ARIMA(p,d,q) when $p = 0, 1$, and the problems encountered with evaluation of the hypergeometric function required when $p > 1$.

References

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